EXPERIMENTAL RESEARCHES
INTO THE PROPERTIES AND MOTIONS OF
FLUIDS.

WITH THEORETICAL DEDUCTIONS THEREFROM.

BY

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PREFACE.

ALTHOUGH I feel personal prefaces are objectionable, I think it due to those who may follow me in this treatise to point out certain limits of my knowledge of the subject treated. I will therefore briefly state the circumstances under which it was written. Earlier than the past five years I had no intention of specially studying the subject, and had certainly no idea of writing upon it. I had previously to this period taken for the amusement of my leisure an experimental examination of the undulatory theory of light, which I could not satisfactorily comprehend. In following up my experiments for two years I found my eyesight impaired, and was advised that it would be necessary to leave these experiments, and also close application to reading, for some years, which I did very reluctantly. One branch of experiment, somewhat relative to my former studies, however, appeared open to me. The theory of undulation of light was generally introduced to our conception by philosophers by similitudes of the motions of water-waves and sound-waves; I thought I would investigate experimentally, as far as possible to me, to be assured our conceptions of these motions were real, upon inductive principles, similar to those I had been employing for investigation of light. In this subject, taking no preconceived theory whatever for my experiments, I soon became absorbed in observations of the motive effects evident in the directions taken by impressed forces in fluids under various conditions of resistance; wherein it appeared to me quite evident that there was yet an immense amount of work to be done in researches in the motions of fluids, before theoretical principles of the sciences of hydrodynamics and acoustics could be fixed upon mechanical principles with any great precision. It was therefore clear to me that in this direction I might, if I had the ability, enter upon fields of research quite as new as in my former studies.

At the commencement of my work, looking as briefly as I could into the works of others in search of purely mechanical ideas, it appeared to me that in science with respect to fluids, as in the earlier periods of some sciences which have become in our time exact, as those of astronomy and chemistry, too great reliance had been placed upon some ingenious theoretical idea which was not clearly and sufficiently supported by direct observations or by experiment, therefore necessarily only brought to bear on facts by the introduction of somewhat arbitrary functions; whereas too little dependence had been placed upon *purely* inductive methods. Thus, to take an instance in the philosophy of wave motions, it was in the first place certain that wind naturally produces waves upon water, as we find evident upon the surface of the ocean. Then from such knowledge it would appear to be most logical for researches into the principles of oceanic waves, to investigate the action of the wind upon a liquid surface; for in this case, if we obtained a clear perception of the principles of this action, by finding causes sufficient to *produce waves,* it would then be quite clear that the same mode of action would *maintain the waves when produced;* and that with this knowledge, if we could attain it, in so far as the production and maintenance of waves were concerned, our ideas would be fairly *complete.* We might then, as a secondary consideration to this, investigate the constant action of gravitation in bringing the surface to equilibrium, as we know that when waves are produced, and the wind ceases, that for a short time, the waves will continue active, and this after action might be *entirely* by pendular oscillations or otherwise, which could not have been so logically considered at first, being only, as it were *secondary effects.* So that if we reverse this order of research, as is general in wave philosophy, and commence our investigations by following the principles of oscillations of fluids through gravitation only, even assuming our ideas actual and not theoretical, this would clearly as a starting-point, be begging the entire question; for in this we have to suppose the waves as *already existent* at the time of our researches; so that the causes and effects of their production, which are also evidently *those of their maintenance,* the important parts of our research, are left quite out of the question. I mention this only as an instance, and I do so with the full conviction that oscillation enters as a principle into wave motions at all times, although most evidently in the dissipation of impressed forces to equilibrium upon the open liquid surface.

For the work before us, wherein I hope some of my ideas may be accepted, these I anticipate of very different values. Thus, the first three chapters are speculative, and even in parts hypothetical; they are generally attempts to apply mechanical principles to hydrostatics, and needed on some points much more leisure than I could command. The first chapter I felt necessary, to offer some theory of the fluid condition that appeared to me consistent with our acquired knowledge of matter and with my experiments, particularly to account for the rigidity and yet mobility of liquid systems. In the second chapter on liquid surfaces I have ventured to differ from the generally accepted theory of *tensile* surface for liquids, founded on the researches of many great philosophers, from Segner (1751) to Clerk Maxwell. My experiments have led me to take quite the opposite view, namely, that such surfaces are *extensile* instead of tensile; except for free films, which are clearly *tensile* by the position of the attractive matter which composes them. This subject I think I may have insufficiently worked out, although my experiments in this, as in most other subjects taken, are ten times the number given. In the third chapter some propositions are offered which will, I am sure, need partial correction. In the fourth chapter and onwards I think my work is more important, and this may be taken quite separately from earlier more speculative parts. The theory I develope of rolling contact of fluids moving upon static bodies will possibly be established by any amount of further research. With this, I believe quite original work, I have taken as great care as I was capable of. In the fifth and sixth chapters I offer principles of conic resistance in fluids which give simple mechanical laws for the class of motions sometimes defined as vortices, eddies, and cyclones. These mechanical laws may be ultimately shown to be general, if not the universal principles, upon which all fluids move by displacement upon themselves; which will open out new theories of fluid motion. The eighth chapter, on resistance of solids, is very incomplete on certain points for want of sufficient research into the works of others and more experiment; therefore it may be considered to be in a certain degree speculative. The ninth chapter, on diffusion of fluid forces, I believe to be important, although it may need more experimental demonstrations than I have given.

In the Second Section, devoted to the discussion of cosmical phenomena in fluids, I have in some parts of this work followed certain ideas proposed by Lenz, Herschel and Dr. Carpenter. By further introducing principles of motions in fluids that I have discovered, I am able to propose certain universal systems of motion for fluids upon the globe; consistent with the distribution of land and water as it exists, and at the same time such as will constantly influence this distribution. I have some hope that these researches may aid in the elevation of the sciences of physical geography and meteorology, now sciences of *observation*, to sciences of *principle*, to rank at some future period with such exact sciences as astronomy and chemistry, and that the principles proposed may be of some value also to systematic geology. I have no doubt, however, that my propositions will need much future correction to adapt them to local circumstances, and altogether, I consider my efforts only the commencement of systematic work in this field of research.

For the Third Section, on Waves, after making many experiments, I found my experimental inferences agree most nearly with ideas first published by M. Flaugergues in the *Journal des Sçavans*, 1789. His principal experiment is, however, very rough, namely, the striking of a liquid surface with a copper rod to generate waves. But his separation of the functions of protuberances and hollows appears to me most rational to observation. Following these ideas, and endeavouring to refine the like experiments, I fell into a track which had already been traversed without my knowledge by the important experiments of Mr. J. Scott Russell, of which I had found no reference in any work on hydrodynamics or physics that previously came to my hands. But as I was working for the demonstration of principles only, I do not regret the want of this knowledge at first, for if I had possessed it at the time it would have materially curtailed the interest I took in my experiments, by which alone I have attempted to study principles throughout this work, and my opinions of wave phenomena, whatever they may be worth, would have been less original. I have, however, replaced

Mr. Russell's work for my own where it appeared more demonstrative.

I have written a Fourth Section that is really necessary to complete my work—*upon sound motions in fluids;* which is withheld from publication, references to which are now unavoidably mixed with this matter. This part was written in continuation of wave motions. I think it may perhaps be an important part, having devoted much time to it, but I commenced it first, and completed it before the important inventions of Telephones by Prof. A. Bell, and its congeners the Phonographs and Microphones were discovered, which instruments I have not had much time to consider, but think they will under experiment materially assist me in demonstration of the principles introduced. I therefore defer the matter of sound motions in fluids for a year or two to repeat my experiments. Some of the propositions of the first chapter were specially written in reference to sound motions, as in the theory adopted I thought it necessary to assume a condition of *quiescent equilibrium* for fluid molecules to demonstrate the action of compound sounds. This would possibly not have been necessary for the proposition here given of the ordinary motions of fluids. Further, in omitting the fourth section, I was compelled to withhold a proposition relative to conditions of *static* in comparison with *motive* equilibrium of molecular motions in fluids, in the first chapter, which would have been incomprehensible without further development in the fourth section on sound; I feel this withdrawal leaves the first chapter, now condensed from what was formerly two chapters, in a certain degree abrupt and incomplete.

My work is thrown into *propositions*, which is perhaps a rather pompous style for such a work, but I mean by these only some things *proposed.* I adopted this system as it appeared to give me liberty to offer speculative ideas freely, and at the same time to keep them in more concrete form, which was necessary for such intermittent work as I was able to carry on when leisure permitted. I intended at first to use these propositions, which were purely, *notes made in presence of my experiments*, only as a scaffolding to erect my ideas, intending to remove this entirely after I had completed my book, supplying its place by some more modern form of writing, but I found this would be more difficult and tedious than anticipated, so that I now leave it in its rough state, and must therefore depend for my readers on the smaller number who look at *things* rather than *forms.* Further, I have not for the same reasons attempted quantitative or numerical deductions, except in a few cases, as I am sure this may be done better when my ideas are proved or disproved, as it would also be better done by others who possess superior educational advantages over myself. Altogether I propose my work as a primitive sketch of the subject from which I anticipate, perhaps vainly, that more perfect work may be reared at a future time by some highly educated mathematician who may care to follow me, and clothe with his skill the rude, although I believe natural, underlying forms that I have brought in some cases to light.

I hoped, when my work was complete in its present form, to have obtained the assistance of some well read, or well instructed student to edit it for me, but after a great amount of correspondence, failing in this, I endeavoured to get my proofs read from press by some eminent men to whom I thought the originality of my researches might be of sufficient interest. In this matter I partially succeeded, and am greatly indebted for valuable occasional critical notes to the kindness of Prof. F. Fuller, who read my first proofs of chapters I. to VII. inclusive, also chapter X. I am also indebted to Prof. Stokes, who only gave me a very conditional promise of help at commencement, for valuable critical notes on parts of chapters I. and III. If I could have obtained a little more assistance of this kind, there is no doubt my work would have been of much greater interest, and I could have avoided many faults. I am also indebted to Prof. Hay for some logical and grammatical notes, and for assistance with the index.

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EXPERIMENTAL RESEARCHES
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PROPERTIES AND MOTIONS OF FLUIDS,
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SECTION I.

THEORETICAL CONDITIONS OF THE FLUID STATE AND MOTIVE PROPERTIES OF FORCES IN FLUIDS.

CHAPTER I.

PROPERTIES OF THE FLUID STATE OF MATTER.

1. Introduction.—*a.* Sir John Herschel tells us that "If there be one part of dynamic science more abstruse and unapproachable than another, it is the doctrine of propagation of motion in fluids, and especially in elastic fluids like the air, even where the amount and application of the original acting forces are known and calculable." This was written at a time when contemporary science regarded fluid as inert matter which moved only as it was moved by exterior forces, so that the directions and values of these forces only were to be considered as active upon *mobile matter in perfect equilibrium.* By our modern dynamic theories of fluids we assume that the molecule of a fluid is *active*, and is never at rest in a state of equilibrium. Therefore the difficulty of research in motive forces in fluids under the impression of exterior forces becomes almost infinitely greater, as we have now not only to consider movements by the impression of forces upon the molecule in a position of equilibrium; but we have to consider its movements in composition with other motions that exist already within its highly motive matter; and although there may be a general equation of the sum of these movements, any system of motion assumed to progress from molecule to molecule only will be irregular both in its velocity and in its course.

These difficulties appear to me to be so great that I cannot pretend to any powers to meet them. I therefore feel bound to follow the track of all writers upon acoustics, hydrodynamics, and chemistry in considering the atoms or molecules of fluid matter to be by some cause in a state of sensitive equilibrium, and motive only so far as experiment alone indicates; at the same time it must impress every thoughtful man who follows the evidences of his times that there exists a perfect continuity of equivalence for the conservation of all forces, although these admit only of observation under special motive forms. This I believe was first ably pointed out by Sir W. Grove in his *Correlation of Physical Forces;* but it is further developed almost to a certainty by the careful researches of Dr. Joule for possibly the most difficult case of correlation, namely, that between mechanical forces and the action of heat. It nevertheless does not appear in this last case, any more than in others, equally certain that such forms of impression of force as we may produce mechanically shall be reproduced by like forms as heat; for in this, mechanical forces themselves may be converted into other forms, as, for instance, the direct impulse of the clapper of a bell produces in the bell vibration, and in the air sound motion; further, we have no possible evidence of any molecular motion whatever, so that we may have by equivalence of energy a dynamic unit represented by any force, as by a pressure, a vibration, a free trajectory, or otherwise.

b. The diffusion and pressure of gases by assumed locomotive or trajectory forces in free molecules, founded on theories of Clausius and Clark Maxwell, present to the mind a very cogent theory for the action of heat in these bodies. But the existence or presence of such free motions appears to my mind to entail immense difficulties for the relative consideration of other forces, as, for instance, in our most exact science of chemistry, where the motions of atoms become, if anywhere, most evident, and the atoms of matter in combination and dissociation appear, as far as outward evidence is concerned, either to approach or to recede *once* only; the same observation will apply to forces of cohesion and of diffusion. It appears also that a state of equilibrium for every particle is required by our theories of the motion of sound in air, where the molecular equilibrium is generally assumed to be perfect and highly sensitive, as in Lord Rayleigh's important work; or for such experimental cases as those considered by M. Plateau in his eminent work, *Statique expérimentale et théorétique des Liquids soumis aux seules forces molé-*

culaires, for capillary and other conditions of assumed equilibrium. There is no doubt that under the condition of perfect equilibrium, a particle would be motive by the constant active presence of exterior forces; but as these forces would be exterior only, they would be subject to general laws of impression only.

c. Upon the above conditions it appears to me that some of the difficulties of conception of a possible sensitive equilibrium of the molecule, as it appears experimentally to exist in fluid matter, may be met in that the same value of the dynamic unit of forces may be found either in *any* form of motion or in the static conservation of its force. This last has been compared to the conservation of energy in bent springs, which has been applied to cases wherein active forces appear to be lost, as where heat forces become latent. Such cases, it appears to me, may be realized by assuming certain physical conditions in the *construction* (if I may so term it) of the atom or smallest part of matter. For this I shall offer some ideas, partly only original, to support the conditions of energy and of equilibrium of the particle, as it is established by the doctrine of position relative to attractive forces, or of *potential energy,* wherein a particle raised or pressed to a static position by a force that acts inversely to, and exceeds its attraction to other matter, may cause the particle to rest in or about this new *static* position, from which it may react at another time with force equal to that employed in its original elevation or pressure. This matter I find most clearly and ably discussed by Dr. Balfour Stewart in his work on *Elementary Physics,* §§ 100, 240, &c.; however, I shall confine these speculative matters entirely to the first chapter, as I take my hypothetical ideas of molecular forces in fluids to be quite secondary to the purposes of this work; I will also endeavour to treat this part of my subject in all cases as briefly as possible.

2. Fluidity of Bodies.—*a.* Perhaps the most simple definition of *fluidity* would be that derived from the Latin, for which we may take *fluens* or *fluidus,* which words will particularly indicate the material property in a body of *flowing.* Taking this definition, and conceiving a fluid body to be alike in all its parts, and that these parts are coherent to each other, which appears physically to be the condition of all conceivable matter, we may then clearly define a fluid as *homogeneous matter that can flow in continuous mass.*

b. If in this manner we take the fluidity of a body to be its physical property of flowing simply as here proposed, which will be in every way convenient for this treatise, then this property of fluidity will be a *relative quality*, dependent upon the facility with which the constitution of the body permits it to *flow* from any cause, that body being considered to be the *most fluid* that can flow under the impression of the *smallest amount of applied force*. Under this condition, if we make no special definition of the mode in which the force is to be applied, we may include in our considerations all forces possible to affect the flowing or fluidity of the body. Therefore these forces may be interior or intermolecular, or be applied exteriorly, as, for instance, the force that produces the fluidity of a body may be generated by intermolecular repulsion by heat, or by electricity, if such forces cause molecular repulsion, as they appear to do, or by molecular vibration, as supposed by some, or percussion and recoil of free trajectories, if it is possible that molecules can possess such forces in continuity. The fluidity may otherwise appear as an effect of exterior forces, as by the action of gravitation pulling the molecules in mass systems to a level surface, thereby inducing equal gravitation; or by pressures, as in the coining of metals, in which case otherwise rigid solid bodies *flow* into the interstices of a mould. Under the impression of *extreme* external pressures every form of matter might be demonstrated to possess some small functions of fluidity or of flowing, the quality here defined.

c. Taking the above conception of fluidity, it becomes evident that when we endeavour to define the exact limits of flowing quality which we may think to be sufficient to qualify a body as a *fluid*, there will appear at once to be a difficulty in defining the exact amount of the *flowing* quality in which this definite form of matter should be considered to commence. This we may conceive, in that it would not be an extremely difficult task taking all known bodies, simple and compound, to arrange them in a long consecutive series upon the principles here given of the flowing quality alone, that the series might extend proportionally to the fluidity each material possessed from the most attenuated gas to the densest solid at some temperature taken. To follow the conditions of this idea we might take, for argument sake, the fluidity of bodies that we might imagine would rest at the extreme ends of such a relative series; say, for instance, a Sprengel pump vacuum off hydrogen for the most fluid end, and steel for the opposite most solid body in which we could trace elements of fluidity. The part of such a series where a body could *just flow* to a level surface under the action of gravitation upon its parts might be possibly near the centre; we will assume that in this centre the mean proportional body may be represented by treacle. Then, in this case, for the definition of a fluid we might conceive of the whole series we have arranged, that on one side of this central line all bodies would be solids, and on the other all fluids. In this case we should certainly find that when we came more particularly to examine the division we had made, that on either side of our selected line we should have bodies that differ very little from the one we have taken, say, for instance, in this case that we have next the treacle, towards the solid end of our series, a mass of the kind of resinous turpentine that exudes from the spruce fir, and on the fluid side some mountain honey. By taking these instances it is very clear the line we have chosen is in no way marked for its special fluid qualities from other matter. If we had taken a point much higher in our series, say copper, which will *flow* readily in all the interstices of a die, under the pressure of a coining press, as a perfect liquid would by gravitation simply, in this case we might have in our series, towards the solid end, say brass, which would *flow* less easily, and on the fluid side, say pewter, that would flow more easily into the die under pressure. It will be easily conceived that the same proportional quality of fluidity might occur in any other part of our series that we might select for the division of bodies, fluid from solid.

d. For another instance of the fluid properties of bodies we may take the action of the separating force heat, evidently the most general cause of fluidity. This we find expands most bodies in approximately equal ratio to the temperature over a certain wide range, the fluidity increasing in some cases proportionally to the expansion. Take for instance glass. This when cold forms a dense homogeneous mass, but as we apply heat after a certain point it passes gradually through all the degrees of visible fluidity until it appears as a limpid liquid, without any definite point of change from the solid to the fluid state.

e. The value of these considerations is, that fluidity may be taken to be a very general physical property of matter in *some degree*, and this opens out to us a road to experiment in some instances, wherein we may draw inferences from the properties of motion in homogeneous solids, where such motions can be followed that may be applied in principle to fluids, bearing always in mind that it is the relative fluid property only of these bodies that we follow. This principle will be taken advantage of in this treatise for a few experiments.

f. If we return to our imaginary proportional series of bodies extending from the most mobile fluid to a dense solid, perhaps the most distinct line that could anywhere be drawn would be between liquids and gases, as the gas appears to assume very generally a new force, that of power of *unlimited attenuation.* But in this the important experiments of Andrews and other experimenters point out to us that the distinction is more apparent than real, and that physically under experiment liquids and gases have more properties in common than the most easily defined liquids and solids under all reservations.

g. It becomes most reasonable to consider upon the above principles, as is generally assumed to be the case, that the perfect state of fluidity does not exist in any material body; all fluids being found to be in different degrees cohesive, viscid, plastic, and elastic; possessing internal construction of parts, or chemical arrangements which do not admit of a perfect mobility of the separate parts which would be necessary for the condition of perfect fluidity. This must naturally be the case, as it does not appear easy to conceive the possibility of the existence of a mass united by its own cohesive forces, by which alone it is known to us, of any unit body whatever, fluid or other, without some such constructive elements as those indicated above to ensure its unity as a system of matter, and having this unity forming a cohesive system held by any imaginary forces; perfect mobility or fluidity would appear to be impossible, as the mobility of any part must necessarily break asunder the cohesive forces which produce this unity in the mass.

h. The above comments are not intended to raise any difficulties in the definitions of fluidity; for the purposes of this treatise many bodies would sufficiently represent the properties of fluidity for the physical experiments that alone will be followed, except for some comparative demonstrations in difficult cases, where help may be obtained from the same motive principles, being possibly induced in other homogeneous bodies or solids. For the general work the definite fluids water and air alone will be taken, for the construction of which kind of fluids I propose and adopt the following theoretical ideas:—

Apparent Physical Homogeneity of Fluid Matter.

3. PROPOSITION: *That a fluid is to impressed forces physically a homogeneous system of matter. That it is composed of separate units, whose forces, attractive, repulsive, or otherwise motive, are equal in each unit, and that in this equality consists the apparent homogeneity.*

a. By assuming perfect equality of dimensions in particles of a fluid, and of each particle possessing equal forces in any system composed of many such particles, the separate particle being free from excess of attraction, so that it may rest in a state of sensitive *unstable* equilibrium with all external forces; then a pressure upon one side of the particle cannot exceed that of the sum of the pressures upon the opposite side without the particle moving towards the minus pressure. A mass composed of such sensitive particles would be a fluid. In the above case it is not necessary to assume the particle as the smallest quantity of matter. It may be such a quantity or a larger quantity, provided the state of equilibrium is perfect.

b. It does not appear that the same sensitiveness of equilibrium will apply to the position of any particle in contact with a solid or another fluid, as the surface of the other matter may possess a system of attractive or repulsive forces that does not *conspire to equilibrium* at contact with the first, so that the equilibrium at the meeting of distinct systems will be less easily defined, as I will hereafter show, than the above.

c. By the above conditions we may assume that a fluid system by the equality of forces about the particles will produce a homogeneous whole, so that any force that has sufficient power to break or disturb the physical construction of one portion of the fluid mass will also have equal power over any other portion. Therefore friction of resistance to a moving body in the fluid will be simply in the ratios to functions of cohesion of the nearest particles of the mass of the fluid to the moving body, and not as a function of the weights of matter superimposed, or pressing about or above the moving body that is resisted. For in the construction of a fluid, if we were to imagine it as a simple aggregation or admixture of equal detached separate particles of loose free mobile matter, however small, not perfectly arranged upon a plan capable of producing perfectly homogeneous equilibrium, or of equal *smoothness* in the mass, it would be evident that in direct ratio the particles might be jambed together or held down by the weight of others above; each particle being a separate identity, they would interlock, and the resistance to impression of force within the fluid would increase. This would be unquestionably the case with separate particles of every form of solid matter under the action of gravitation or of pressure, whether whole or in division. For instance, powdered chalk may be compressed to a very resistant solid; or if we were to take a pile of smooth papers the friction of moving laterally one of the lower sheets would be nearly as the weights of the sheets above it. It is not difficult to be assured that entirely different conditions to the above hold with every possible fluid.

d. It was possibly under the influence of a theory prevalent in the seventeenth century—that a fluid was a mass of detached particles, representable by a mass of fine dust, motive or quiescent, without connective functions—that Newton investigated the properties of a fluid as infinitely fine matter endowed with repulsive forces to assure its equilibrium, the particles of which could repel each other with forces that were inversely as their diameters. In this theory resistances in a single fluid were shown to be as the general specific density of the fluid simply. The experiments offered to prove this are to be found in the scholium at the end of the seventh section, second book, of the *Principia*. For our present purpose these experiments show the perfect homogeneity and internal equilibrium in this case of liquid matter, as we find no palpable increase of friction shown by resistance, by increase of pressure, or by the weight of liquid above a body moving in it. In the experiments in the above scholium to which I wish to draw attention, Newton loaded small balls of wax with lead until they descended very slowly in water. In this way he found that gravitation acted in equal accelerative ratio in *deep* water as in *shallow*, or that, deducting the resistance of the inertia of the fluid in contact as a constant in the ratio of its specific density, these bodies fell as in vacuo.

e. In the experiments of Coulomb upon the viscosity of fluids the perfection of the homogeneity and internal equilibrium of liquids is equally assured by the equality of resistance under the adhesion of the liquid to a solid at great differences of pressure from superimposed weights. Coulomb caused a circular disk of tin plate of about five inches diameter to oscillate by reciprocal rotation by the torsion of a fine wire at various depths below the surface of water, carefully measuring the amplitudes and times of the separate oscillations. With this apparatus, which is said to have been admirably constructed, he could discover no difference of resistance, after proper correction for the surface of the wire was made, whether the disk oscillated near the surface of the water or at a depth of five feet, although the difference of superimposed mass, and consequently pressure, was immense. Perhaps this is the best experimental evidence that we possess of the perfect physical homogeneity or structural equality that evidently exists in so perfect a fluid as water under great difference of pressure. I do not think, however, that this homogeneity will *entirely* account for the phenomena given by Newton and Coulomb, of equal resistance at all depths, but it does so very nearly. I will, therefore, supplement this by another proposition further on (9 Prop.).

Construction of Atoms forming Fluids and other Bodies.

4. PROPOSITION: *That a simple gas is a fluid composed of atoms, or the smallest divisible parts of matter; these atoms being infinitely tough and infinitely elastic bodies endowed with polar attractive forces by which they symmetrically unite.*

a. The atom may be to human perception a particle infinitely small. It is said that a ten-thousand-millionth part of a grain of one of the densest bodies, gold, can be made a visible quantity to the eye, and that this quantity may be more than multiplied by itself to go to the limit of detection of its presence chemically. So that even supposing this to be the limit of size for the atom, which it evidently is not, the mind is lost in the conception of it, even within manipulatory range of its smallness. Nevertheless the distinct characteristics of atoms as natural bodies may be made evident in many ways, particularly in their power of combination with other atoms in definite proportions, which are found to be generally in equal quantities or multiples of their unit volumes in the gaseous state of the body.

b. It is said to be demonstrable that a solid body, however small, may, by division in parts that may remain in partial contact, fill a space, however great, so that assuming the atom sufficiently small, the gas may be a body of atoms in contact, and in this form possess all its observed or much greater tenuity.

c. Every known hard body has an elastic surface; if the atom has such a surface, which deflects under pressure by some function of the pressure to the distance of surface impressed, or that the deflection is inversely as the square of the pressure, *this deflection commencing upon contact by a quantity of pressure infinitely small.* The pressures to the volumes of gases could as well be accounted for by this means as by any other, without the necessity of supposing the atom to possess propulsive forces or to act where it is not. There is no physical reason to assume the atom being infinitely *hard* as is general: in assuming it infinitely *tough* and *elastic,* this would equally insure its permanent durability, or it might possess an infinitely hard nucleus and an infinitely elastic surface, which I think is most probable.

d. It is easily seen by the smallness assumed for the atom, that a very small absolute material deflection of its surface would materially decrease the volume of a physically measurable mass.

e. The elastic surface forces of the atom may be *increased* or even *caused* by heat. But it is not necessary to assume that the same amount of heat force per atom should produce an equal elastic envelope about it. There is strong evidence that this is not the case. We know that it takes a much higher temperature to render gold gaseous than it does mercury, and this a much higher temperature than hydrogen, whereas at the gaseous temperatures of these bodies the elastic forces may be nearly equal.

f. It appears to be quite possible that the increase of elastic force about the atom caused by heat is constantly equal to the amount of heat present, so long as the body remains in the same state, although the general dimensions of the body may indicate the reverse of this. Thus, the experiments of Arago and Fresnel show, that although the exceptional body water increases in bulk from 4° Centigrade downwards as well as upwards, by defect as well as by excess of temperature, that both in the time of its descent while it remains liquid, and in its crystalline form as ice, the refraction index of light passing through it increases at all times in ratio to the loss of heat, as though the water continuously contracted, or that the structure of the water became more dense. In this case it appears to me that we must assume so far the action of polar forces, that the refractions are relatively proportional to the molecular structural density, and irrespective of the molecular interspaces shown by the measurable mass of the outer volume.

5. PROPOSITION: *That in a gas the elastic exterior forces of the atom may be partly absorbed at the points of contact by attraction, and form thereby a more dense gas, or liquid, or other material body.*

a. Two volumes of like or of different gases may form a bulk equal to two volumes at equal pressure. In such cases the atomic elasticities are assumed to remain about the atoms in exterior contact, except as they are mechanically pressed together; such mixtures we have in the compound *atmosphere* formed of oxygen and nitrogen.

b. Compound gases may form mixtures at equal pressures which are in volume fractions of their separate volumes, as half, one third, two thirds, &c. Thus two volumes of hydrogen and one of oxygen, that is, *three volumes* united so that they form vapour at say 120° C. occupy the space only of *two volumes.* If we imagine that at the point of contact of the atoms of oxygen and hydrogen the cohesive forces of these bodies (which as chemical forces we know to be great) exceed or in some way *absorb* the greater part of the inter-elastic resistance between the atoms, the elasticities that remain will be now *exterior* only, or principally upon the surface of the tri-unit molecule formed by the force of combination.

c. Two atoms of a gas may unite by polar forces, or otherwise, in a similar manner, to form the bi-unit atom or molecule, and this bi-unit may have exterior elastic surface forces only or principally, as we may possibly presume from chemical combinations is the case with hydrogen, chlorine, and other elements. It is possible that atoms have polar points where attractive forces are only or mostly active, which are special to the particular kind of atoms, but that the elastic forces are equal in all directions about the atoms.

d. As all gases expand equally by equal increments of heat forces from the same temperatures, it is clear that the exterior elasticities here proposed will be equally affected by heat for all simple atoms, so that all bodies in separate atomic conditions of 4 prop. *e* are *equally elastic upon the atomic surface.*

e. It is possible that heat may be the entire cause of the surface elasticity of the atom as just proposed. The atom being without this force infinitely *hard,* and infinitely *attractive* to other atoms; this is consistent with the known properties of chemical attraction and of heat to separate atoms, that is, to expand matter. In this case all matter to be palpable must be coherent (attractive) in its parts in excess of a large amount of its elastic surface resistance, the separate atomic or free elastic state being invisible in all matter. It would also follow that without heat all matter in contact, by this assumption, would coagulate and become a perfectly dense solid mass.

f. If we assume the elastic forces equal about every atom of matter, a probable condition, then the density of matter will vary directly as the attractive forces have power to overcome the elastic surface resistances.

g. If we assume an atom to possess a permanently elastic surface impressionable by contact with another atom by a *very small* force for very small distance, also that one atom is attractive to the other, and that atoms of different materials have different attractive forces for other atoms, which is entirely consistent with our chemical knowledge, and that the elastic surface compression of the atom, as just proposed, diminishes in distance from it in inverse proportion to the strength with which attraction increases between the centres of the atoms—then in this case it is possible that *chemical attractions will be forces active directly proportional to the nearness that the atomic centres can by their attractions approach each other through the resistance of the elastic surface;* the chemical forces overcoming the elastic resistance in few cases entirely, if in any.

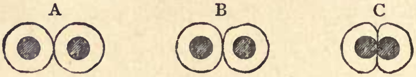

Fig. 1.—Theoretical Atoms.

h. Thus in the diagram above, A may represent a slight atomic affinity (attraction), but insufficient to overcome any part of the elastic surface resistance, the condition of a permanent gas under no pressure. B, a greater chemical attraction, or an ordinary cohesion. C, the greatest possible cohesion, in this last case the chemical attraction entirely overcoming the elastic resistance, so that a perfect bi-unit compound is formed. If such a bi-unit is possible, its atoms would be afterwards inseparable, and it would be to science a new element, so that possibly this last is a limit of *positive* attraction never reached, or reached only in a few cases wherein we find that two chemical elements, as nearly as modern analysis can isolate them, possess properties that are very nearly alike.

i. If the action of heat is the cause of the elastic forces about the atom under heat expansion, the atoms B may be separated as shown at A by heat under no pressure. It does not follow from the above that a mass formed by such attractions as represented by B for two planes of the atom should be of density proportional to the distance of atomic attractions into elastic resistance *per se*, as we may naturally conceive that any mass may be by further attractions of its molecules rendered more or less porous or crystalline, or otherwise molecularly arranged to possess a different structure.

j. It will be readily seen by the above conditions (*g*) that chemical attractions at a certain temperature acting upon two atoms may much exceed the elastic surface resistance, or they may be of such character that these forces of attraction and elastic resistance are in equilibrium by slight indentation of the elastic surface. In the first case the chemical compound will be *stable*, in the second *unstable*.

k. Further, we may assume that compression, by bringing the atomic centres nearer together, will strengthen the atomic attractive forces proportionally, so that in this position the attractive forces may overcome the elastic surface resistances, and form a compound that will be stable, or stable in a certain degree, as a gas may become a liquid or a solid. This may also be effected either by cold, or by cold and pressure.

l. In a static mass or compound, heat forces acting in opposition to the attractive forces of the otherwise stable unit may increase the elastic force, and separate the atomic centres for a certain distance, that is, expand the outward body, yet still, by the superiority of the attractive forces the unit will be to a certain extent stable in its expanded state. On the other hand, in an unstable compound the equilibrium may be such that any increment of heat that may instantly overcome the attractive central force will entirely release the atomic elastic forces. This same effect may also occur from motions of freedom by release of pressure, or by frictional separation, or by vibration. This is the condition of unstable compounds of the explosive class. It is possible that at some low temperature every mass of material is stable, and at some higher temperature every mass is unstable.

m. The attractive forces of the atoms of matter appear to be distinctly different for different elements with respect to themselves and to other atoms, in many cases the densest matter possessing the strongest attractive forces, as with gold and platinum. The material with the strongest elastic forces known may possibly be nitrogen, as this holds its units powerfully apart in the atmosphere, and does not frequently enter into stable compounds, or form hard bodies where it is present in relatively large quantity.

n. When an atom is in such a state of equilibrium with respect to other atoms that its elastic force exceeds the attractive central forces that surround it, it will form *as stable a material or mass as the densest solid,* and the only difference between this state of the body thus formed from that of a solid will be that compression upon its elastic exterior form will react with equal force upon removal of the compression it at first received; whereas compression in a system, with great excess of central attractive over elastic resistance, will increase the central attractions, and be less reactive through elasticity; this last case may be evident for the conditions of a gas, and by the next proposition it will be equally so for a liquid, which may resist pressure with greater force than a solid composed of the same elements at lower temperature.

o. If the atomic surface is infinitely tough and elastic, this, as here proposed, will give perfect elastic reaction, and will be a sufficient cause to preserve the atom as an intact body. The absolute depth of elastic surface from the entire size of the atom will be very small, or certainly not a trillionth of a millimetre. Under the ordinary conditions of pressure of contact a visible elastic body, as india-rubber for instance, would possibly bear more wear than any hard body, and the same would occur with the elastic atom.

p. As a further condition, it is not quite impossible that the atoms of all matter are surrounded with an attractive atmosphere of another matter, ether or some other body, but this is not necessary to support the theory of an elastic, or, if necessary, an infinitely elastic surface to the atom.

q. If the attractive forces between atom and atom act by means of a material elastic envelope which is a part of the atom surrounding it within a certain radius, and the forces of attraction are active by means of this envelope with intensity varying inversely as the squares of the distances from *centre to centre* of the atoms, and that this same entire radius is the extent of the radius of the elastic forces surrounding the atomic nucleus, and that this is active *as resistance* inversely as the square of the distance from the *surface* of an assumed solid nucleus—then the atomic nuclei of two atoms could never touch, as the powers of resistance would become infinite at a point of approach where the attractive forces would only be proportional to the distance approached towards the centre, which, being occupied with perfectly dense matter, it could never reach.

r. The simultaneous action of attraction and elastic repulsion of this proposition may be roughly shown by the experiment of a floating soap-bubble upon carbonic acid gas. The attraction of the earth draws the bubble; the elastic gas resists the attraction, and a point of equilibrium is established for the bubble, where it rests.

Construction of Liquid Molecules.

6. PROPOSITION: *That a liquid molecule being formed of atoms, or of the smallest divisible units of matter, these atoms are united by polar or concretionary forces to form globular molecules of equal dimensions among themselves, which are possibly very large compared to the separate atoms of which they are formed.*

a. As we know that there are solid bodies or masses composed throughout of like materials or elements, it is certain that there must be modes of attractive aggregation by which the large units of matter unite in some special manner to form such masses. In this case the extreme elasticity of the atomic surface would be absorbed to static equilibrium by the attractive forces of the atom, by my theory of a gas (5 prop.). Chemistry furnishes us with abundant proof of the union of matter from such a state of division that it is at first invisible, but that it forms by aggregation or condensation liquids or solids. Indeed matter in its atomic or most divided form appears universally to possess such combining qualities unless these are held separate by other forces.

b. By this proposition I assume that if there exist a mode of atomic aggregation, the effects of attractive forces within a definite radius about separate centres, to which the atoms are attracted, or by which they in any way combine to form a concrete but separate system, that such a system formed of separate equal small masses or molecules, by the separation of parts caused by local attractions within a certain radius, would be a *mobile system*, and in this respect possess functions of fluidity, or it would possibly be a liquid.

c. If the system of atomic aggregation by consecutive attractions is linear or continuous upon consecutive parts, or interlacing or outwardly crystalline, it will by the mode of adhesion of its parts, or of these interlocking with each other, form a solid, or in some instances form a molecular liquid first by separate unit aggregations, and then by further aggregations of such units, a solid.

d. The system here proposed for atomic aggregation to form a liquid mass, may be continued in principle to form liquid drops, when the system is free from the near cohesive forces of other liquid mass. Thus atomic aggregation may form concrete molecules of such size that a mass of these separate molecules will produce a visible vapour, although the dimensions of each concrete molecule in the vapour is separately too small to refract light, or to be visible by any power of magnification possible to human art. Assuming such molecules to form a visible vapour, the aggregated contact of these molecules would afterwards form a liquid if there were not present the interference of air diffused among them. But with such interferences molecules in a visible vapour not held by great rigidity of heat elastic surface forces, proposed (5 prop.), may again separately aggregate and form visible liquid drops, by a similar, although, I anticipate, less active and less regular attractive force of aggregation than that described above; so that this second form of aggregation will not establish the uniformity of dimension in the globe or drop produced, that is possible for the molecule in the first case formed of equally distributed atoms of its elements.

e. It is possible that a concrete molecule of a liquid may have for its nucleus an atom of another matter; as, for instance, water may have for its nucleus an atom of nitrogen or other matter that will be attractive to both the hydrogen and the oxygen in its aggregated system when it forms the liquid molecule; the position of this atom of nitrogen before the molecular concretion assumed being established in the mixed gaseous state of air and vapour by polar forces to be considered in the next proposition.

f. For evidence that the system of a liquid is molecular, or of such detached parts as to have interstices, we may refer directly to experiment. Thus if we mix with the liquid *water* an equal volume of another liquid *alcohol*, the union of these liquids does not produce the bulk of the two separately before admixture, but a less bulk, proving that the one liquid passes in this case into the interstices of the other. The experiment offered to prove this, is to have a tube of the form shown below, consisting of two bulbs, with a prolonged tube at the end of one and a stopper at the other. If the lower tube and bulb be filled with water, and the upper one with alcohol, the stopper may be pressed down air-tight upon the liquids, and the tube will be left quite filled. If this apparatus be now reversed two

Fig. 2.—Specific Density Tube.

or three times so as to mix the alcohol with the water, and it be then inverted so as to leave the fine tube upwards, there will be seen a vacuous space, indicating that the fluids have decreased in volume by admixture to this extent. It must be observed that the mixture of alcohol and water is not a chemical mixture, as by applying heat the spirit will evaporate and the water will be left. In this matter we could have arrived at the same conclusion by noting the separate specific gravities of absolute alcohol and of water, and finding the difference of specific gravity in the mixed compound, but this experiment shows the conditions more clearly. The compressibility of fluids is possibly proportional to the porosity or separation of the molecules.

Some Evidence of Polar Forces in Fluids.

7. PROPOSITION: *That a fluid under the action of chemical attractions which are at the time producing molecular changes will diffuse the chemical attractions throughout the fluid with forces proportional to the polar or directive vigour possessed between the combined or compound molecule and the more simple uncombined molecule that has not been under the special chemical action. A liquid or a gaseous system will not be in symmetrical polar equilibrium until the chemical forces are distributed equally in all parts of the mass.*

a. The conditions of attraction and repulsion (elastic force), which ultimately give static position to a molecule in a fluid, may possibly be best observed under conditions of chemical change. This proposition is introduced to give evidence of polar or directive action during this change.

b. A liquid in effervescence, or in ebullition, or under chemical change, is obviously not in a state of symmetrical equilibrium as proposed for a static liquid in the last proposition. There is nevertheless most probably a constant tendency to equilibrium, which is more or less complete in parts of the mass, but so long as there are polar molecular forces in a system not in attractive contact with opposite polar forces in other molecules, the system will be unstable.

c. That liquids possess polar forces of the same kind as those we witness in a globular piece of loadstone, may possibly be best inferred by the action of a corrosive liquid upon a solid which it has the power to dissolve, in cases where a newly constituted liquid is formed that is a chemical compound of the two bodies present; the same conditions will also apply where the body is dissolved without chemical composition.

d. If we place a corrosive liquid in contact with a solid that it has power to adhere to and to dissolve, the solid will be dissolved into the liquid, and the original polar system, or cause of attraction, which produced the cohesion of the solid, will be changed or dissociated by the polar or attractive forces of the liquid. This is clearly the case, for if the corrosive liquid has equal affinity in all directions to the solid, this affinity would cause the liquid to unite to the surface of the solid, and remain in this position attached to it simply, so that when the surface of the solid was completely attached to the liquid, and thereby covered by a stratum of it, no other molecules of the cohesive liquid, for want of space to approach, could come in contact with it. It is possible that this really occurs in certain cases, as with silver in hydrochloric acid, zinc under aerial oxidation, &c. But for the solution of a solid body in a liquid it is necessary not only that the molecules of the corrosive body should adhere, so as to unite with the solid, but that this adhesion should also at the instant possess some *mode of force* by which the new molecule formed by combination is immediately detached from the solid in order that another molecule of the corrosive liquid may be able to approach and to unite with the surface of the solid in the same manner as the first particle approached and united with it.

e. It is necessary to complete the total effect observed, in addition to the causes offered above, for the solution of a solid in a free liquid, that the corrosive particles should have a mode of approach and a mode of retrogression, as we find a solution of a solid in a corrosive liquid will often maintain the liquid in all parts of approximately equal strength. Now if a liquid were a homogeneous system without molecular construction, it would approach the solid upon which it acted *en masse*, and there would be no change of parts except quite locally. It therefore becomes probable that the molecule of the solid, at the instant it is attracted, has its polar forces displaced at the point of contact by the more attractive polar forces of the liquid molecule, so that now the elastic forces of the solid atom react and throw this off the solid mass, to which it was before united by the elastic forces set free from polar cohesion.

f. We may also conceive that by the same system of forces acting with less intensity, when the newly formed molecule is free, that this molecule will possess new polar forces which may cause it to be in unstable equilibrium in the liquid, so that it will be driven in like manner to the first departure from other molecules that have not been in contact with the solid, and the neutral molecule formed after combination would *back out*, as it were, from the presence of the solid to permit the corrosive attractive molecule to approach. This being the case, a molecule, after combination with an atom of the solid, would not find rest in polar position until it reached the most distant part of the liquid system from the solid it at first corroded. We may conclude, nevertheless, that the intensities of polar forces may be very different in the direct action of a corrosive liquid upon a solid to the intermolecular polarity in the liquid afterwards, as the polar forces in the new molecule may be very weak. For many reasons, particularly from the fact that dense compounds do not mix, I anticipate the polar forces in a neutral liquid to be excessively weak, or even outwardly impalpable.

g. What I infer from the principles of the above, by the conditions under which the action of a corrosive liquid upon a solid forms a new liquid, that is, a newly constructed molecular mass, that such may be the general formation of other liquid molecules either of simpler or more complex structure. The forces of atomic polar cohesion being distributed until a symmetrical liquid is formed possessing molecular polar forces which act directively, and produce its general cohesive and other physical properties.

h. Under the conditions discussed above, the final disposition of the molecules of a liquid would be by symmetrical arrangement, consistent with the polar forces of its separate molecules, and this would hold for all static liquids, that is, liquids not under the influence of chemical changes, as it would hold also for the liquid as soon as the chemical change ceased. The evidence of this being the case would no doubt be difficult to prove, except by inference that nearly all bodies that we are able to observe in a solid state, if pure, assume symmetrical forms (crystals), and it is most probable that such symmetrical form as we witness in the crystal is a continuity of like symmetrical or orderly construction in the liquid from which it is derived. In the case of solids that appear to be homogeneous, Dr. Tyndall has devised a beautiful experiment, namely, the dissolution of the centre of a pure apparently homogeneous block of ice by the focus of a burning-glass, in which experiment, as the ice dissolves, its crystalline structure is rendered evident by the well-known forms of ice crystals that start into view.

I may here remark that I do not assume that liquid symmetry is necessarily or generally *crystalline.* I assume that this is the special condition of solidity, but that the molecules of liquids are held in position by polar forces which are equivalent to the same polar forces that produce a crystalline or solid mass. The probable symmetrical forms of liquids I will discuss further on.

i. The only known evidence that may possibly exist of the symmetry of the system of a liquid may be inferred from the similarity of effect upon light to absolute crystallization which we witness in certain liquids, as, for instance, in sugar solutions under polarization. Perhaps many liquids under severe tests would also exhibit polar forces to light. In such cases I presume the polar forces are internal, and do not affect the general principles of attractive cohesion in the liquid molecule, which is exactly or very approximately equal in all directions, and that these polar forces are no more than sufficient to place the molecule in a state of symmetrical equilibrium if free from external forces.

Uniformity of Cause of Fluidity for all Matter.

8. PROPOSITION: *That the fluidity of liquids is dependent upon the presence of gases, or of liquid vapours held by attractive forces upon the molecular surface and intruded intermolecularly.*

a. If every concrete molecule of a liquid (6 prop.) were on a very small scale constituted exactly as the terrestrial globe we inhabit, it would be surrounded by an atmosphere of gas or vapour. If we imagine that the central attractive force of such a molecule towards other molecules as here imagined were very weak, say proportionally to its size in relation to that of the earth, then a number of such molecules thrown together in a space free from gravitation would each maintain its atmosphere with only slight impression upon it by the weak mass attractive forces of the other molecules against it. This is equivalent to the conditions I have imagined for the atoms forming a gas, by these atoms being surrounded by an elastic surface (4 prop.). If we can apply the same principles to the larger molecule of a liquid as the gas, assuming much greater central density and comparatively larger central mass of the liquid by atomic concretion (6 prop.); then we could conceive that there would be uniformity of motion in gases and liquids by impression of like forces upon or within these fluids as regards the friction of displacement of their molecular systems *inter se* by their surfaces of contact being composed of like matter, and that motive resistance would then only vary as the density of these fluids. That such equivalence of principle exists in fluids I shall endeavour to show by experiment as we proceed.

b. For the principles of the proposition I assume that the gas is a finer division of matter (as offered, 4 prop.) than the liquid, possibly separately atomic, or united in its free state by weak polar or other forces in bi-atomic or tri-atomic separate molecules (5 prop.), the atoms being perfect solids, infinitely tough and infinitely elastic at their surfaces; and that the liquid is formed of molecular concretions, which may be crystalline, conglomerate, or otherwise united, so that by absorption of elastic surface forces of the atom upon certain points of contact, they are relatively to the gaseous matter large combined molecular masses (6 prop.). By the present proposition, in the construction of a liquid molecule the molecular aggregate is assumed to be surrounded by an atmospheric system of gaseous atoms which are *adhesive* upon it through the continuity of the attractive forces diminishing from the centre upon which the entire concrete liquid molecule is formed. The gas that surrounds the large liquid molecule may be either a separate gas, or a gas formed of the same material as the liquid by a difference only of a more open arrangement of the atomic parts, in which case the surrounding gas may be termed a vapour. This vapour being in adhesive contact upon the liquid molecule, a more perfect gas may in some cases further surround this exteriorly.

c. In this theory of a liquid, it will be seen that the difference of the sizes of the molecular concretionary nucleus, and its more attenuated surrounding gaseous parts, would be alone sufficient to account for the same equivalent mobility in a liquid system as that observed in a gaseous one, as the gaseous system would be *exterior* to the larger liquid molecule. For we might in this case either assume that the gaseous matter, held by weak attraction or cohesion, acted by the freedom of its parts, caused by their fineness and separation, as a *lubricant to the motions of the larger molecule* of the liquid proper, or that the liquid molecule *floated* as it were in the more attenuated matter of the gas. Upon this principle the whole system of matter of the liquid as before proposed would be moved only by direct contact of solid parts, and the phenomenon of repulsion would not be a *necessary principle to be assumed*, or any greater initial motivity to exist than the conditions of known polar forces in fluid matter would warrant. The separate atoms in the gas, by their smallness and interspaces being assumed to be arranged to globular form around the liquid molecule would produce an extensile elasticity in the liquid proportional to the number of surfaces in contact.

d. For the passage of molecular matter from a gaseous to a solid state, and *vice versa*, it is not necessary to assume the intervention of the liquid state, or certainly not in all cases. But it is almost certain that all matter may exist in a gaseous as well as a solid state under varying conditions of temperature and pressure. Of the passage of the gases directly from the solid under atmospheric pressure such instances may be taken as the evaporization of iodine, camphine, ammonia carbonate, and some other bodies wherein gases are thrown off solids directly at low temperatures. Under greater pressures, at melting-points, with the same bodies a liquid may be formed as a gas under less pressure, as is the case with arsenic. It is probable that the liquid is generally a *re-formation*, the gaseous being the first form of molecular separation; the liquid concretion being by a new arrangement of polar force (6 prop.) partly dependent upon the vapour pressures upon the evaporating or dissolving solid. This may be inferred in that if the vapour forces are removed as quickly as they are formed, as in the case of evaporation of snow in dry winds, the intermediate state of liquidity does not intervene. And the same may be inferred in the melting of zinc, aluminium, iron, and other metals, where these bodies inflame if not retained by a flux or by a coating of their own oxides sufficient to form a solid or liquid pressure immediately upon them.

e. It is further possible that liquids of all kinds either form gases by evaporation of their own molecules, or imbibe certain quantities of such gases as may be presented to them, as may be consistently adherent to their systems. Mercury, the densest of all permanent liquids at ordinary temperatures, has been found to emit vapour at its free surface sufficient to discolour gold placed above the surface. Faraday has shown that this evaporation occurs in mercury down to a temperature of 4° Centigrade at a *sensible distance* from the surface. It is probable that vapour of higher density exists at insensible distances, that is, *intermolecularly* and upon the *immediate surface* of the molecule of mercury, so long as heat force maintains the mercury liquid, and probably after this.

f. This intermolecular space, if I may so term it, in liquids that

I assume to be occupied by vapours or gases is possibly a nearly constant quantity. This is inferred from the quantity of gas that a liquid absorbs, that is, as I assume, attracts to surround its molecules, and is proportional to the pressure upon the gas. And as the density of a gas varies approximately as the pressure, it follows by this proposition that a given liquid always absorbs or collects by the radius of its attractive forces a constant volume of gas *per massa*, whatever the pressure, according to Dalton's law: by my theory *the quantity being that represented by a certain depth of gas upon the area of the surface of the liquid molecule.*

g. Where a liquid by loss of heat forms a solid, the solid so formed, although generally of less bulk, is a body *unlike the liquid;* it is generally *compressible by small forces.* In this case I assume that the perfectly rigid solid atoms that formed the symmetrical gas, or the liquid, by structural arrangement or polar force about the concretionary liquid molecule are now crystallized, as it were, upon the molecular surfaces in more compact form by suppression of a part of the atomic elasticity in the gas or gaseous envelope, so that the atomic atmosphere is now concrete upon the solid molecule. The cause of the permanent atomic surface deflection in this case being derived from the force of chemical attraction present by loss of heat-elasticity, so that what was formerly the liquid molecule is now the solid molecule, the gaseous envelope being now deposited upon it; this gas has therefore no longer the symmetrically distributed structural matter, or the force, to offer the resistance it offered before.

h. We can imagine that the dense molecular system of matter that a liquid is proved to be under pressure, that if it were formed physically of molecules of equal forms which were as particles of dust, such an equal system would form an immobile mass; but if we can conceive that such a system can by its molecular attractions, or otherwise, compound with it another less dense molecular system whose individual atomic forces are as the diameters of their atomic masses, that these will disjoint, as it were, the equality of the more rigid system, and cause its molecules to float or move in its elastic atmosphere easily; the intervening gaseous or vaporous system being by its tenuity more mobile than the entirely concretionary molecular one. In such a form the relatively large molecule of a liquid would be free, or very nearly so, of polar forces, assuming the attraction forces to be satisfied, and to be equal in every direction upon the gaseous envelope.

i. It is possible that by the gaseous intrusion which occurs in the admixture of air with water, that we have water as a very mobile fluid, at ordinary temperatures, the air acting, as before mentioned, as a mechanical lubricant to the larger molecular aqueous system proper. However, this would not exclude the condition I propose, that water may be mobile principally from its large molecules being surrounded by their own vapour. That the fluidity of water depends in a great measure upon the presence of air, is shown in the researches of Donny, in his celebrated experiment, which is as follows:[1]—A clean glass tube a metre long, closed at one end, is bent in its centre so that it forms two arms, inclined at an angle of about 60 degrees to each other. The tube is for about two-thirds of its entire length filled with water, and the open end of the tube is now nearly closed over with the blow-pipe. The tube is placed so as to keep the closed end lowest, and in this position the water in it is boiled for an hour or more, so as to expel as much as possible of the air that is contained in the water. When the water in the tube is still boiling, and the space above it is well filled with steam, the opening in the tube is closed by the blow-pipe, and hermetically sealed. When the steam condenses there is nearly a perfect vacuum above the water. If the tube be now placed in such a position that one of the arms remains full of water, and this arm be tapped several times with the finger-nail or any light body, so as to obtain surface adhesion of the water to the sides of the tube, the full arm may now be gently raised to an equal angle with the one partly filled, and the water will not run down to a level surface in the two arms, but remain in the first as before the inclination, as shown in the engraving. The water will also be found in any movement of the tube to move very sluggishly, showing that it possesses little fluidity in this nearly pure state. It is very possible, that, if water could be entirely deprived of air, it would be a gelatinous body or a solid.

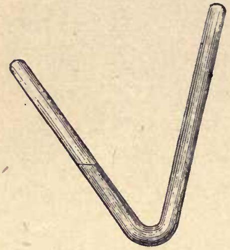

Fig. 3.—Donny's Vacuum Tube.

j. The above principles of liquid construction that I propose may be shown diagrammatically by systems of coherent molecules surrounded by a gaseous atmosphere, this atmosphere being of an absorbed gas, or of the vaporous gas of the molecule proper. But

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[1] *Ann. de Chim.* iii. xvi. page 167.

if it is surrounded by the gas, this will produce an hydrostatic pressure about the molecule, so that the vapour assumed to be nearest

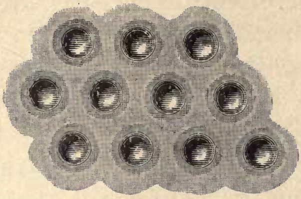

Fig. 4.—Theoretical Molecular condition of a Liquid.

to the molecule will be absorbed by the molecule, and the gas only remain surrounding the vapour. Or if the absorbed gas be one of great adhesive force to the liquid molecule, a part of the gas may be liquefied upon its surface by near attraction. The system proposed is shown diagrammatically above of liquid concretionary molecules. In this illustration the molecule is supposed to be surrounded by a vapour close to it, with a general atmosphere of gas surrounding the entire system as it would be under pressure.

k. Taking a purely hypothetical view of the subject, I do not see great improbability that this system of gaseous atmosphere about the molecule does not extend to all matter including solids. In which case we may conceive that, as the diamond is harder than graphite, that this may be, that its elastic surface force or vapour force is less, or was less by near attraction through the great pressure existing at the time of its formation into a set solid, thus giving a closer or more attractive mode of atomic aggregation, and a greater suppression of elastic resistance. In the same manner a hydrocarbon may be formed of a molecule of carbon surrounded by an atmosphere of hydrogen partly condensed upon it to form a liquid as an oil; or in another case, a molecule of carbon may be surrounded with an adhesive atmosphere of hydrogen or of oxygen to form a gas. This might be true assuming carbon to possess extremely small, extensile, elastic, or evaporative force at the surface of its atom.

l. If molecules of matter, particularly plastic and liquid matter, are separated by vapour force, capable of a certain amount of condensation upon the molecule, *then expansions by heat forces will be as powers of evaporation of the separate molecules, and minus heat contractions, as the powers of condensation,* which are generally inversely equal, so that a liquid or solid molecule at a certain temperature engenders a certain vapour or elastic force about itself, but at a higher temperature, by the same continuous process, which may be by the strengthening of elastic forces at the surface of its atom, it may be wholly dissipated as vapour, either in a system of finer molecular division or in certain cases by heat forces and relief of pressure or attraction, possibly in separate atomic division; the separate atoms being then held by their own forces in symmetrical polar positions, beyond which further expansion could not occur by any amount of heat force.

m. In the construction of liquids I have taken the surrounding vapour forces in this proposition to be equal, therefore the central forces of attraction or adhesion of the vapour to the liquid molecule will be alike on all sides. I anticipate, nevertheless, as before proposed, that there is a certain amount of polarity or directive force in every molecule of matter to other molecules, which I have before inferred for liquids by the systematic polarization of light by sugar and other solutions. But as this probably does not generally in the free liquid molecule exceed the effect of polar forces of the earth in relation to general attraction of gravitation, I conceive the average liquid molecule to be equally attractive to its vapour or gas on all sides, as the earth may be to its own atmosphere, although the molecules and the earth may have directive or attractive polarity quite independent of this for position of its parts in relation to each and to other matter. The smallest separate elements of the earth or the molecule may possess polar forces of much higher powers relatively to their masses than the compound larger bodies or molecules of which they are the parts. This is evident by the nature of chemical attractions in divided matter.

n. Perhaps further evidence of the elastic surface forces about material systems may be found in Newton's experiment of pressing two convex surfaces of glass together, where visible contact is attained with difficulty. In this case the elastic or molecular atmospheric force is, as in all other cases, assumed to be of invisible matter.

o. One subject that particularly concerns us practically is that there may be a system of unity in all fluids, which may even extend to homogeneous solids, so that experimentally air may be replaced by water and *vice versa*, which will in many cases permit us to follow motive causes by visible effects. This principle, upon assumptions of the proposition, will be made use of in this treatise as occasion may require.

Sensitive Mobility under Pressure.

9. PROPOSITION: *That the mobility of a fluid varies directly as the elasticity in the atomic forces surrounding its molecule. The elasticity being in sensitive equilibrium proportional to the pressure upon the fluid.*

a. In the above I am compelled to use the word atomic to distinguish a finer system of gaseous matter that may surround a gaseous molecule, but I do not assume that the atomic system is necessarily *separately* atomic, although I see no reason that it should not often be so. For a liquid molecule we may assume a gaseous envelope only, as offered in the last proposition.

b. This proposition is offered to meet a difficulty, as it appears to me, in explication of Newton's and Coulomb's experiments given 3 proposition, *d*, *e*, for although the homogeneity or equality of resistance at all depths in a liquid is clearly demonstrated, this equality does not appear to be rational with an equality of attractive or repulsive forces in separate molecules, or equality of mobility of contact of the molecules or parts simply *per se* from any cause. For if we consider the mobility from any cause equal about the molecule under all conditions, we must neglect the effect of pressure of the mass above a lower molecule, which pressure being derived from material parts of the same mass, should act as a resistance to the freedom of motions of displacement to the lower molecule. It therefore appears to me probable, that there is a natural line for fluids as defined by the proposition, by which the molecules of fluids are in more *sensitive motive equilibrium under greater surrounding elastic pressure.* This, it appears to me, would also very well account for one cause of a kind of immobility, that is very evident at the surface of a liquid, which is seen to move as a concrete system, when small objects of great specific gravity float upon it, some other conditions of which I will consider in the next chapter, in discussion of surface forces.

c. Assuming the molecular elastic surface by equality of central attractions perfectly smooth, which I think most rational, we may find evidences that mobility of contact will be relatively greater under greater pressure in solids, which would be from the greater sensitiveness of compressed surface elasticities upon contact. Thus we may lean a light body with a hard point, as the point of a pen in a light handle, at an angle of from 20 to 30 degrees, upon a perfectly polished hard surface of glass; but if we press the upper end it will instantly slip down, or if we lean a heavier body with the same surfaces of contact, it will not maintain itself at so great an angle to the surface.

d. Another simple experiment will demonstrate the same fact. If we cut three or four pieces of clean smooth paper, of exactly the same size, and place them lightly upon a clean surface of glass, one by itself, and two or three superimposed; and then raise the piece of glass at one end to form an angle, at which the pieces of paper begin to slip, the heavier parcel will begin generally to slip first. The same will occur with pieces of moderately stout flat metal and with metal foil, and in vacuum as well as in air.

e. It is possible that the velocity of sound through bodies is wholly due to the sensitiveness of compressed elasticities acting in direct lines, caused by the strength of cohesive attractions into the surrounding elastic surfaces of the atom or molecule in these lines; so that the velocity of sound is great and sensitive in steel, glass, and other hard bodies; but slow and insensitive in gases, being most insensitive in hydrogen. These conditions on the whole will be subject to the molecular construction of the body taken.

Mode of Molecular Aggregation for Liquids.

10. PROPOSITION: *That the molecular system of a liquid, assuming each molecule surrounded by its vapours, has its molecules symmetrically arranged by polar forces in such a manner that the greatest number of molecules, or molecular systems, may be contained in the space that the liquid occupies.*

a. The molecular system here proposed would be represented as in the engraving below at Fig. 5, the large liquid molecules and their vapours being represented by circles so arranged; and not as at Fig. 6, where the polar forces retain the molecules in a vertical series, although I see no reason that some forms of matter should not be so arranged or changed to such form as by cooling, where bulk is increased thereby, preliminary to crystallization.

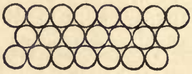

Fig. 5.—Molecules.

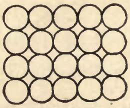

Fig. 6.—Molecules.

b. The same form of molecular construction as that shown at Fig. 5 has been illustrated for that of a liquid by Newton, Bossut, and others. It is here introduced to consider a condition that renders it most probably the actual system, which is, that for liquids (and fluids generally) lateral pressures upon containing vessels are as the heights of the column of liquid above any given point. If the molecular system of a liquid were as that shown Fig. 6, this would not be the case, as every vertical range of molecules would be supported upon the lowest molecule, and very small lateral pressure would keep the column vertical, so that the pressures upon the sides of a vessel would be small and that on the bottom great. But if every molecule were held in its position by forces that were not vertically axial, these pressures would be equally lateral forces, so that the limiting lateral surfaces would aid in supporting the system. And in this, the whole system being assumed elastic and mobile in all its parts, would be as a flexible system, and all parts at equal depth would receive equal pressures.

c. Thus let A A′ be a lateral surface, and all the shaded molecules press by their gravitation downwards, and the unshaded molecules rest against the lateral surface. Then the lateral particles would be pressed outwards by the vertical force of the contiguous molecules, and it would be clear that to press these molecules by this lateral surface further into the system, we should require a force sufficient to lift up all the particles above. In this manner the lateral particles would be jambed against the sides of a vessel, and the jamb, in a perfectly elastic mobile system, would be a pressure equal to the gravitating force of the mass above. In the same way it will be seen also that this jamb would produce an equal upward pressure, or pressure in any other direction than that which merely supports the weight of the fluid, if the area of pressure were restricted by the form of the vessel.

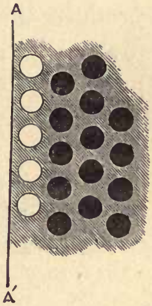

Fig. 7.—Molecules of Liquid against a Vertical Solid.

d. The above hypothesis is consistent with the fact that a continuous fluid resting upon or pressing against a plane surface exerts a force directly perpendicular to this surface, this force acting as a pressure equal to the greatest pressure per area upon any part of the fluid that can be compressed by the force of gravitation at equal depth. Thus if we perforate a hole through a thin vessel containing a liquid, the liquid is projected directly perpendicular to the surface of the vessel, with a force of projection equal per area to the weight of the liquid above the hole, that is, equal to the greatest pressure per area that gravitation can effect in any part of the mass system of the liquid at equal depth with the hole. This principle is rendered important in consideration of directive motive forces which will be hereafter considered.

e. If such symmetrical molecular action as here proposed exist, this may either be derived wholly from a general principle of cohesion which draws the greatest number of molecules about any single molecule, the principles of which I will discuss in the next proposition, or from this and polar attractions conjointly active at several points of the molecule, the principles of which may be inferred from the disposition of matter to take crystal forms.

f. The same form of molecular arrangement as that illustrated above would possibly occur from pressure upon a system of globular molecules. But not necessarily so unless there were present the polar forces assumed. As, for instance, a vessel filled with rapeseed or with globular shot would not have its separate units so arranged by chance, or unless immense trouble had been taken to produce this arrangement, whereas with polar forces acting symmetrically at certain points of every molecule this might be the necessary or only possible arrangement.

Equal Universal Cohesion of Liquid Molecules.

11. PROPOSITION: *That the cohesive forces of liquids are caused by molecular surface attractions, which are active in producing adhesions of the greatest superficial area of contact possible upon all the molecules of a liquid system, of which each molecule forms a symmetrical part.*

a. The above proposition, as far as I have been able to observe, is a *law of liquid cohesion*, that is particularly valuable when applied

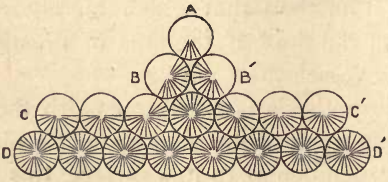

Fig. 8.—Molecular Attractions to form Mass.

to the conditions of surface forces which I will hereafter consider, the matter beng now taken for the unity of masses only.

b. Let the figure above represent any exterior surface of a molecular liquid system having attractive cohesive forces. Let the cohesive attractive forces reside in every molecule, and act upon the nearest molecules of its own mass system, and further let these attractions be equal in every direction from the centre of each molecule to its area of contact, and for a certain space surrounding this, as shown by radial lines from the centres of the molecules in Fig. 8, neglecting any possible influence of local polarity, which I assume would be very weak in the liquid molecule, and suppose, also, all action of gravitation suspended as a separate force acting upon the system; then will the molecule A be out of central equilibrium to the system, and exert a direct attractive force upon the system towards the molecules B B′, and the molecules B B′ will exert an attractive force towards the plane C C′ and towards each other with a small force only towards A, so that these molecules also will be out of central equilibrium. For the conditions of the molecules C C′, these will exert attractive forces towards D D′, the mass system, and lateral forces towards each other; they will therefore be out of central equilibrium but in *lateral equilibrium.* Under these conditions they will represent the greatest area of radial forces possible for the surface of a liquid system. The molecules of the plane D D′ as a part of the mass will be surrounded by equal radial molecular forces, and will therefore be in perfect central equilibrium at every point of the molecule.

c. Now returning to the consideration of the motive forces derived from attractions in the molecules A and B B′, these will act as certain forces in ratio to their attractive *area* upon the lateral equilibrium of the plane C C′, and indirectly through the plane C C′ upon the perfect equilibrium of the plane D D′, and if these planes are perfectly mobile and capable of extension, these molecules A and B B′ will sink into the mass system and form molecules either in lateral equilibrium in the surface system C C′, or of perfect equilibrium in the plane D D′; the law being satisfied *that the molecule has found the greatest area of surface adhesion possible for a free system of liquid matter.*

d. In the above proposition it must be distinctly observed that the force of gravitation forms necessarily no function of the system here proposed; the condition of this for liquid surface will be hereafter considered. A liquid, if its molecular attraction or cohesion is sufficient to support its mass, forms a smooth surface in any position, as for instance a varnish laid on a vertical surface or an inverted horizontal one, and the same will occur with water or other liquids. The splendid experimental researches of M. J. Plateau may be used to assure us of the principles of the above proposition, that the molecular cohesion that we witness in liquids is a force of continuity of physical attraction or cohesion of the molecules which forms the liquid system; and that this attraction or cohesion is independent of the density of the mass, or nearly so. The important experiment which proves this is shown in that a free molecular mass of liquid in equilibrium is in no way influenced in its motion by the presence of denser matter within its mass system.[1]

e. The most important experiment, by which the general equilibrium of forces in a liquid mass is shown by M. Plateau, is by taking two liquids of exactly equal specific gravity that are found to unite only in a slight degree with each other, and by placing a mass of one of these liquids of any form in a central position within a larger mass of the other. In this case the central mass forms a perfect globe. The liquids employed by M. Plateau in these interesting experiments are pure olive-oil for the one and a mixture of alcohol and water of exactly the same specific gravity as the oil for the other. By injecting the oil with care in the centre of the mass of dilute alcohol by means of a glass syringe the oil is found to draw itself together, and a perfect globe is formed, which is supported as a free body in the diluted alcohol.

f. That the above is not a *mass attraction* M. Plateau's important experiments before alluded to show, as he finds that by placing a stout plate of iron of nearly the same diameter as the globe of oil in its centre, by means which he points out, the presence of this heavy mass is found not to deform the external figure of the perfect globe in the slightest degree.

g. M. Plateau attributes the perfect equilibrium of forms which his experiments demonstrate for free liquid matter to be caused principally by *surface tension* of the mass system as originally proposed by Segner in 1751; and his experiments in some cases that he offers go very far to show that this would represent a sufficient cause if active to the necessary extent demanded, the interior inertia of a mass system being assumed to be in such a general state of free equilibrium as to be entirely directed by the surface tension. It nevertheless appears to me that as a general system of cohesion

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[1] *Statique expérimentale et théorétique des liquids soumis aux seules forces moléculaires,* 1873, by J. Plateau, § 13.

exists in all liquids, as shown clearly by the phenomena of traction, in which the parts of liquids follow each other with evident force, as they also in the same manner adhere to solids, so that the surface tension cannot be the entire cause of cohesion in the system of matter of a fluid any more than of a solid. On the other hand, if the molecule seeks the *greatest area* of surface attraction in a perfectly free liquid, as here proposed, this principle of cohesion alone will ensure the perfectly globular form for any free unit mass of liquid of the same cohesive system subject to molecular forces only.

h. I do not think the conclusion I arrive at, that molecular masses may be held by cohesive forces simply, *if this be true*, detracts in any way from the value of M. Plateau's splendid experimental researches. I only think that the symmetry of molecular forces in a liquid has more to do with the symmetry of free molecular aggregate forms, in any case, than surface forces taken simply as such. I conclude this among other reasons, in reference to the theory of tensile surfaces, by the *slowness* with which an undoubtedly *tensile film* of a soap-bubble contracts when the interior of the pipe by which it was blown is left open for it to do so. The tension of the two liquid surfaces of the film in this case, the exterior and interior being active as lineal attractions upon the resistance of the small volume of air in the interior of the bubble only, shows that with large surfaces of tension, and very small mass to be moved, the tensile force judged by its active velocity in this case must be *extremely weak.* Whereas the motive forces that form the globular mass of oil by moving its heavy bulk quickly to equilibrium of figure, in M. Plateau's important experiment just mentioned, indicate a much greater constant force in the liquid, for it to be able to overcome the general inertia of its large mass, than could be due to the influence of any weak surface force, such as we find able to act only feebly upon the small mass of a film, having exposed surface on both sides open to this action.

i. If cohesion be powerfully active in the manner I propose, the same molecular aggregative forces as are actually observed would be produced in free molecular matter, as by any form of surface tension, or even of *surface distension* of a liquid; if the central mass be held together by the general cohesive force that all liquids possess. It would be also clear that the globular form would be the only one that could satisfy the conditions of perfect equilibrium in a free mass upon principles proposed *a.* Under these conditions also, inclosed masses of more or less dense adherent matter, would no more interfere with the system, than if the force were considered to be an exterior tension upon the surface, unless such masses protruded, when the conditions would be altered for all cases. It will be further evident that the proposition I have made above for the cohesive system of a liquid really becomes active as a surface force; but it does this independently of assuming a special condition necessary for the surface molecular cohesion possessing greater activity than is otherwise indicated by experiment; which may possibly be attributed to a superior viscosity at the surface for water, but could not be so for oil, where cohesion is found to be equally active, as M. Plateau's experiments otherwise show. See works referred to, § 269.

j. Perhaps some evidence of the law offered in this proposition may be discovered by taking the conditions of a body in which we can scarcely imagine any distinct conditions of surface force to be present, or at least to be sufficiently active to overcome the general cohesion or viscosity of the body taken. The best illustration that occurs to me would be the very viscous liquid, melted glass, which we know may be drawn into very fine threads, by strain upon the internal cohesion of its system without any disturbance from surface forces being scarcely imaginable to be active upon it. Now, if we take a thread of glass and hold this in or near a flame, until it begins to melt, it will be found that the melted part instantly *seeks the greatest area of intermolecular cohesion,* that is, it assumes a globular form, if not prevented by the force of gravitation from doing so by the position in which it is held.

Special Qualities of Water and Air.

12. PROPOSITION: *That the fluids water and air, possibly by the molecular elasticities of their systems and by polar forces, possess a certain amount of elastic rigidity of the same kind as that observable in gelatinous bodies.*

a. By the molecular structure proposed for a liquid (8 prop.) every liquid system, by the elasticity of its molecular surfaces will possess a certain amount of flexibility of mass derived from its chemical and physical qualities; which I assume to be different for liquids of different kinds. Besides which I imagine that there are present weak polar forces in all fluids (10 prop.) that will produce a certain amount of rigidity within the radius of mobility of a liquid or other flexible system. From these conclusions, and by experimental inferences that water and air possess the property of *rigid flexibility* greater than other fluids relatively to their densities, that is, in those that I have examined, and as I intend, as before mentioned, to follow experiments in air and water principally, I wish to distinctly distinguish this property by a word, for which I propose *gelatinity.* This property has been already inferred to be a quality of water in the bent-tube experiment of Donny given page 24 *i,* as observed in the sluggish movement of the water in the tube and its rigid state when the surface of it is rendered adherent. The same property may be observed in a less degree when the water is in a more fluid state by natural aeration. The property that I suggest appears at all times to give to the water a kind of static mass rigidity greater than its general fluidity, or the mobility of its system, or its density in relation to other fluids would indicate.

b. The property indicated above has been generally included in the term *viscosity;* but this does not very clearly define it, to my mind, as by viscosity, we understand from general definitions, a certain clamminess or stickiness after the manner of varnish, treacle, hot glue, or oil. Bacon gives a definition of this when he says, "Holly has so *viscous* a juice that they make birdlime of the bark." The physical state of both water and air at ordinary temperatures appears to me in experiments to be better expressed as above, by *gelatinous* than *viscous.* The gelatinity I conceive to be somewhat after the nature of cold jelly scarcely capable of settling, and of a higher degree of mobility. This property is apparent in water in the adhesion with which it supports the spherical conditions in drops resting upon solid or liquid surface, in which the drops do not appear to enter into or adhere with any similar property to that observable in really viscous bodies, unless some force is applied. Further, after adhesion is effected with water by any means to a surface it becomes complete, and equally resists removal as we find is the case with gelatinous bodies. Some experiments of Count Rumford led him to infer that a pellicle or film was formed at the superior and inferior surfaces of water, by that which he concluded was a special natural cohesion of the fluid at the surface, and by this he accounted for that peculiarity of surface resistance by which bodies that are specifically heavier than water will float upon its surface. This property of *gelatinity* that I propose to adopt, will offer the same qualities of surface resistance, as we find that *jellies* resist at the surface, but viscous bodies do not. Reserving for the present the discussion of some causes of surface resistance, I may mention one of Count Rumford's experiments. He poured a quantity of sulphuric ether upon the surface of water in a vessel, and placed gently in the ether several bodies of greater specific gravity than the water, as a needle, particles of tin, and globules of mercury. These all descended through the ether, but floated upon the surface of the water. It was quite observable that these bodies pressed down the surface of the water, and little hollows or bags were seen beneath them. These floating bodies were made quite clean, and the water adhered persistently to them when they had once entered it. Indeed similar resistances were observable when the like bodies quitted the water. It is therefore from no material repulsion inherent in these bodies, but quite the reverse of this. A particle of water will rest upon water surface similarly to a solid body, as some experiments by Brewster show, and that which appears quite contrary—the water will creep up and adhere to the same bodies which resist its surface. This is quite explicable by the same principle of gelatinity that promotes and supports continuity after contact by a kind of static firmness of surface which rejects first contact. The gelatinous principle in question is evidently due to certain conditions of cohesion not present to the same extent in all liquids. The same cohesive principle is possibly maintained in ice, as is witnessed in glaciers where the ice forms stand up with remarkable rigidity, and are quite brittle if struck, yet they bend to the rocks past which they are compressed under the constant force of gravitation. The gelatinous construction of water that I propose is one cause of a special form of resistance in pipes to the free flow of water, upon a principle of fluid motion to be considered. It also gives to fluids which possess it, a special kind of rigidity by which continuity of surface is very persistent, and this is one cause of the tendency to division in motions of undulation where the surface is compressed horizontally by any small forces. These general conditions will be hereafter considered.

c. The experiment by which I tried, to investigate the relative comparative amount of gelatinity in liquids for my own satisfaction, and to endeavour to discover the cause of the extreme static rigidity of water under certain conditions, which I will discuss further on, was to cause the liquids to flow over a small bridge in a channel, and to measure the heights of the head above the bridge relative to the velocity of flow. To compare water particularly with some acknowledged viscous fluids I selected two liquids, one a varnish, a little of which placed between the fingers held them together with considerable force, and for the other linseed-oil. I selected these two liquids, oil and varnish, as I found them used previously in experiments to determine the viscosity of fluids. I constructed a small apparatus, consisting of a channel of zinc about seven-eighths of an inch square, and a foot long, which was connected with a reservoir so as to obtain a regular flow in a current of the liquid from an aperture of the width of the trough, and one-eighth of an inch high. A bridge across the channel was placed inside at one inch from the end furthest from the opening, which was made of a thin piece of zinc. This bridge stood up from the bottom of the trough a quarter of an inch. Above the bridge I placed a micrometer-screw, by which I could read to the thousandth part of an inch, and which was connected with a point to reach the surface of the liquid. I tried the two viscous fluids named, and the water, after cleaning and thoroughly drying the trough between each experiment. In the flow along the channel I found the head of water at its highest point, which was nearly over the bridge, measured by the micrometer screw taking the average of ten experiments about ·13 of an inch; the head of the highly viscous varnish measured about ·12 of an inch; the head of linseed-oil, ·11 of an inch; showing the water to be the most rigid or self-supporting, which I assume to be derived from a certain molecular polarity which produces the property of *gelatinity*. I shall be able to show in experiments further on that water in some cases acts as a colloid. In Coulomb's torsion experiments to discover the *viscosity* of various bodies by the resistance offered to the circular oscillation of a tin disk, oil was found to be *seventeen times* as viscous or intermolecularly resistant as water. Therefore the quality of rigidly standing up or *beading* in the above experiment, which is greater in water than in oil, must be from a fluid property in excess in the water.

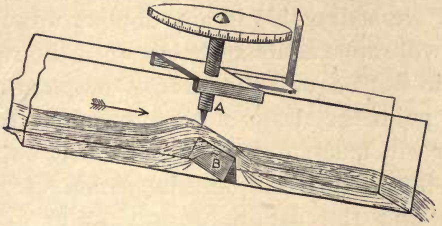

Fig. 9.—Gelatinity Apparatus.

d. The *gelatinity* or static mass rigidity of air, which I also ascribe to a weak polarity, may be shown in the slowness with which its elasticity will react in a free state. This may be seen in the following very simple experiment, by taking one of the thin tissue india-rubber balls or balloons which are blown out for toys for the amusement of children. The one I experimented upon was 9 inches in diameter. If we strike this in any oblique upward direction in quiescent air it will move against the air until the resistance of the compressed air in front equals the force of momentum given to the ball, and at this point the elastic static force of the air in the balloon and the free air will react, and the balloon will be reflected on its course; that is, it will not complete its paraboloid, in ratio to its projectile force and the action of gravitation upon it, in proportion to the quantities of resistance it receives *seriatim*, but will fall to the ground at an angle inclined to the direction from which it was projected.

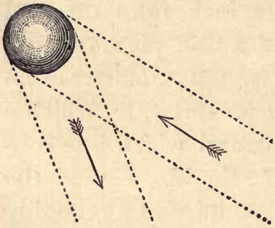

Fig. 10.—Aerial Gelatinity.

If the term *gelatinity*, or the reasons which induce me to make use of this word, should appear ridiculous to some, I wish only the property of static mass elastic rigidity of water and air to be observed as important to the elucidation of some phenomena to be hereafter considered, which gives these fluids in some cases static forces that appear very relative to homogeneous solids.

13. Remarks.—*a.* The part of this chapter that relates particularly to the static equilibrium of the molecule in a fluid was written to offer the necessary conditions for my conceptions of sound motions, in which I take the general principles of equilibrium for fluids that have been assumed to be present from Newton to Airy, only that I have substituted permanently *elastic* forces, which are assumed to act within the limits of the body of the molecule or atom, in place of *repulsive* forces, which are assumed to act without matter at indefinite distances for which these elastic forces may be considered to be motively equivalent. The fourth section of my work being now withheld for reasons stated in my preface, these matters as regards other parts of this work might have been omitted, only that they are now unavoidably mixed with the entire work, and that they give the theoretical ideas upon which

I assume a general state of molecular equilibrium may exist in a fluid. I admit at the same time that the subject is one beyond my powers of demonstration.

b. The mobility of gaseous fluids in free molecular trajectories, which I assume would be antagonistic to any theory of constant equilibrium of each individual molecule, appears to gain reasonable strength from the evidence offered in the beautiful experiments of Mr. William Crookes, of passing electrical currents through tubes or vessels exhausted, nearly as far as possible, of air. In these experiments radiant matter is projected from the negative pole, which is not only made visible by the radial direction of the rays of light, but also evident by the performances of work by the force of the radial projections. There are, nevertheless, conditions in this case which do not by any means assure us that the radiant matter observed is of the residual gas in the tube, as we find it is generally assumed to be, or even if it were this, the motivity observed might be constantly induced by electrical forces supplied directly from the battery, so that this could be scarcely imagined to be a case of *free* trajectory of the residual gaseous matter present. However, we must admit that it is an evident case of one form of projection of molecular or atomic matter, but whether it is of the residual gas in the chamber or of the matter of the negative pole is quite another question. In sparks thrown off conductors in experiments with our common frictional electrical machines, it is quite clear that pieces of the metallic conductor are projected, which inflame and produce large sparks in the air. If we reduce the pressure of the surrounding air so as to produce the ordinary exhaustion of a Geissler's tube, Mr. Crookes has found that *platinum* is then strongly projected from the negative pole, so that it darkens the glass all around it; showing that this is evidently still a *metallic projection.* But Mr. Crookes has found that in higher vacua there is no evidence of projection of the matter of the negative pole.[1] In this last case it is quite possible that the platinum itself under the nearly perfect release of pressure that these higher vacua produce, is severed into separate atoms at the instant of its projection, and that this as elastic matter (4 prop.) forms during the continuity of the electrization a perfect gas (*platinogen*), and whether the gas so formed is deposited upon the positive pole or the containing vessel, or not, will depend upon the strength of elastic forces due to its electrization, into its material

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[1] *Phil. Trans.* 1879, § 628.

atomic attraction for other matter present. In this instance also the constant radiation may be established from the negative pole by a system of convection currents or vortices in the vacuum chamber, by which the gas performs a definite circuit. I assume in this hypothesis that the metallic gas is invisible; but that it excites by its friction the medium of the fluid residual air or gas into which it is projected, and, in like manner, by its friction it also excites in its return current the body of the vessel in which it is contained, to cause its phosphorescence, the return current being in all cases possibly more condensed than the gas projected from the negative pole radially in the first instance. So that upon the whole I do not see evidence that this is a motive projection of the residual gas in the tube, but rather that this gas is the medium in which the projection occurs. It is reasonable to assume, nevertheless, that the projection will engender convection currents of the gas present during the atomic projection of the matter of the negative pole, and that such currents may probably return the matter projected to this pole in some cases after projection, or, if the resistance of the medium is great and condensation occurs, deposit it near this pole.

c. In Mr. Crookes' earlier important experiments with light and heat in his well-known Radiometers we have all the general properties of the forces employed materially changed from conditions given above for intense electrical forces, as in the radiometer we have to deal with forms of force that exhibit much less intensity of action. For radiant heat and light, such as is active upon a radiometer, we may possibly assume a force only sufficient in these experiments to affect the equilibrium of the fluid (vacua) inclosed in the vessel in a manner similar to that which was formerly assumed for radiant heat and light by the undulatory theory, and that is now assumed for it in other cases when discussing spectroscopic matters, phosphorescence, &c. However, it appears evident that undulation would not produce the direct forward mechanical force observable in this case. But possibly the equilibrium of the molecule may be conserved if the rays of light or heat induce the formation of direct molecular *polar rays*, as I conceive may be possible in free elastic residual gas upon conditions stated in 7 prop. *h*, so that such rays may act as forces directly from the vessel to the mobile vane, with quite as great or even greater force, from the assumed elastic rigidity of the atoms forming the ray than could be assumed for the propulsive force of the free molecules present, whose weights must be conceived very small, even if they possess innate forces to be projected entirely in one direction at one time, and their velocity must be immense to give by this projection of their masses, momentum sufficient to overcome the inertia and resting friction of a relatively enormous solid body, many millions of millions of times the molecular weight of the projectiles, such as we must assume is the weight of the connected vanes of a radiometer, even assuming that a molecule or any other body could impress force and retain its momentum equally for reflection back to the sides of the vessel, with force and velocity equal to the original projection, which in this case it must be assumed to do or the whole of the projectile matter would soon rest against the vane. In this difficult case some writers appear to hold the projectile-molecule theory, or corpuscular theory as it was formerly called, for radiant matter, and the undulatory theory *also* for separate cases; this appears to me quite inconsistent, or at least so until some process of reconciliation has been offered for these separate quite distinct principles of molecular motion of radiation forces. In the assumption of the constant presence of polar forces offered above we have the conditions of a form of motion that is evident in some cases which we have not for any case of free motive propulsion of a molecule or any other body, except it is, as in the case given above *b*, induced by the constant force of electricity.

d. Upon the conditions offered above the equilibrium of fluid matter may be conserved under the impression of exterior forces that may affect its interior construction during their impression, by inducing at the time greater activity of polar forces. The molecules of fluid matter being acted upon in such cases as masses of soft iron to electrical induction, but in this case by the unknown actions of heat and light. If we assume these forces as *polarity inducing*, or strengthening forces that pervade the system of matter upon which they are impressed, and that the atoms or the molecules of bodies are symmetrical, and endowed with like polar forces, the union of such atoms or molecules in equilibrium can produce none other than a symmetrically constructed mass. This state appears also as far as possible evident when by vision we can first detect the construction of pure matter, as for instance in the symmetrical crystalline structure of all the metals, iodine, sulphur, &c., and the same by inference for certain liquids which are shown by the polariscope to take the properties of crystalline bodies. So that it is only when we lose sight of structure, by the condition of fluidity, and when the mind is left in darkness, that we can rationally *assume* symmetrical structure of matter to disappear; and it would almost appear that if matter changes its *structural laws* at this precise point, that it has been in some way created relative to the faculties of man to permit the free action of his imaginative powers.

e. For the general conditions of equilibrium in a fluid, I think that whatever may ultimately be consistently thought to be the principle of projection of radiant heat force, in any form, that the same will be applied to light, as these forces are so evidently united in their motive principles; as the conditions of similar refractions, reflections, and polarizations evidently show to my mind. Neither do I think that it is probable there are *two forms* of heat motion, that of radiation and that of conduction, as it is much simpler to suppose one mode of action for one force; *conduction* very possibly being a form of less intense *radiation*, which is restrained by resistance of the inertia of the matter affected by it. Extending this idea a little, it is also most possible that one form of motion exists for both heat and light under all conditions. If I should at any time be able to discuss my experiments on light and heat, from which I was unavoidably diverted, I will return to this matter. It would here be quite out of place to discuss the subject further than is necessary to support my propositions of the constitution of a fluid against what I imagine may be a transitory science (1880), in so far as its theoretical deductions are concerned, but not its practical uses.

f. For the equivalence of heat to mechanical force, we may assume that if heat forces act on separate atoms and molecules as proposed (4 and 5 props.), or by any means so as to increase the elastic forces of the atom or molecule by a system of movement acting contra to forces we may call chemical attractions, as I have suggested, here is possibly work that will fully represent the heat lost in its performance, whether it be in separating the millions of surfaces of the atoms of a powerfully cohesive mass of platinum, slightly against its powerful chemical cohesion, of near central contact of its atoms, equally, to the force shown in dissipating solid carbonic acid to vapour extension where chemical cohesion is represented by a small force. The greatest motive effect probable from heat in any case will be explosion, or elastic vibration, which will become finite by the influences of surrounding resistances or in new conditions of elasticity. It further appears to me that the most important physical function of heat is not to produce the palpable exterior surface forces we witness as personal temperatures; as such exterior forces may be present or not, for in this I assume that if the atomic attractions *inter se* become negative or minus quantities with respect to the elastic surface force that I propose (4 prop.) the heat force as *temperature* is entirely lost to perception, or becomes latent as it is termed. Whereas if the heat force is only sufficient to increase the elastic pressure, and not to overcome central attraction, such pressure becomes by movements producing separate frictions on every atom, active in the general expansion and outwardly possibly *vibratile* upon the elastic envelopes of the atoms or molecules, disturbed from the equilibrium of rest surrounding them, and is thus as temperature force, rendered motively free to communicate vibration as radiant heat from body to body, so long as the heated body is changing its state by either increase or decrease of heat. If the body expands, and heat force is lost as palpable temperature, the elastic forces have placed and may possibly retain, the atoms in positions in which the central attractive forces of the atom act exactly as a body placed upon an elevation above the earth. The active energy of the heat force apparently lost being conserved by a position that can restore the outward, palpable heat, by friction of *attractive precipitation* in the atom falling to its former radius of attraction. This matter, as before mentioned, has been ably discussed by Professor Balfour Stewart so far as regards the potential of molecules.

CHAPTER II.

CONDITIONS OF LIQUID SURFACE:—CAPILLARY ACTION—TENSION OF FILMS—BUBBLES—BOILING—EVAPORATION.

14. Definition. It will be convenient, before the discussion of surface forces, to define the words *Tensile* and *Extensile*, as I wish to use them in this chapter and henceforth. By *tensile* I intend a disposition of the parts of a system of matter to draw themselves together, as a stretched drum-skin does. By *extensile* I intend the reverse of this, or the disposition of the parts of a system of matter to separate and thereby to engender external pressures.

15. Equilibrium of Surface. *a.* The still surface of a liquid at a distance from any object presents to the vision a perfectly level smooth plane. The perfection of the equilibrium of a liquid surface may be demonstrated by the following simple experiment. Take a dish of clean water, the surface of which is visibly still, and prick the centre of the surface with a fine needle; this will instantly send forth a wave extending in circumference from the centre to the edge of the dish, which may be seen by surface deflections along the shadow of a window bar, or the edge of any straight object placed against the light. Upon removing the needle from the surface of the water, a similar phenomenon may be observed. I may note that in this experiment the entire point of the needle to be immersed need not exceed ·01 part of an inch in diameter, that is, occupy a space upon the surface of the water greater than ·0001 part of a circular inch. The diameter of the circular dish in which I made the experiment was 12 inches. Therefore in this case the liquid surface was thrown palpably out of equilibrium by a force of increase or decrease of area of surface by ·00007125 part of the whole, neglecting capillarity, which was possibly nearly equal in the elevation of the water about the needle to the volume of the needle immersed; the point of the needle in this case penetrating the water for a very small depth only. This puncture of the surface produced a very visible motion. It is probable that the same form of motion would occur with ·01 part of the absolute displacement, therefore $\frac{1}{100}$ part of the surface strain, but in this case we should need special optical means for its detection. By further experiment to discover the limit of smallness of displacement necessary to produce a visible disturbing influence, a glass thread of ·00025 part of an inch diameter was withdrawn from a still liquid surface. I could in this instance by attentive observation detect no surface disturbance by reflection at a distance greater than 3 inches from the point of immersion.

b. I have already considered some of the conditions upon which the rigidity of a liquid surface may depend, namely, the minus elastic mobility of the surface, and also the gelatinity as a functional quality, particularly of water (12 prop. *c*, page 36). These principles previously considered, if true, relate to the surface as a part of a general mass. The conditions I now wish particularly to follow relate to the surface as a separate system. The great resistance of the surface of water, as shown by Count Rumford's experiments, mentioned in the last proposition, assures us that we need the co-operation of every auxiliary force that may act to fully account for the great surface rigidity we observe in certain liquids, particularly in water.

Extensile Forces on Liquid Surface.

16. PROPOSITION: *That every level surface of a liquid is held in equilibrium by attractions of the surface molecules to each other, and to those beneath equally per area of the molecular surface in contact; that such equilibrium acts as a condensation upon the surface of a liquid, thereby increasing the molecular surface density.*

a. Let a liquid be constructed as proposed 8, 9, and 11 props. last chapter, then any plane surface of the liquid may be represented by a plane of molecules AA′, Fig. 11, whose lateral attractions (11 prop. *h*, page 33) will be equal between every molecule, and

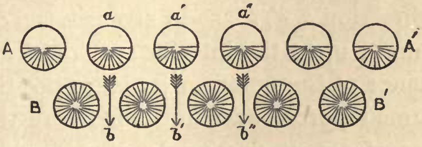

Fig. 11.—Surface Extensile Forces.

therefore place this part of the system in lateral equilibrium, and in gravitation equilibrium also if the surface be horizontal, as before proposed.

b. To represent the action of cohesive forces by which a plane becomes *more dense*, let the plane BB′ be one of a liquid system having similar globular molecules in every direction, so that it rests in perfect equilibrium with surrounding molecular forces upwards, downwards, and laterally; then will the molecular attractions of the plane AA′ be out of equilibrium in vertical direction, as this plane will have superior attractions in proportion to the extent of molecular surface of the cohesive liquid system downward towards the plane BB′, and *no* upward attraction of cohesion. Now as the molecules AA′ fall upon the elastic interstices of the plane BB′, as shown for three places by the arrows *b*, *b*′, *b*″, indicating the equation of the direction of the attractive molecular surface forces in the molecules *a*, *a*′, *a*″. These attractive forces will therefore be equal to *pressures* that are constantly active between the molecules B to B′ to press between them, and thus to condense the surface of the liquid here assumed as a mobile molecular plane.

c. By the same forms of attraction as discussed above, the molecules A A′ will also by their downward attractions, which act as compressions upon the first surface of lower molecules B B′, render this lower surface also, by the molecular intrusions in the stratum below, *more dense* than any lower part of the liquid system, and this again by like compressions more dense than one still lower; the equilibrium about every molecule being at the same time preserved as far as possible. This is quite irrespective of gravitation.

d. In like manner, by the molecular conditions proposed (11 prop. page 30), and of elastic pressure proposed (10 prop. page 28), every liquid surface upon which there is exerted any pressure whatever by a motive plane, will be similar to the surface pressure of *a*, *a*′, *a*″ intrusive in the direction *b*, *b*′, *b*″, as in all cases the exterior molecule will be driven into or upon the interior system of molecules against the elastic resistance of the polar, or pressure positions of the molecules in the assumed porous system. Therefore the lower surface, and the lateral surfaces at any angle of a liquid contained in a vessel or pressed by a *smooth plane*, will be under *compressile strain* relative to the central equilibrium of the liquid system.

e. The demonstration of the conditions here assumed of condensation of the aerial surfaces of a liquid may be inferred in an experiment as early as Descartes,[1] and although this resembles those already offered (page 36, *b*) by Count Rumford, it may be here considered for certain particulars.

Fig. 12.—Action of Needle Floating on Water.

f. An ordinary sewing needle placed upon the surface of still water, upon which it floats in apparent contact with the water, does not break the continuity of surface or the strength of internal cohesion of the water, to follow the mass attraction of the floating needle. In this case the needle has nearly eight times the specific gravity of the water; therefore to be able to float upon it, it bends down the surface of the water surrounding itself a quantity equal to nearly eight times the bulk of the needle, so that the needle lays in a trough of the deflected surface. If we imagine in this case that the weight of the needle presses the molecules $aa'a''$ in the previous diagram (Fig. 11) into the surface of the water, the elastic resistance of the surface will be increased by the pressure, as there is no space for these extra molecules without disturbing the whole surface system; at the same time the lateral liquid retains its equilibrium by an outward rounding. This experiment further gives, as before mentioned, an idea of the general elastic cohesive rigidity, or gelatinity, as I term it, of the water, which in this case offers resistance for a distance below the absolute surface to make room for the indentation in which the needle is visibly buried, as shown in the sectional illustration above, which represents the needle enlarged $2\frac{1}{2}$ diameters. The gelatinity, as a resistance in this case, may be partly derived from the continuity of like, although diminishing, intermolecular pressures to the surface pressure which acts similarly to a continuity of surface forces upon the internal water under the strain as proposed c above; in this manner producing a density varying inversely as the square of the radius of pressure for a certain small depth. The expansions of the lateral surfaces to the hollow formed by the needle are rendered nearly free of extensile force by the curvature and continuity of cohesional contact until the equilibrium of gravitation in upward hydrostatic pressure in the

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[1] Les Météores. *Œuvres de Descartes*, vol. v. p. 187.

near parts, equals the surface resistance of the hollow surface, so that the needle, as I have found, by the elasticity of the system deflects the surface water rather less than a weight of water equivalent to the weight of the needle.

g. When the needle once enters the water it descends to the bottom with great velocity, from which we might conclude that the cohesive forces of the water, after the surface is once penetrated, are much weaker than at the surface. I anticipate that there is not a great difference in the general cohesion of its system, in the depth in relation to the surface, derived from differences of molecular attractions (5, 6, 8 props.) causing differences of density, except that there is more perfect equilibrium in the depth. The greater part of the visible difference in the case in question being the motion induced in the water by the needle after it has entered it; the principles of which, I will hereafter discuss in some more demonstrative phenomena; this proposition being entirely confined to the conditions of the surface in relation to the static liquid interior, the subject which I am endeavouring to investigate.

h. In the above conditions of the floating needle, if the surface of the water were already under tensile strain, as is now popularly concluded (1880), it would then be clear that the weight of the needle would immediately increase this tension, and unless the special cohesion of the surface were by some undefined cause very great the needle would immediately penetrate it. This would be illustrated by the instance generally offered of a stretched drum-skin; if this were stretched very tightly, a very moderate blow with a heavy body would break through it, but if it were loose or flabby it would resist a much greater blow, as the blow must be *plus the tension* present in the first case. In the case of a tension also the surface would remain nearly level when struck; in the case of molecular condensation which would have a tendency to extension (equivalent to looseness) the surface would be deflected by the blow of the heavy body without separation as the water surface was shown to be by the weight of the needle.

i. To observe the greatest amount of deflection of a liquid surface by a weight placed on the surface film it is necessary that the heavy body should be perfectly smooth and perfectly *clean*, as any particle of projecting matter would disturb the molecular arrangement. With distilled water, 60° Fahr., I have found that a straight polished clean steel wire with hemispherical ends, of one inch in length and

·05 of an inch in diameter, would just float with stability if undisturbed until oxidation commences. If there was the smallest particle of grease upon the wire a much less diameter only would be supported. If the same wire was dipped in acid so as to produce even slight oxidation it would also enter. The method that I have found best suited to clean the wire, after polishing, was to dip it in a saturated solution of caustic potash, to rinse it in clean water, and wipe it on a very clean linen cloth; it would then be perfectly supported. If it is not wiped it oxidizes slightly in drying. But I have found that if the same wire be afterwards touched by a greasy body, or one not perfectly clean, as for instance a hair of the head, ever so slightly, it will descend immediately upon being placed on the surface of the water. The necessity in this experiment for the polished and perfectly clean surface is that the water should at once perfectly *adhere* to the wire, and so retain as far as possible the molecular surface positions. I will make this matter more evident by another experiment further on (20 prop., *k ante*).

j. To endeavour to investigate the depth of surface that would be rendered more dense, therefore more resistant, by the action of cohesive forces in the liquid conjointly with the effects of gravitation upon a liquid surface, the last being assumed very small, I constructed a small scale-pan of microscopic glass that was of about ·01 of an inch in thickness, and 1·05 of an inch in diameter; this was suspended by three fibres of silk, and then immersed in a beaker nearly full of water, the pan was attached to a delicate scale-beam which turned distinctly with ·01 of a grain. By slowly lowering the surface of the water I found as this surface descended to the pan, that a resistance which first disturbed the equilibrium of the beam was observed at about ·08 of an inch below the surface; this resistance increased, but did not appear to equal ·01 of a grain until the pan came within about ·003 of an inch of the surface. In the above case the resistance, I believe, was largely due to the general gelatinity of the water; for if the pan of glass was moved upwards to the surface very slowly, resistance was detected with difficulty until it was very near the surface.

k. The general condensation and tendency to extensibility of a liquid surface may possibly be better inferred by the activity at this surface shown in inducing the solution of a solid in a liquid, by the elastic pressure near the surface causing a more perfect contact than elsewhere. This may be readily seen when any soluble body of long cylindrical form is partly immersed in a liquid which has power to dissolve it.

Fig. 13.

l. For this experiment I have tried successfully a cylinder of fused caustic potash, and a clear sugar-stick from a confectioner's. The cylindrical rods of the soluble bodies were cut in two by the solution after a certain time at the surface of the liquid; this effect is known to occur in vacuum, and is general in all soluble bodies. The illustration represents the form produced by the solution of caustic potash or sugar in this manner; in which it may be observed that near the surface plane of the liquid, the narrowest point of the engraving, there is a conic pointing; which I have found generally to commence at about ·08 of an inch below the surface in water. This depth may be the practical limit of surface extensibility by condensation for water, as it is about to this same depth that a surface of water may be deflected by a small heavy body resting upon it before submersion.

m. The conic pointing in the above case would appear to indicate that the surface condensation acts as a pressure from the surface downwards varying inversely as the square of the distance for a certain depth (·08 of an inch), lower than which depth, the general cohesive forces of the molecules of the liquid have not their inertia much disturbed by the surface forces; possibly by reason that their polar forces (10 prop.), although these are assumed weak, yet exceed at this distance the influence of surface forces due to attraction in the area of cohesion of the extreme surface molecules as defined *a* of this proposition.

n. In the above experiment it may be observed that there is a general conic pointing of the entire cylinder at a very small angle, as observable in the lower part of the engraving, which is from no function of surface forces, and is therefore quite independent of the principle discussed above. This general conic pointing is produced by the solution of the solid in the water being of higher specific gravity, and falling over the lower parts; the solution having less solvent power in proportion as it is saturated. This by itself would produce an entire cone extending to the bottom of the solid, and not the sudden conic pointing at about ·08 of an inch in depth in the water, as shown in the engraving.

o. The same conditions of surface extensibility by condensation appear to be evident in the extensile force with which oils or spirits spread over the surface of water with the great rapidity observed. In this case I imagine that the oil or spirit takes the position of the attracted surface molecules of the water, and that these liquid molecules by the superimposed attractive matter become now the interior of the surface system, where they fall to equilibrium as lower molecules, and become as other deeper parts of the water, except that the surface still retains a certain condensation in ratio to the differences of specific gravity of the water and the lighter fluid now above. I imagine, however, that in this case there is a certain amount of chemical action, in which the oil or spirit has strong attraction towards the critical fluid or vapour which I assume to be superimposed over the water at all times (8 prop. *e*, page 22), although this vapour is of too small a specific density to produce the same state of equilibrium as that of a denser fluid, that is of the oil or spirit mentioned above.

17. **Capillarity,** *defined as the action of a liquid which causes it to rise and hold itself up or to be depressed against a solid about the surface gravitation plane, may be attributed to five causes.* 1. *The force of extension by attraction into the liquid surface.* 2. *The attractive forces of the surface molecules seeking the greatest area of contact.* 3. *The elasticity of surface.* 4. *The adhesion of the liquid to the solid.* 5. *General cohesion of the liquid system. These conditions, although relative to each other in a certain degree, are taken separately in the following propositions.*

18. **PROPOSITION:** *That aerial surface forces of condensation shown in the last proposition, supported by lateral molecular pressure* (10 *prop.*), *both tending to a like extensibility, are causes of the rise of liquids against vertical solid surfaces, where the liquids are sufficiently free from cohesive intermolecular attractions to be able to touch the vertical surface at the horizontal gravitation surface plane of the liquid.*

a. If a vertical surface rising above the aerial plane of a liquid be neutral, as regards attraction for the liquid, the liquid will then rise against the solid, by the effects of molecular condensations producing extensile surface forces, as shown by the following diagrams:—

Let A B, Fig. 14, be an aerial surface of a liquid, and B C be the surface of a solid neutral as regards its attractive forces to the liquid, except by the pressure that it receives from the elasticity of the liquid mass. Then, by the condensation of aerial surface (16 prop. *a*) horizontal surface pressures will press the molecular elastic system from A towards B, and hydrostatic pressures upon the vertical solid

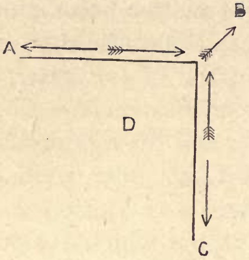

Fig. 14.—Diagram.

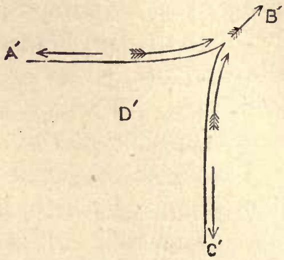

Fig. 15.—Diagram.

acting laterally will also, as hydrostatic forces, act vertically (10 prop. *c*, page 29), and will tend to press the molecular elastic system at the surface upwards to B, which will be the point of least resistance of a uniformly cohesive porous system under pressure, and the entire system by the composition of forces will be deflected in the direction of the arrow near B. This will occur although a surface rests in equilibrium (15, art. *a*) if the downward attraction of the surface molecules (16 prop.) cause a condensation by the direction of their attractions, and this condensation will produce an extension at the boundaries of a free surface.

b. The equilibrium of extensile surface forces for the inside of a flexible cubic cavity, under a state of free extensibility, may be shown exaggerated theoretically for one side at B′ in the second diagram, Fig. 15. If the vertical solid plane were continued rigid, as by upward continuation of the vertical side, B C, Fig. 14, the surface deflection would then act upwards only against this plane.

c. Under the conditions of porous molecular construction proposed (10 prop.) there can be no doubt that hydrostatic pressures will *tend* to extend and support the surface, B C, Fig. 14; but this may be otherwise partly demonstrated by pressure upon soft or hard substances proportionally to their fluidity, as defined 2 art. *b*. Thus, if we move a flat board horizontally in water with its plane normal to the direction of motion, the pressure upon the water causes it to rise upon the near forward part. If we press a long roll of butter at one end this end expands, or if we press a malleable metal the same will occur; this last instance is very important in mechanics, as the whole process of riveting depends in principle upon the malleable surface being expanded by the pressures or percussions of the blow of the hammer being active as extensile force through condensation near the point of incidence, and although the plane C B be assumed static, in this case of hydrostatic pressure, yet by the molecular porosity and elasticity of the liquid, the pressures against the plane will give the same tendency to extension as if the plane were in a small degree motive.

d. Under the conditions of this proposition, *a given liquid should rise from a level plane equally upon all bodies*, the rise depending entirely upon the molecular forces of the liquid, and not upon the solid upon which it rises; but these conditions will be modified in certain cases, as for instance if the liquid by any cause, as by the strength of its molecular cohesion, does not by its adhesion wet the solid at its aerial plane, then the same conditions of extensile surface force would cause the liquid to round outwards, that would otherwise cause it to form a hollow against a plane of resistance to the free action of its surface extensile forces. The strength of the extensile force, although small, would at all times compel a liquid to take either the round or the hollow form.

e. A liquid will, by its forces of surface extension through attraction into its mass, constantly tend to drift the matter of its surface outwards to the most free circumscribing area, where it will produce from this cause a small upward curvature against any vertical solid. But as this curvature increases, the molecules will be proportionally more nearly circumscribed by contact with others, and consequently be also more nearly in vertical equilibrium (11 prop., page 30); so that a molecule in a hollow will approximately resemble a lower molecule, there being less excess of attractive force in one direction. It is *partly* upon this principle that by the superior strength of surface molecular attractions into a level plane (16 prop.) over the more nearly circumscribed attractions of a hollow plane, that any light solid body placed upon a liquid near the edge of the containing vessel will drift from the side it receives the greatest normal pressure to that upon which it receives the least, or from the level surface to the area of greatest curvature against the solid surface of the vessel. The excess of surface curvature caused by the presence of the body will also accelerate the drifting to the most concave side, as the conditions of 20 prop. will show.

f. If we take this proposition in conjunction with the last (16 prop.) the physical law of pressure of a liquid against a vertical surface becomes more simplified for the rise of a liquid against a solid than the conditions required for the assumed surface tension of a level aerial plane of a liquid, and of hydrostatic pressure below this plane. As we find under the conditions now offered, the hydrostatic pressure to be a continuous process affecting the entire depth, therefore acting, although not quite equally, for all depths; the cohesive surface forces of the aerial plane acting also in the same direction in some ratio inversely proportional to their distances from the surface, thus strengthening the surface forces under conditions given in (16 prop.), so that the entire pressures are positive. Whereas, if we imagine a tension upon a level surface, this must be, as assumed by Segner, a force superior to the hydrostatic pressure at the surface plane, *or it could not act as a tension;* therefore upon a vertical plane rising from a liquid we should have, by the theory of surface tension conjointly with the conditions of hydrostatic pressure rising from depths below the surface, the pressures diminishing upwards to a *certain line* until they cease to act and become neutral at this line, whereas above this line there would be negative pressures, increasing inversely, or as minus pressures up to the aerial surface; conditions which appear to me, although supported by the theoretical deductions of many great philosophers, are too complicated to be possibly imagined to be possessed by the same physical body, in which we can distinguish no difference of parts and only small differences of conditions as active at the surface that are not general for the mass.

g. The above paragraphs relate only to the extensile pressures of an open liquid against an open vertical plane, the liquid surface being assumed of extensive area. Otherwise if the surface area were closed by contiguity of other vertical planes, or if it were surrounded by solids, then any elevation of the liquid against such solids by causes discussed above, would abstract a certain portion of the liquid from the part of the close area that would be at the time below the surface plane, and this abstraction would have a tendency to contract the capillary area, although the surface forces were acting as pressures upon its boundaries; but by the next proposition I will endeavour to show that the cohesive forces direct the surface pressures upon the vertical solids lineal to their planes, and this will limit the action of the surface extensile forces as outward pressures, particularly in very close areas.

19. PROPOSITION: *That the cohesive force, by which the molecules seek the greatest area of attractive surface* (11 prop.), *is a cause of the continuity of the rise of liquids against solids particularly in tubes and close areas, where the liquid surface is of limited extent, and supported by near resistances.*

a. Assuming extensile forces under the conditions given in the last proposition to cause a certain small elevation of a liquid against a vertical solid, the cohesive forces of the molecule seeking the greatest area of cohesive contact would *support* such elevation as a rigid system, but by its own action it would not elevate the system of a liquid from its level plane in equilibrium of repose; but conjointly the extensile and aerial cohesive forces proposed would cause this elevation. It may nevertheless be observed that these forces would act in no case otherwise *than as surface forces*, and would not be the entire causes of the maintenance of a liquid high up in a fine tube, as this will depend principally upon adhesion and mass cohesion of the liquid to the tube upon conditions to be hereafter considered.

b. The extensile force at the boundary of a liquid surface, considered in the last proposition, will account for the circumstances of deflection of a liquid against any vertical solid plane. The plane in this case may act in one of two ways according to the cohesive forces manifest at the surface of the liquid in relation to the attractive forces for the solid. If the solid body has no attraction of adhesion to the liquid, but an apparent repulsive force towards it, the liquid by its molecular cohesion will not rise against the solid, but will establish a rounded edge upon its own mass against it, as we find mercury does against glass. In this case the molecular forces of the liquid seek equilibrium by attraction to the greatest molecular area (11 prop., page 30) in their own mass conjointly with molecular surface extensile forces. The lateral supporting vessel, not entering into the system of attractive forces, is to the liquid *negative*, or as the continuity of the aerial surface to a plane of cohesive equilibrium, therefore the cohesive mass of mercury acts as the oil to the dilute alcohol in M. Plateau's experiment mentioned (11 prop. *e*, page 32). In the case of mercury in a tube we have outwardly a flattened nearly globular surface in gravitation equilibrium with equal surface extensile forces, the same forces producing nearly cylindrical edges in all cases in open areas as when resting upon a level non-adhesive plane.

c. In the case of mercury in a tube, there being assumed no attraction to the solid by the continuity of attractive forces in the liquid, this is drawn down by general cohesive forces upon itself according to the conditions shown (11 prop. *b*, page 30), under such circumstances there is no necessity to assume *repulsive forces* between the liquid and the solid, as the excessive intermolecular cohesion of mercury is sufficient in itself to overcome other weaker forces for the depression, in this case observable; the glass being assumed to be quite neutral to the mercury, or even if it were *inferiorly attractive to it* (*negative*) relative to its own intermolecular attractions, the same would occur. If by the conditions present any liquid is unable at first by its adhesive force to touch a vertical solid at its aerial plane it will form a rounding surface.

d. If, on the other hand, the vertical surface of resistance to the extension of a liquid plane is a solid, to which the liquid is freely adherent, the force of this adhesion may be equal to that of the intermolecular forces of the liquid *per area*, or greater or less than this. We will take a case in which we may assume them greater. Then, by the superior attraction, the liquid surface in equilibrium will by its attractive forces, form a hollow circular curve inscribed in the angle of contact of the vertical surface of the solid and the horizontal surface of the liquid (11 prop., page 30), if we neglect the influence of gravitation, which would enter with a certain force into composition with the molecular surface forces proposed.

e. Now taking the above conditions, but admitting the constant action of gravitation to be sufficient to materially reduce the surface rise of a liquid system against an adhesive solid. If we assume a surface extensile by condensation, it may be equal as a supporting system, and give extensile rigidity to the surface plane of the liquid; that is, it may be inversely proportional to the depressing force of gravitation, and the molecular attractive forces of the liquid surface system simply may be manifest through the composition of all other forces present, and the area of surface be a circular concave by the free action of its molecular surface attractions alone. This would appear to be a very general condition of capillarity in small areas. Under these conditions a liquid resting against a vertical attractive surface may be theoretically satisfied by principles shown in the following diagram:—

Let A be a liquid mass, the general conditions of which will be as those proposed in (prop. 16).

f. Let B be a vertical surface having equal molecular attraction or adhesion upon the molecular system as upon the liquid at A. If we omit the action of gravitation, and assume B to be wetted by the liquid to ensure cohesion; then the surface molecules seeking the greatest area of radial forces about their centres as given in (11 prop.) would be in equilibrium at *a b c d e*, the liquid being contained within the angle produced by the lines A B by an inscribed quadrant of a circle.

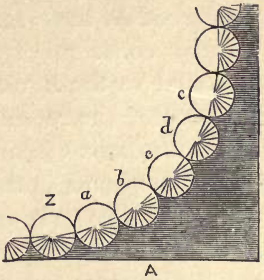

Fig. 16.—Cohesive Surface Forces.

g. If gravitation forces were active in a superior degree to surface extensibility by condensation, or resistant solids were too distant to support the general cohesive surface system, then the first molecule *a*, *held on a plane* of repose, would not be in symmetrical equilibrium with the next molecule upon the plane *z*; this molecule *z* would be therefore pressed outwards from the angle upon the plane, so that the curvature would be more nearly elliptical, and the whole system would be under elastic strain, in which the extensile strain upon the surface of the liquid would permit the support of a certain mass of liquid only in the angle by surface molecular forces against the action of gravitation, which would be active in bringing down the entire system to a hyperbolic curve.

h. If, on the other hand, we assume the point *z* to be firmly supported, this would maintain the liquid hollow with greater rigidity, or if another plane were elevated above the liquid vertically at *z*, this being opposite to the first at the same angle as A B, these angles would then conjointly maintain the liquid with greater rigidity by completing a semicircle of curvature of the liquid surface between them. It will be also seen that, as by my theory, the surface molecule always seeks the greatest area of surface contact with a certain force, and that this is best satisfied by the smallest curvature, and by this also the surface extensile forces are deflected to positions lineal with the solid against which the liquid rests; so that this extensile force becomes nearly inactive upon the solid to press it outwards. At the same time by the strength of cohesion the surface hollow tends constantly to decrease its radius with a certain force. It is on this principle that two free globular solids floating upon an adhesive liquid, when near, are drawn together by the strength of surface cohesion in the liquid molecules seeking the greatest area of contact by internal curvature. The same also occurs when the floating bodies have attraction for the liquid inferior to the intermolecular cohesion of the liquid, where an outward rounding is established in the liquid against them, as in this case also by the depression of the rounding surface the molecule of the liquid finds the greatest area of contact in the approach of the floating solids. On the other hand, in the case of two free bodies floating on a liquid, the one adhesive to it and the other non-adhesive, the hollow liquid surface of the one coming against the rounding liquid surface of the other, the greatest area of molecular contact is then found in a plane, which, as a rigid system of surface, acts repulsively to the approach of the two bodies (18 prop.) which would upon approach cause, by the opposition of surface curvatures, a less area of molecular contact.

20. PROPOSITION: *That the cohesive forces of liquids are directly proportional to the nearness of the parts. So that upon a concave or a convex surface the cohesive forces constantly tend to make the radius of curvature less, acting with elastic surface forces equal to the strength of the cohesion of the liquid to produce this result.*

a. Referring again to Fig. 11, page 45, we may assume that the plane of surface molecules A A′ will be held in attractive positions by the plane B B′; the molecules A A′ intruding by their attractions within the plane B B′ upon their lower sides, but that in the open space above the surface A A′ there are present no such intrusions; therefore the tendency of the surface molecules would be to draw themselves together if not resisted by the attractive forces acting in the lower plane. Now, if upon the level plane these molecules exert a certain lateral attractive force towards each other, although this force be effectively neutralized by the attractions of the lower plane, yet if this plane by any cause be deflected so as to become concave, the surface molecules would then exert more lateral attraction from their nearness, and if the deflection be convex, they would exert less lateral force from their greater distances. We may further conceive that in relation to the plane below, the greater the concave deflection, the nearer the surface molecules would arrive at a state of circumferential equilibrium (11 prop.), therefore press less by their plus attractions upon the lower plane, and the more the surface was deflected in a convex form, the less would be the lateral attraction of every surface molecule about its centre, and the more it would press downwards, so that the level surface would represent a condition of cohesion under a constant strain although in equilibrium, that a greater curvature, either concave or convex, would partially release in one direction.

b. As I have assumed every molecule of a liquid system to be surrounded by elastic forces (8 prop.), the surface system of liquids must necessarily be elastic also. Further, as in cohesive elastic masses, if one plane of strain is under deflection by a force that neither increases nor decreases the general equation of forces in the mass, then the elastic compression on this plane must be somewhere compensated by an extension in another part. Thus, if we assume the attractive forces upon the molecules of a concave surface film in a capillary tube to be a self-contained system, then by the conditions given in the last proposition, if this surface attracts the molecules so as to produce a hollow with a certain force, which at the same time compresses the liquid laterally at the surface, there will be somewhere in the system an equivalent extensile force which establishes the elastic equilibrium. This matter may be roughly demonstrated by an experiment as under.

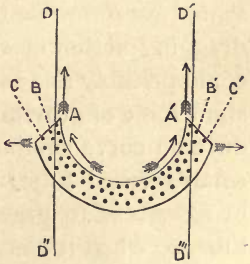

Fig. 17.
Elastic Position of Surface Forces.

c. If we take a piece of vulcanized india-rubber of the size represented in the engraving, say 1¼ inch by ¼ inch, and of any thickness, and make dots over the surface with a pen at equal distances in three lines, and now bend this into a hollow as in the engraving by a force upon the india-rubber, assumed to represent the lateral attractive surface forces of the molecules in the concave area, inducing them to make the greatest area of contact (11 prop.); then the molecules on the surface plane A A′ as they are drawn together will be condensed, and have thereby a tendency to extend upwards upon the surface D D′ at the termination of these planes, and the molecules below the surface, retaining their polar or pressure positions, will have also a tendency to distend outwards from the axis of curvature, so that on the surface molecular plane under internal curvature there will be a kind of tension caused by attractive forces, but at the terminal edges of this surface there will be an extension which will press the liquid further up the surfaces represented by D D′. On the planes below there will also be an extension that will press the liquid against the tube, the extensions at the upper edges and lower planes producing an inflection of the surface plane. Therefore that the entire liquid acts as a tension to draw the sides of the tube together and rise in the tube, is that the surface molecules seeking the greatest area of contact act always with force to decrease the area and increase the curvature, at the same time the surface extension A A′, Fig. 17, being directed upwards by the cohesive surface forces lineal to the plane of the vertical surface, and the lower extension B B′ C C′ pressing matter towards the periphery of the tube to *supply* the surface extensions, but with small force only. It is important to observe in the above case, that the cohesive forces which draw the surface molecules to closer curvature, and thereby, by the elasticity of the system, cause them to *appear* to act as a tension of the surface upon the vertical solids against which they are placed, act at the same time as an *extension of this surface at its boundaries* in the same manner as indicated 18 prop. *b.* It will be seen that the theory here proposed, in which the contraction of capillary areas is the result of the elasticity of the surface forces only, is quite opposite to that of Monge, wherein the surface molecules are represented by *lintearia,* the separate molecular tensions resembling the dragging action of the links of a free hanging chain. If a liquid could possibly rise in a tube under such surface tension, with the constant force of gravitation pulling normal to this, there would be produced a curve similar to that observable at the capillary surface in a small vertical tube; but exactly the same surface form is produced in *opposition* to gravitation when the liquid is *suspended* in a fine tube, or when it rests horizontally in one, so that the surface curvature observable in capillary action in close areas is independent of gravitation, or it may be so.

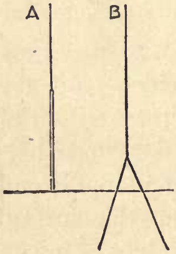

Fig. 18.
Internal Capillarity.

d. By another experiment I was able to observe the extensibility of a liquid plane immediately below the surface. If we split off a very thin scale of talc, this will be found sufficiently flexible to be folded upon itself, and if the fold is rubbed down upon a smooth surface with the thumb nail, the talc will be broken and remain quite close, so as even to possess a certain elasticity of closure. If we now cut the doubled talc with scissors, into a square of half an inch or so, leaving two projecting points at the ends of the fold, we may suspend this, by two fibres of silk, so as to leave the free edges downwards. If the talc planes are now lowered in water, upon first contact the water will instantly rise between them, and if the planes be open they will at first be drawn together, but if we continue to lower them we find that at a certain point they will open out, and as we lower them more they will open more, the effect being the same as that produced upon the dots on the india-rubber in the previous experiment, where the lower dots are opened out in the same ratio as the upper ones are closed.

e. Upon these principles every concave surface system will have an elasticity which will tend to press the liquid against the tube or close area, at a plane not distant below the surface, and at the same time the cohesive surface forces will constantly bend over the extreme surface; under these conditions the extensile pressure would increase the adhesive and cohesive forces in the same ratio upon the solid (18 and 19 props.), and the liquid would be drawn by near attractions through pressure and its increased density to the area of nearest contact by continuity of forward pressures until the elastic surface forces became in equilibrium with the resistances and the general cohesion of the system of supply; or, if the system were vertical to this strain, and to that of the resistance of friction and of gravitation conjointly; as, for instance, if a free drop of adhesive liquid were placed in a conical area, it would move towards the vertex of the cone by the forces here defined being more active in the smaller radius of surface curvature, as also by the surface density impressing the greatest lateral force, every movement at the same time by surface deflection increasing the curvature; or, if it were connected at the base of the cone with a mass of liquid, it would move towards the vertex until the liquid surface forces equalled the resistance of cohesion, friction, and gravitation combined.

f. If we now take the conditions of a *convex* liquid surface; in this case the surface molecules will be farther apart, therefore their attractions to each other will be less. But as the cohesive forces to the mass will be now further out of equilibrium with radial forces, these attractions will be proportionally stronger on the mass system beneath. Further, these outer molecules will, by the angle of contact on those beneath, have greater *force of intrusion,* so that as the convexity increases the surface extensile forces will increase also.

g. The free aerial elastic surface forces here assumed for convex areas by the conditions offered above, will have a certain tendency to extensibility; and if we consider them as being impressed on a certain stratum of neutral liquid they will be active upon one side, that is, upon the upper or aerial side of the stratum taken. Suppose the extensile force upon the aerial surface by itself to be equal to a force that would increase a small circular area by $\frac{1}{4}$, then the stratum of the liquid considered would be in extensile equilibrium from this cause when its exterior superficial area exceeded its interior by $\frac{1}{4}$. It is clear that in a thin stratum this would produce a regular spherical curve of rapid curvature, if the surface stratum were free to extend in the ratios of the extensile forces present.

h. By the conditions just proposed exterior extensibility of a limited small mass would find static equilibrium at a certain outward spherical curvature; and if we were to consider the position of any lineal plane of a liquid surface at this curvature in equilibrium as equivalent to that of an elastic lineal solid body in a static position, we might then compare it with this body. For instance, taking one direction of the spherical surface of the liquid at its assumed curvature of equilibrium to be represented by a narrow semicircular steel spring of perfect flexibility, as a part of a watch-spring sufficient to form the letter C, then we should find that, to straighten out this spring to a flat plane, it would take a constant force nearly proportional to the distance moved in straightening it about its axis of curvature.

i. If a liquid surface under the conditions given above were free from lateral attractions, the extensile surface action of liquid (16 prop.), acting rapidly, would extend the upper surface considerably, and bend it over at the boundary edges of this surface. Thus if the liquid formed a thin stratum over a small part of a solid horizontal plane, the edges of such stratum would be projected forward to the front of the liquid over the area of contact upon which it rested. Under these conditions the surface extensile forces would be placed in equilibrium with the forces of adhesion of the liquid to the solid, and when the liquid became in equilibrium with the action of gravitation upon the system it would cease to move forward outwardly over the plane, but retain its beaded edge stationary. This will be the ordinary condition of a small volume of liquid resting upon a level plane, deposited without pressure, as, for instance, dew or drops of water upon glass. There is no doubt, that, by increasing the attractive force for adhesion of the liquid to the solid plane, as, for instance, by chemical action, as by the presence of an alkali spread over the glass, the circumstances may be modified, but not sufficiently so as to preclude the possibility of clear observation of the principles here shown, as we may clearly conceive from the action of a drop of water falling even upon water, near the capillary edge of a vessel, as observed by Brewster, in which case the drops roll over the surface, and do not enter it at once nor spread upon it immediately.

j. The following diagram will represent the compound conditions of surface extensibility, cohesion, and attraction in two cases. Let

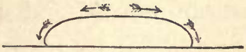

Fig. 19.—Drop on a Plane.

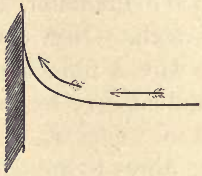

Fig. 20.—Capillary Edges.

Fig. 19, represent the surface extension of a liquid, the cohesive forces being brought into equilibrium on a plane surface of a solid; the same conditions hold where the surface is in a tube if there is present no excess of external attraction over mass cohesion to the vertical solid, as before observed, so that the liquid cannot touch the aerial plane. Fig. 20 represents by the direction of the arrows surface extensibility, attraction, and general conditions of capillarity against a vertical attractive solid, or one at first neutral and previously wetted to produce a hollow surface flexure. The surface *extensile* forces in these illustrations are of the same values in both cases, although derived from different causes, the internal extension being caused by *crowding*, the external by *intrusion;* they are also conditions common to the same liquid: in Fig. 20 they act directly normal as a pressure upon the vertical plane, upon which they are deflected; but in Fig. 19 they act parallel to this plane, and in like manner are deflected by its adhesion; the intermolecular cohesive forces of the liquid (11 prop., page 30) being at the same time strengthened by approach. I anticipate that inflection of a liquid surface to a very small internal curvature by capillary action produces a relatively powerful extensile strain by increase of density upon the extreme surface; so that in this case the inflection will be equal at the surface, to increase the density of the liquid system, to the same surface conditions as are presented by a considerable exterior pressure upon a level liquid plane.

k. The similarity of elastic deflections, therefore, of extensile forces in a curved surface under the two conditions, where a surface is made round and hollow, may be shown experimentally by raising and lowering a floating body on an open surface of water; this may be conveniently done by lifting the needle or wire described 16 prop. *f*, which will form a very interesting experiment quite in opposition to the theories generally assumed for the conditions of small dense floating bodies.

Take a polished wire as before, of an inch in length, and of about ·04 of an inch in diameter, well cleaned with potash liquor, and wiped on a clean cloth. Now carefully attach near to each end upon one side of the wire a fibre of cocoon silk by means of shellac varnish This attachment may be immediately dried if we place a hot body a short distance over it. The wire may then be suspended horizontally by the fibres to any support. Place the suspended wire in the centre of a vessel, and pour water into the vessel until it nearly reaches the wire. If we now take a syphon formed of a fine glass tube, and filled with water, and place one end of this deep into the vessel already described, and the other end in another vessel containing water; then by raising or lowering the second vessel we may very slowly raise or lower the water to or from the suspended wire. If we now carefully raise the water, the wire will make contact and float exactly as in the experiment illustrated in section, Fig. 12, page 47, the silk fibre remaining quite loose. If we now gently lower the water, the wire as it comes above the surface will draw the water up with it, as in other capillary phenomena. When the elevation is at a certain point, the same form of curvature, but inverted, will be produced at the surface as we witnessed in the depression. This is shown in the illustration, Fig. 21 above, which may be compared with Fig. 12. These operations of lifting and lowering may be repeated any number of times with care, without removing the wire from contact with the surface. This experiment is valuable in another way, in that it shows that molecular surface displacements which produce capillary

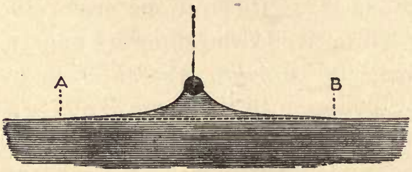

Fig. 21.—Floating Needle lifted from Water.

phenomena are not instantaneous, for if they were so it would be impossible to raise the needle above its normal contact plane upon the liquid surface.

l. Under the conditions here proposed, that of increasing the elastic rigidity of surface in a liquid upon a solid, by increasing the radius of curvature contra to the naturally extensile surface of a liquid, either by crowding or intrusion, it may at the same time be observed, that the capillary action will depend mostly upon the elasticity of the system of the liquid, which may be taken in conjunction with the strain produced by inflection. A less rigid liquid system producing less surface compression at the same curvature, which would, I anticipate, in all cases be for the same liquid, proportional to its density. Thus water at 4° Centigrade would rise higher than at any other temperature by this cause, except perhaps, at the instant of ebullition, when there are present other extensile forces yet to be considered. By the same conditions also of elastic rigidity, water will rise higher than other liquids, admitting the conditions proposed for this fluid (12 prop., page 34), the same being also evident for its support upon the surface of solids (16 prop. *d*, page 45).

m. Some theoretical deductions of Professor James Thompson of Belfast, since verified, tend to show the important law, that the melting-point of a body which expands in congelation would be lowered in temperature by pressure, while a body that contracts in congelation would have its melting-point raised by pressure. Now water is of the class of bodies that expand in congelation, and it has been since shown experimentally by Sir W. Thompson that at a pressure of 16·8 atmospheres the freezing-point of water was reduced $-0^\circ{\cdot}13$ C. M. Despretz has shown that in fine capillary tubes water may be lowered to -20 C. without freezing. If this can be taken as a measure of the surface molecular compression caused by extensile force in the inflection of elastic surface for water which prevents its freezing in the tube taken by Despretz, it would indicate that the extensile forces under these conditions are as great in the tube in question as could be relatively produced by 128 atmospheres upon a level surface, supposing the surface compression to retard the freezing, and assuming the freezing to commence at the surface, as is commonly observable. I only offer this case as one quite hypothetical, as we are not very certain of all the conditions of freezing or of crystallization. Crystallization of water or other liquids appears to me to depend very much upon the facility of motions of the molecules to permit reformation to new polar positions (6 prop., page 15), and some other conditions not yet recognized by science.

21. Proposition: *That surface forces of extensibility and cohesion affect the surfaces only of liquids for a very limited depth. Therefore capillary action, so far as its action at a distance below the aerial surface is concerned, is dependent upon the cohesion of the general system of the liquid and of its adhesion to the solid.*

a. The attractive forces that may be conceived active through adhesion alone would fairly represent the effects of capillary action for certain cases, but such adhesions are not easily accounted for unless they are *attractions to mass,* and in cases of capillary attraction it is possible that mass may act only as a weak force. The important experiment of M. Plateau mentioned, 11 prop. *e,* page 32, shows that mass acts as a *very weak force* for the cohesion of liquids, if it acts at all, as the globe of oil in the case given in the paragraph referred to is not deformed by a heavy iron ring, which it evidently would be under any form of active mass attraction in the liquid about the solid. Therefore I suppose that the rise in capillary tubes is almost entirely due to cohesion (11 and 19 props.) and surface extensibility (16 prop., page 45), previously proposed, with such components of elasticity as are produced by inflexion considered in the last proposition. The whole of these forces being assumed to act contra to gravitation in the elevation of a vertical capillary system, but I assume that the *maintenance of a tall volume of the liquid* in a close area above gravitation equilibrium is largely due to the adhesional and cohesional forces now proposed, as the surface force is active only within the liquid at a position very near the aerial plane.

b. In the case of glass and water we have evidence that the adhesion of the water to the glass after intimate contact is a superior force to the intermolecular cohesion of the water, as, in the separation of a clean disk of glass from the surface of water, the line of fracture or separation is always at a distance from the surface of the glass; this distance being uniformly at about ·12 of an inch in the water below the line of contact. It must be conceived that this is within the *radius of attraction* of the glass from some cause, as the liquid system being homogeneous must evidently break in the weakest plane, or that which receives the greatest gravitation strain upon its cohesive system. Further, one sheet of glass will lift another below water, and a sheet of glass wetted with water will retain a certain stratum of water upon it even if placed in a vertical position, without the water flowing down, which is evidence of the intimate cohesion of the thin surface of water being superior to the force of gravitation upon it.

c. Newton has briefly discussed capillary action, and has given the ratio existing for distance between two vertical plates of glass to the height that the water will rise between them, in which he says this[1] "*will be reciprocally proportional to the distance very nearly; for the attractive force of the glasses is the same whether the distance between them be greater or less, and the weight of water drawn up is the same if the height be reciprocally proportional to the distance.*" This is perfectly consistent with the observed action of adhesive forces that are active from some cause nearly inversely proportional to the square of the distance within a very limited space. Newton further shows that the height that water will rise in a slender glass pipe will be reciprocally proportional to the diameter of the pipe, and will equal the height to which it rises between two plates of glass, if the semidiameter of the interior of the pipe be equal to the distance between the plates. These conditions appear to be quite true and clearly definable as components of adhesional forces for the fixed position of the liquid after contact or elevation within the tube. Upon the certainty of these conditions it appears to me probable that the adhesional forces in liquids most probably extend beyond the first line of contact by proportionally strengthening the cohesive forces of the liquid molecules by attractive forces *inversely proportional to the square of the distance from the surface of contact, and which may be taken as a whole or by consecutive attractions with like effects.* These adhesional forces are probably derived from the intermolecular pressures caused by the pressure of the surface of the solid against the liquid. According to conditions of 10 prop. *c*, page 29, by which the hydrostatic pressure upon the surface of a liquid resting against a solid causes intermolecular intrusions of the same kind as proposed, 16 prop. *a*, page 45, for aerial surfaces; these intermolecular intrusions acting by a certain amount of suppression of elastic intermolecular space, increase the molecular attractions, and thereby practically extend the forces of cohesion to the solid, as adhesions which are found

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[1] *Opticks*, 31 Query.

to act inversely as the square of the distance for a small distance, probably for ·14 of an inch as offered experimentally for surface condensation 16 prop. *k*, which would be active for ·28 between two plates, or ·56 in a tube of this internal diameter.

d. It is thought that capillary action is better explained by molecular attractions sensible only in insensible distances, first proposed, I believe, by Monge, and supported by Young and Laplace. I do not see that this helps the matter. It is no doubt nearly true as regards the attraction *within* the solid against which a liquid rests; but practically it is not true to experiment in the liquid itself. If the attraction of glass for water were active *within the water* at insensible distance only, the disk of glass just mentioned, as being lifted from a surface of water, would not be affected by the adhesion of the glass, but would separate the water so as to leave only an *insensible stratum of water upon it*, as it is quite clear that, in the assumed equality of the adhesive system of the entire water, near and at a distance from the solid, or of an assumed consecutive cohesion, sensible only at insensible distances in all parts, the glass would not lift *any stratum of gravitating matter greater than the extent of the surface attraction of the glass from all causes*, and would therefore separate from the liquid at its highest plane, so that if the glass were wetted the distance of attraction would be so small as to be imperceptible.

e. Newton has also recognized both the cohesive and surface forces for the liquid, in the same query, for he says that "If a large pipe be filled with ashes well pressed together, and one end of the pipe be dipped in water, the water will rise up slowly, so that in the space of a week or fortnight it will reach the height of 30 or 40 inches. The water rises up to this height by the action *only of those particles of the ashes that are upon the surface of the elevated water, the particles which are within the water attracting and repelling it as much downwards as upwards.*" In this case the adhesive forces of the entire tube would support the mass of the water, *raised at first by the surface forces.* Newton's proposition appears to define the matter clearly without stating the causes of surface forces, the theory of which has proved and still remains the great difficulty to all his followers.

f. We may conclude, nevertheless, that the theory of capillary rise in a tube would be defective without consideration of surface forces, as we are assured that the adhesive force simply, as it is uniformly distributed over a solid will not of itself wet or spread a liquid indefinitely when it is placed upon a level surface, where there is present no gravitation force to resist this extension such as we find present in the case of a vertical tube; this it should certainly do if the capillary action were due to its attraction to the solid simply at its nearest plane; for upon the level plane it would act under no restraint from gravitation, but be aided by it, and the principle of its first attraction would proceed indefinitely over the plane until the liquid was equally wetted by a very thin stratum. Whereas the spread of a liquid upon a level plane is quite different, if there is no chemical action, the terminal edge is in no case a thin molecular stratum, as it is observed to be universally under capillary elevation (20 prop. *i*).

g. The conditions of tensibility assumed for a liquid, as already discussed, and represented by a stretched drum-skin, have been sometimes taken from the conditions of films, which can scarcely be relative to the surfaces of masses, as the position of the attractive matter, therefore of the attractive forces, of the molecules are upon a different plane; those in films being placed lineal to the attractive matter, those in masses normal to it. If we assume capillary surface simply tensile as the film would be, then it would be clear that such tension would be pulling the liquid towards the centre of a capillary tube, and thereby lower the liquid by decreasing the possible attractive forces which would otherwise exist naturally against the solid surface of the tube, together with the hydrostatic pressures present.

22. PROPOSITION: *That the rise of a liquid against the surface of an adherent solid is proportional to the support the elevated liquid finds in the closeness of the parts of the contiguous solid and the liquid cohesion conjointly.*

a. By conditions proposed, a convex surface of a liquid may find equilibrium in curvature (20 prop. *h*). If a liquid rise against a concave solid the exterior surface of the liquid will by its convexity be approaching equilibrium, and be thereby of more open structure than if it rested against a concave surface, which I have already assumed will be in a state of lateral extension. Under these conditions a liquid resting in a concave surface will be more free from lateral pressure to move upwards; for although its radial compressions will be strengthened by the *intrusive pressures* the lateral or *circumferential pressures* of the molecules will be less. Therefore there will be greater freedom for elevation by the general conditions of capillary action for the rise of a liquid against a concave solid than against a convex one.

b. By 11 proposition, page 30, the particles of a liquid seek the greatest area of contact. By 16 proposition, page 45, the surface molecules exert efforts of intrusion which make the surface extensile. By 21 proposition a solid may act by adhesion with limited force as a mass of liquid in its attraction to the surface of the molecule. Now suppose a surface extensile, as I propose, the liquid will by this means have its attractive forces strengthened to rise against a solid, if the solid present a greater surrounding area of attraction to the surface molecules than these molecules possessed in their position of lateral equilibrium upon the liquid surface. Further, the molecules will rise proportionally to the extent of attracting surface that from all causes lifts them upwards. Now, if we take a solid body of almost infinitely small cylindrical area, and place this vertically in a liquid, in this case, as the molecules rise against the small cylinder they would have to be placed in such a form as to be compelled to follow the outline of the cylinder at the liquid surface. It is clear in this case that the *surfaces of contact* (11 prop.) per area of each molecule would be less than when the molecules rested in lateral equilibrium in the plane of repose. Therefore by the same principles of 11 prop. the surface liquid would not rise at all, as the molecules would be thrown further out of static equilibrium by such rise.

c. If, on the other hand, we place in an adhesive liquid, a solid body that protrudes vertically above its surface, of a small concave area instead of a convex cylindrical one, then will the concave present a greater area of molecular contact to establish the elevation of surface liquid than that presented by the aerial plane surface, and the molecular surface extensile force will be better supported by the near parts, so that the elevated liquid molecules will be in more perfect static equilibrium by this elevation.

d. If the attractive force of a liquid from all causes be measured against an open vertical flat plane of a selected solid rising from the liquid, this measurement may be taken as the unit of attractive force the liquid has to that particular solid, under free surface expansion and mass cohesion, constituting open capillarity. A liquid against such a surface will rise less, as just shown, than upon any concave surface (*c*), and more than upon any convex surface (*b*), the concave surface by inclosure being assumed to support the expansive surface force by radial convergence upon the liquid, just as a convex surface will act inversely by its support receding radially from the convex rigid area.

e. To investigate the general principles of the combined effects of open capillary attraction into area of contact I took as a first experiment a clean sharp razor. This I fixed in such a manner that the cutting edge was quite vertical to a liquid surface, the razor being immersed sufficiently to ensure general contact for the portion below the surface. After I had entirely immersed the razor in the liquid, and agitated it to ensure contact, I then dried it and found upon second immersion in the liquid at the position given for the experiment, that from the very small amount of surface oxidation now formed, adherence was almost instantaneous and perfect.

f. The liquid that I found best adapted for this experiment was pure water, to which one half per cent of acetic acid was added to give attractive force. This acidulated water made a permanent dark line of oxide to the extent of capillary attraction upon the surface. The rise upon the sharp edge was only just visible above the liquid surface plane. The general elevation was as in the diagram below, the water line being shown at A B, the rise being greater in the hollow of the razor, falling at the angle which turned for the back, and rising in the centre of the curvature at the back; this last rise appears to be due to a mass attraction. I consider that there was, however, a certain amount of chemical action which increased the capillary attraction and gave a general compensation of forces in all the curves; but the effect of capillarity was roughly indicated for vertical curvature.

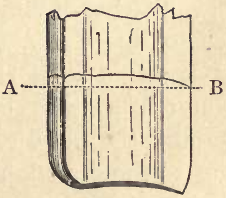

Fig. 22.
Capillarity upon a Razor.

g. To trace more particularly the effect of curvature and extent of near surface, I drew out with a spirit-lamp from a small glass tube, fine tubes of various sizes, until I had a good assortment. Selecting four of these by measuring them with a micrometer under the microscope, I obtained tubes from parts of longer ones which appeared under the microscope by the reading of a micrometer very approximately in series ·00025, ·0005, ·01, and ·1 of an inch in external diameters. These I fixed vertically by sealing-wax at about one inch apart to the edge of a card which was about five inches long by two inches wide. Taking a pan of clear water at a temperature of about 26° Centigrade I supported the card so that the ends of the glass tubes were each about half an inch under the surface of the water. Seeing by a hand-magnifier that they were thoroughly wetted on their surfaces, I left them in the water for about twenty-four hours.

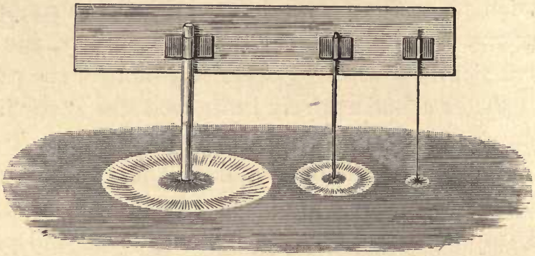

Fig. 23.—Capillary Surface Reflections.

The following day, on raising the glass tubes, I found them equally wetted. They were now left immersed about a quarter of an inch only in the water. In an hour's time, when the upper parts were quite dry, I read off the capillary action to the exterior of the tubes in the following manner:—

h. Placing my eye at an angle of about 15 degrees to the surface of the water, I obtained a bright reflection from the vertical image of a solid straight-edge which showed clearly the elevated water near the tubes; the reflections appeared as rings of light around the glass tubes. By a little adjustment of the eye according to the direction of the light, a point could be found where the first part of the palpable angle of deflection rose from the smooth liquid surface. By placing a transparent opal glass rule above this, the reflection of the rule was thrown into the water, and the position of the lines of the first elevation of the water were measured off by the greatest diameter of the reflected light, approximately. These measurements were as follows:—

	inch.	inch.	inch.	inch.
Exterior diameter of glass tube,	·1	·01	·0005	·00025
Diameter of area of deflected surface,	·89	·5	·21	·15

i. The glass tubes under the microscope all presented a bore of about one-third of their entire diameter. The hyperbolic curve of flexure of the water surface against the tube was approximately proportional to the dimensions of it, as theory demands for attractive forces; the heights would therefore be relative to the breadths of deflection; giving thereby the elevating force of the system.

j. With a solid glass rod of ·25 of an inch in diameter the surface deflection extended ·45 of an inch from the surface. I measured this by the reflection of a vertical window bar by moving the eye until the reflection from the water ceased to have its outline indented by the capillary deflection near the solid glass rod. The greatest extent of surface deflection by a plain solid that I have been able to produce, was ·47 of an inch nearly.

k. In considering the entire attraction from all causes I found that with a larger tube, as one ·5 of an inch in diameter, I produced a ring of only slight excess of diameter over one ·1 of an inch, plus the difference of diameters, so that beyond ·1 of an inch diameter of a mass, there would appear to be but little measurable difference of capillary force from curvature.

l. To examine concave surfaces the following was tried for me by a friend, as I had not time. Four tubes were selected, and carefully ground away to nearly half their diameters, leaving about 185° of the internal circumference as nearly as could be measured. The ground surface was afterwards polished. With these sections of tubes fixed on a card in the manner described for the previous experiment, the result showed the rise according to the following table measured approximately in fractions of inches:—

Fig. 24.—Section enlarged.

Diameter of half tubes.		Capillary rise.	
Outside.	Inside.	Outside tube.	Inside tube.
·210	·152	·060	·155
·160	·105	·057	·255
·110	·069	·054	·310
·072	·030	·052	·415

m. In making two of these tubes of equal size for experiment it was found that there was a slight variation in the rise of water from the above; but the difference was too small to need a repetition of the experiments to ensure the general principles, which is my desire alone to follow in this treatise, and not the investigation of quantities.

n. By the above experiments we may conclude that the adhesive forces, including capillary action, evidently act in every direction from a point upon a solid surface equally. This may also be demonstrated by the following experiment:—Cement two clean glass plates together with an intervening slip of thin glass between them along one pair of edges as shown above, A; leaving the opposite edges near B open. If the plates be now thoroughly wetted with water containing a little alkali and then placed erect in the water the water will be elevated against the surface of the thin slip of glass A in volume inversely equal exactly to the depression at the open edge B.

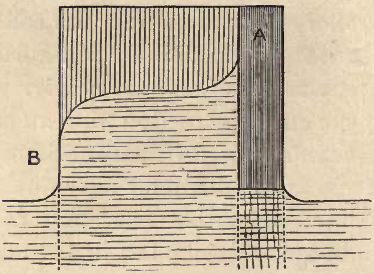

Fig. 25.—Capillary against a Supported and an Open Plane.

Extensile surface strain producing drops in small jets.

23. PROPOSITION: *That the extensile force active within a plane or convex aerial liquid surface, is a cause of the separation of a fine liquid stream into drops.*

a. The expansive aerial surface force upon a plane or convex surface will be proportional to the relative area exposed into its curvature and mass for the entire expansion; and in composition of forces the expansion will be active proportionally to the distance of effective penetration of the surface force in the stream (20 prop. *h*, page 62), so that small streams will be measurably affected by it. But in larger streams the active surface forces in their power to induce any movements in masses will be almost infinitely small.

b. The mode of action of extensile aerial surface force on small jets may be shown by the following diagram:—

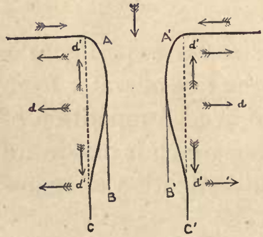

Fig. 26.—Extensile Surface Forces on a Fine Jet.

Let A A′ be a very small hole in a thin plate with a liquid above, the flow of which to the hole will be shown by the direction of the arrow above it. A B and A′ B′ will be the outline of issue by the composition of internal forces to form a vena contracta, which will be hereafter considered. Let an extensile surface force deflect the issue outwards, and enlarge the stream as shown at C C′ in the ratio of circumferential surface expansion. This expansion will take place also longitudinally upon the direct lines of the stream, contra to the central cohesive forces of the jet here assumed to be small, and by expansion the stream will divide into separate units or systems in equilibrium, as drops; the direction of the forces being as shown by arrows in the diagram above. In the engraving the projection is shown downwards; exactly the same effect will be produced if it is projected upwards, so that this action is independent of gravity.

c. If we could assume, on the other hand, that an aerial, lineal, or convex surface of a liquid in mass were *tensile*, the stream at issue should have a force of contraction exactly opposite to the above; therefore by this immediate contraction a cylinder of less diameter than the size of the hole would issue. Further, the stream would continue in the same form as at its issue by the constancy of the tensile strain upon it supporting its general cohesion; this same force would also tend to constantly decrease the cylindrical circumference of the stream, and keep up the continuity of the same form in any small free cylindrical jet projected downwards, and thereby prevent division, so that with a tensile strain, the water would fall to the ground in thread-like masses instead of drops.

d. By the principles of extensile surface strain, which I assume to actually exist on the surface of plane or convex masses of liquids, we may conclude, that if this force were active upon a fine falling liquid stream, that the central cohesion of its mass would be broken if the longitudinal extensile surface force were sufficient to overcome its cohesion in a very small unit of time. In this case, by the expansion, every free part of the falling liquid would be projected separately in drops, as soon as its central cohesive force could be released from surface strain. Further, every jet under any conditions in falling or rising would have this tendency to separate into drops.

e. The value of the extensile surface force of the stream in its power to break the mass cohesion of a jet would depend upon the size of the jet, for it is quite certain that if we assume the cohesion of the liquid as a certain force, and the surface extensibility as another force; then as the surface of a body constantly increases in inverse ratio to its decrease of mass, or in proportion as the diameter decreases; there will be a certain size of jet wherein the surface extensile force is in equation with the cohesive force, and for smaller jets than this the surface extensile force will be the *greater*, so that the stream must be separated by this, if the force acts immediately. Therefore fine streams from this cause will be detached into drops *immediately* on issue; and although we may imagine that the cylindrical issue will have extensile surface forces active upon it, everywhere alike, it will, by the extensile force present, be in such a state of unstable equilibrium upon the whole surface of the cylinder, that the smallest disturbing cause in the air or otherwise would tend to release the surface forces to equilibrium, which could only be found for a free liquid body in the globular form, or in free drops. In this manner waterfalls from great heights, as the Staubach Falls in Switzerland (1000 feet), have their waters divided into spray of fine drops before the stream reaches the bottom of the precipice from which it is projected. The same also occurs in principle in the projection of water in large fountains, as at the Crystal Palace, where the vertical issue from a pipe of apparently 2 to 3 inches diameter is reduced to fine spray in about 50 feet of projection.

f. By the conditions offered (16 prop. *b*, page 46), the extensile surface forces of cohesion will be in equilibrium (11 prop., page 30) when the external curvature of a cohesive system equals the disposition of these forces to distend (20 prop. *b*) by molecular pressure upon the central cohesion, so that at a certain size a drop of water would be in perfect equilibrium of cohesion in relation to its surface distension. In such a condition exterior molecules (11 prop. *a*) would not be able to intrude themselves into the molecular drop system without pressure, as the internal resistance would be in equilibrium with the external separate molecular pressures upon it, and in this case the drop would not further enlarge by contact of like liquid upon it, without pressure. Possibly this is a condition that nearly holds in rain-drops, so that vapour and smaller drops do not easily further compound in the static drops after they attain a certain size in equilibrium with cohesive and surface forces active upon them. Whereas, in the formation of hail, this not being subject to fluid surface forces, condensation may continue to enlarge the volume of the hailstone to any extent.

g. Tension of surface has also been inferred as necessary to the production of spherical drops in free aerial fluids. This might be, or not, equally consistent with the effects observed; as the cohesive force of liquids proposed in 11 prop. is quite sufficient in itself to assure us that the free drops will be globular whether the surface be equally tensile or extensile, if the cohesive forces exceed the surface forces, which they in all cases appear to do. In the spheroidal state of water upon a hot plate I anticipate that the surface must be undoubtedly expanded by the heat, and can be withheld by nothing except the cohesive force of the liquid system, which is surrounded possibly, by a surface expansion into elastic critical surface fluid, the dryness, and therefore the non-conducting power of which, and the presence of surrounding vapour prevents excessive evaporation.

Motive continuity of surface extensile strains in surface movements upon liquids.

24. PROPOSITION: *That in the extensile surface of a liquid, disturbances by compression upon the surface that produce a greater area, will be persistent in extensions over the surface in proportion to the force of extensibility of the free surface.*

a. As a free liquid surface under the action of gravitation is found to be a level plane, every possible disturbance will necessarily increase the area of this plane.

b. We are indebted to the valuable researches of Mr. J. Scott Russell[1] for the very important discovery, that a protuberance raised upon a surface of water in a parallel channel by any means, will go forward in the direction of its original impulsion for great distances, even for miles, without greater decrease of altitude than can readily be attributed to molecular friction of a moving system, so that the surface curvature once produced is maintained with considerable force. In this matter we may conclude that *if the surface force were extensile it would support a protuberant wave. If it were tensile the tension would constantly be active in lowering the wave.* I wish to call attention to this fact only upon principles of surface tensibility. I will return to the consideration of the general principles of this nearly frictionless wave further on.

c. By another interesting experiment offered by Mr. J. Scott Russell (page 377 of the same reports) a wire a sixteenth of an inch in diameter is inserted for a very short distance in the still surface of water in a large tank. The wire is then moved forward with a velocity of about one foot per second. It is found that the surface of the water in front of the wire is rippled up in small waves, of which 12 to 20 may be distinctly counted; whereas, following the

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[1] *British Association Reports*, 1844, page 319.

direction of the wire, the water is nearly still, or only affected by very faint undulations. The rippling or crumpling up of the surface extends to about three inches in front of the moving wire. It would appear to be necessary in this case that the water must possess an extensile surface force to form this undulatory surface in front of the direction of motion under the impression of the small moving body at so great a relative distance from it. If the surface were tensile, it is presumable that it would open to the impression of the small moving body, and the distance ruffled in front would be very small. We may conclude that as soon as the surface is ruffled in small undulations it loses a great part of its rigidity, and that this is the cause that the extensile compressions can be detected for three inches only before the moving body. If it were not for this crumpling it is presumable that from the constancy of extensile strain its rigidity would be made evident for a great distance. I will return to this matter further on when I take into consideration the phenomena of waves.

Tensile forces in liquid films.

25. PROPOSITION: *That a free film of a liquid as a free homogeneous cohesive body will be under tensile strain in proportion to the strength of attractive forces of its molecules that causes its cohesion and by its adhesion to surrounding areas of attraction.*

a. Let *a a'*, Fig. 27, represent a film, which, for the principle of the proposition, may be a single plane of molecules, and let this plane be distended between two solid bodies. Then will the cohesive forces in the film be attractive centre to centre diametrically, and the film will be in *tensile equilibrium.*

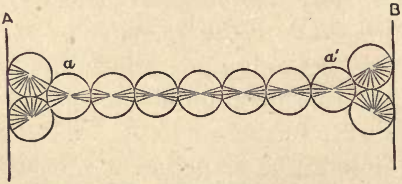

Fig. 27.—Diagram Tensile Forces in Films.

b. Let the terminations of the film, *a* and *a'*, each rest upon two molecules in the manner shown in the figure; then the area of attraction of these molecules will be greater than the area of attraction between any two molecules of the film, and these will by their forces pull the film to the planes A and B, or exert constant *tensile strains* upon the film by principles discussed (11 prop., page 30) in the attractive forces of every molecule endeavouring to find the greatest possible area of surface attraction.

c. If we imagine the film to be of two or more series of molecules, as shown in the diagram below, Fig. 28, then will the same amount of tension be established as that just discussed, as the attractions about the exterior planes will be equal and the medial planes will be neutral. It is presumable that in every film there are many hundreds or thousands of planes. The attachments also will extend to a cone of such molecules of many times the width of the film, and resemble the attachment of the molecules shown Fig. 27 upon the planes A B. The general conditions of the force of tension of a film of water may be discovered by the following simple experiment.

Fig. 28.—Diagram Films.

d. Project a small jet of water into the air and ascertain the force of its projection by the extent of its trajectory against the constant force of gravitation acting upon it and the resistance of the air. Again, project the jet in such a manner that it may adhere to a surface of water in contact with it, so that it carries up a film of water with it. The film will now under this condition suffer such restraint to its trajectory as its tensile force will cause, approximately.

e. The apparatus I have used for this experiment is constructed as follows:—A supply can of water is placed at some fixed elevation; a small caoutchouc tube is connected to the can to supply a jet. The tube is of about ·25 of an inch in diameter. In the free end of the caoutchouc tube a piece of glass tube of the internal diameter of the outer tube is fixed. The glass tube is drawn off to a fine point of about ·025 of an inch internal diameter. The can of water is adjusted to various heights by a cord and pulley above the jet, which is placed just within the surface of water in a trough. The glass tube which produces the jet is held down to a light frame under the water surface, so that it can be tipped to any inclination, and by the setting of an adjusting screw, if so tipped, it can be restored to the same position afterwards by the weight of the frame. This arrangement is shown in the figure below, A being the tipping part and C the adjusting screw. With this apparatus the orifice of the glass jet can be at first set to an angle to the horizon with its point just below or at the water surface, and when the water is flowing through it, it will project a stream entirely above the surface. But if it is started below the surface, and again brought to the same point of rest, it will carry up with it a film of water which will restrain by its cohesive force the projection of the stream. I found with this apparatus, by keeping the point of the jet at a constant small depth below the water surface, that is at about ·02 of an inch, that if the jet were placed erect no film could be formed; that if the jet were placed at a great angle to the horizon a film would be formed, if the force was small; but at a very low angle the force might be very great, and the film would restrain it. Placing the can at the different heights above the water surface the following angles were found to perfectly restrain the film, and the following projections were observed:—

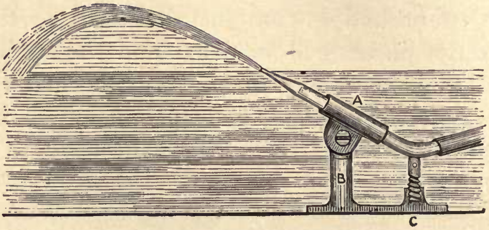

Fig. 29.—Apparatus to Project Films.

f. First experiment. Height of surface of water in the can above the water surface in trough 8 inches, angle of projection, 30 degrees to the horizon. Height of projection of free jet in air 1·5 inches, distance 5 inches. Immersing the jet for an instant and then restoring it to its place so as to carry up a film of water: height of projection with this film ·5 of an inch, distance 1·25 inches.

g. Second experiment. Height of water in the can above the water surface 11 inches, angle of projection 25 degrees. Height of projection of free jet in air 1·5 inches, distance 7·5 inches. Immersing the jet so as to carry up a film of water: height of projection ·38 of an inch, distance 2·26 inches.

h. Third experiment. Height of water in the can above the water surface 26 inches, angle of projection 18 degrees. Height of projection of the free jet in air 1 inch, distance 17 inches. When carrying up a film of water, height of the projection ·12 of an inch, distance 3 inches as nearly as could be measured.

In these experiments the cohesion of the film being equal to the restraint of the projection into the area of the film; the entire force of restraint of the film will vary as the difference of forces necessary to describe the greater over the lesser range.

i. The film in these cases is quite persistent, and remains as long as the water in the reservoir is kept at the same height. Perhaps the above by calculations, with further corrections for collateral conditions, may give an approximate value of the tension of a film of water; as the force of trajectory is easily computed in equation with the restraint the jet suffers by the film *per area.* It is possible that an approximate value of the cohesive force of the water *in mass* might be ascertained by the restraint to projection exerted, into the transverse area of the film, in the experiments described above, if the film could be rendered motionless, so that it could receive no supply of liquid from the distension of the surface upon which it was projected. In these experiments, however, the film will be found to be in revolution by principles of motion to be hereafter discussed, and these revolutions will of themselves produce a strain upon it in the plane of its distension, and at the same time by the tangential force of the revolution, withhold the liquid surface upon which the projection is made, so that the cohesive forces ascertained by the means I propose will be only approximate.

j. I have endeavoured by tracing all the conditions in my power, to obtain an index of the cohesive force of water by the means indicated above. My results gave me the cohesive force equal to a cohesion *per area* that would support a hanging column of water of 17 inches in depth. I have, however, reason to suppose that this even may be too low an estimate; but I withhold all details, as I have no doubt such a proposition would appear extravagant, when we find the cohesive force of this fluid is generally estimated at about $\frac{1}{7}$ inch only by some, and by others, on the dynamic theory, that there is no cohesion whatever in fluids, but the reverse of this, separative action.

Extensile surface force of a liquid under evaporation.

26. PROPOSITION: *That the surface of a liquid under evaporation is by this cause constantly extensile, in proportion to the rate of evaporation.*

a. It is well known, that water either under the pressure of a gas or under the vacuum of an exhausted receiver, is constantly undergoing evaporation at its surface, until the gas becomes saturated, or the vacuous space becomes charged with an amount of vapour, the elastic force of which, by itself, produces a certain surface pressure; this evaporation or vapour force being in known ratio to the temperature. When the water surface is free the vapour that is formed at its immediate surface plane may be conceived to exist as a superior stratum of a more attenuated molecular form of water resting above the liquid proper, which may be necessary to be superimposed above the water for it to remain quiescently in static equilibrium as a permanent liquid, on conditions of surface equilibrium pointed out (16 prop., page 45).

b. Under the above conditions we may imagine that the surface of the liquid must be by some means endowed with a force that can produce the expanded fluid which we term a vapour. It is also probable that this force is partly communicated to the liquid by the vacuum or gas that rests above it before a superficial vapour force is formed sufficient to stop excessive evaporation. Under any conditions, the vapour being in a more extended state than a liquid, as its specific weight clearly indicates, must at the *instant of its formation* upon a free liquid surface by any cause, be endowed with a force of *expansion* which will at first be *at or within the liquid surface,* that is, until it rises from it, and therefore it must expand *in or at* this first surface *in contact.* It also appears probable the directive force of this expanded surface will cause by its adhesion an *extension* in a less degree of the immediate liquid surface beneath. That is, if the first surface is by any means *extending,* this surface by contiguity of the same system of matter, will extend the next stratum beneath in a less degree also. This expansion in liquids now assumed to take place under quiescent evaporation, may form a stratum of critical liquid of the kind demonstrated by Andrews, as a visible phenomenon in evaporation and condensation of gases to liquids brought about by the withdrawal of heat, and application of pressure in confined areas. But in the case assumed above for an open liquid, the *critical surface* is proposed to be of very small depth, and possibly more dense and elastic than under conditions where it forms a sensible quantity of critical fluid.

c. That a liquid surface has superimposed upon it a different molecular system of matter to the body of the liquid, is very clearly indicated by the differences of chemical affinity with other bodies that exist at this surface, which is demonstrated by the chemical and physical differences which the two positions in and on the liquid indicate. Thus alcohol, ethers, and essential oils as turpentine, spread instantly over the surface, whereas they unite very slowly, or not at all, with the mass when entirely submerged. I have found it convenient to consider this surface molecular system, which I assume to be equivalent to the critical fluid of Andrews, as separate from the liquid system proper, in which the surface molecule forms a part of the liquid mass, as I find a liquid has molecular cohesive forces quite independent of any superimposed vapour systems, as shown for fluid glass (11 prop. *j*, page 34), as also in M. Plateau's experiment of oil in dilute alcohol already discussed; but I anticipate that the critical surface fluid prevents at all times the immediate evaporation of the liquid by constantly exerting a certain pressure upon the surface molecules to support its equilibrium, so that the conditions are not quite according to those defined, 16 prop. *b*, page 46.

d. The entire expansion of a liquid surface may possibly be seen most clearly when the liquid is in a state of violent evaporation, as in boiling. In this state the heat forces, whatever they may be, which cause the expansion of the water into vapour, will be most active, and the surface expansion, if real, most direct. At the same time any possible expansion of the surface will be withheld by the general mass cohesion of the liquid beneath, and by the air or vapour or critical fluid pressure above, with a force in proportion to the cohesion of the liquid at the temperature at which it is taken, from all causes; so that the outward or visible surface expansion can only occur when the extensile force of the surface exceeds the cohesive force, together with the exterior pressure upon it. In boiling, the superimposed pressure is assumed less than the tenacity of the cohesion of the liquid beneath, for any minute surface depth of the liquid that we may term a *film*. This film will therefore, under the strain of distension, be so far out of equilibrium in the system, that it will be vaulted upwards into the less resistant media above at the instant that the expansive surface force overcomes the cohesion to the mass beneath. *And the vault or bubble so formed will represent at the instant of its formation as a free surface, the ratio of extreme expansion of the extensile force of a liquid surface in contact with its vapour, from all causes.* At the extreme temperature of the water the vault or bubble when formed will break, as soon as its convexity permits the expansive force to overcome its mass cohesion; such breaking being generally rapidly accelerated by the rising vapour which again accumulates from the surface beneath.

e. In the case of heat applied to the bottom of a vessel of liquid, wherein the vapour is rapidly formed upon the lower surface, at slightly higher temperature than the superimposed liquid above, the rising bubble of vapour, itself created by surface extension at the sides of the vessel, will, by its specific density being less than that of the liquid in which it is set free, have an upward projectile force. It will, therefore, when it reaches the surface, aid in projecting a surface vault or bubble, or it will otherwise, by a kind of intrusion within the extensile surface, expand as the nucleus of a bubble.

f. If the surface temperature be lower than the boiling-point, that is, if the surface is not in a state of distension sufficient to overcome the cohesive force of the general liquid beneath, the small rising bubble may simply burst at the surface or rest upon it, or even be condensed, if the surface temperature is much lower.

g. In the case of water being heated as uniformly as possible, bubbles or vaults may be seen to rise from the upper surface only, although the greater number will receive the first impulse of projection from a smaller bubble rising from the lower or lateral heated surfaces, as before proposed. This principle will be easily conceived mechanically, as the upper surface extensile force will be lineal with the cohesive force beneath, and therefore very static. But the lateral force of a small bubble rising by its buoyancy against the surface film will be immediately active, normal to the extension of the surface system, and, therefore, use its force directly to separate the more extensile surface plane of the liquid from the less extensile one beneath.

h. As evaporation carries off a large amount of latent heat, it is difficult to maintain the extreme upper surface of a liquid at so high a temperature as the heated mass beneath; but by causing water to boil by heat applied entirely above its surface, as by radiation from a mass of red-hot metal, bubbles from extensile vaulting of the liquid at its superior critical point may be observed forming near the surface without any indication of a deeper disturbance.

i. The conditions for the formation of a bubble when the surface of water is expanded by the force of evaporation, and thereby placed in unstable equilibrium, although still retaining its level plane, may be compared to a straight vertical rod supporting a weight above it, in which case the strength of the rod vertically in equilibrium of rest may be very great, but under any force that causes at first a slight flexure, it will lose all its rigidity and give way, so that afterwards a less force may bend the rod to considerable curvature, as the bubble of the surface film may be assumed to be bent immediately after its release.

j. In this manner, if a liquid of prescribed area, supported by surrounding solids, is kept perfectly still by shading it from motion and falling particles, its critical vapour resting quiescently in equilibrium as perfectly as possible, may have heat applied to raise its temperature far above the ordinary boiling-point before surface-vaulting or boiling will commence, or such an amount of evaporation as generally accompanies ebullition. In the case of pure water that has been deprived of air, some experiments of Donny have proved that this liquid may be raised to a temperature of 135° C. without ebullition. That this is particularly due to the quiescent state of the surface may be seen in that water thus heated will by a touch of a needle, a particle of dust, or any other small disturbing cause be thrown into a violent state of agitation, which will not again cease until the temperature falls to 100° C. This fact shows that it is ready to *distend* at any instant if the continuity of surface presents at any point a greater convexity than the level plane; which it must do in certain local positions by any disturbance caused by intrusion of other matter at its surface.

k. Under the above conditions there will be two temperatures for boiling, one in which the surface is agitated by rising bubbles or by mechanical means, and another in which the liquid uniformly or surface-heated, boils by vaulting of the quiescent surface, when the distention overcomes the molecular cohesion. As this quiescence can only be relative to exterior disturbing causes which are always present in a greater or less degree in a moving atmosphere, the last temperature is probably very high and cannot be discovered.

l. Capillary tubes placed in a bath of water so as to be gradually heated, will elevate less water as the temperature rises, probably from the minus elastic force of the critical fluid that rests on the aqueous surface; but at the instant of boiling the expansile surface force overcomes this resistance and the water darts up the tube.

Local extension of surface in the boiling of liquids.

27. PROPOSITION: *That the vaulting of a liquid plane to form a surface bubble will be in instant equilibrium when the exterior surface* *of the bubble equals the duplicate or triplicate or quadruplicate area of the surface from which it is projected.*

a. By the conditions offered 16 prop. page 45, a liquid surface will be in equilibrium when its molecules are in regular stratified planes, possessing in this state of equilibrium surface extensile force. If an elevating force act upon such a plane, equilibrium cannot be restored unless the plane be expanded to quite double its previous area, so that the one plane of molecules is intruded into the planic series of the other, or, if the elevating force be greater, a triplicate area may possibly be produced, or a quadruplicate, that is, a free bubble. In this last case a duplicate surface may be produced by direct intrusion of the nearest molecule beneath, and the duplicate of this again by another like intrusion to form a free bubble, whereas the triplicate area would cause a general frictional derangement of the system; hence it appears that duplication is the most general principle.

b. Upon the principles offered above, a bubble at its elevation will at first subtend an arc of about 180 degrees, the number of aerial molecules in the vault being the duplicate of those upon the level surface of the liquid, or the bubble may subtend, as I find by calculation, an arc of 218° 56′ 30″, the superficial area being the triplicate of the area of surface upon which it reposes. The diagram above will represent the system theoretically, where there are double the number of circles representing molecules in the arc to those on the base. This form of duplication will occur in any number of superimposed surface strata that the extensile force may elevate to form the bubble, upon the same principle of equal intrusion as shown for a single hypothetical series in the diagram.

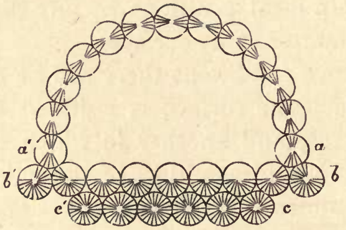

Fig. 30.—Diagram Molecules of Bubble.

c. By the equality of central attractions, as shown for internal angles (19 prop. *f*, page 57), for capillary action, the meeting plane of the bubble upon the surface will be deflected; every molecule using the efforts of its attractive forces to find equilibrium of radial attraction, therefore the edge of the bubble when it rests on the plane will be elevated, and the centre of the included area will be depressed. There will be a constant tensile strain over the bubble by its union to denser matter, as shown 25 prop. *a*, and this will tend to depress it; but as this lowering will cause greater compression on the air included, it will generally reduce the film in thickness only, until it breaks.

d. The following diagram will show the character of bubbles, of instant static equilibrium, Fig. 32, and, after action, of extreme tensile strain, Fig. 31, by the action of gravitation on the plane on which it rests for very large bubbles; this form, however, is not general.

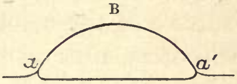

Fig. 31.

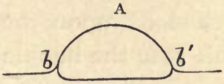

Fig. 32.

e. A bubble may rest in static equilibrium on the surface of a quiescent liquid if there is sufficient vapour force above it to prevent evaporation, and if it be constructed from the surface film which it covers, as in this case, the positions of molecules *a a′* and *b b′* of the diagrams above will support by their attractions the circumscribed area of capillarity.

f. In water and most fluids the bubbles assume instantly the duplicate area figure. In the agitation of cold fluids this is also general, but very large bubbles, that have sometimes little or no vapour support, become occasionally extended as shown in Fig. 31.

g. The boiling of liquids as before proposed occurs when the vapour force between the molecules of the liquid proper exceeds the vapour force superimposed upon the liquid surface. So that if we increase the vapour pressure above a liquid, its temperature may be increased in like ratio without boiling.

h. In blowing soap-bubbles it appears that these may be increased to indefinite size by the internal pressure of the breath, so that these bubbles do not follow the conditions of this proposition, *a.* In these bubbles the water is combined with a viscid substance which permits laminated displacements that under the tensile strain are in rapid motion, in which they take interlaced diffusional forms, the conditions of which may be observed in the polariscope by the colours and forms produced by light passing through them.

i. Quite independent of the mode of construction of a bubble resting upon a like liquid plane, it will possess generally a duplicate area to the plane beneath, as this is a figure of equilibrium between its tensile forces and capillarity of support.

j. The density of the surface of a free bubble may be made evident by its surface elasticity and non-cohesion with a like dense surface in which it may be brought in contact, and in which the surface molecular positions would have to be changed for it to enter. This I have found may be shown by a pretty experiment with a small bubble of soap solution blown at the end of a small pipe, from which it may be jerked off, upon the surface of the solution in a vessel below. In this case the bubble will often, after contact, *rebound* for several times as a free elastic body. This bubble, when it enters the liquid surface, will occasionally take for an instant of time the triplicate area to the surface on which it reposes.

Hypothetical conditions of quiescent evaporation.

28. PROPOSITION: *That the extensile force of a liquid surface under quiescent evaporation at low temperatures may form points of evaporation that will occupy limited areas of the entire liquid surface, these points taking the functions of bubbles for the escape of vapour.*

a. By the conditions of 26 prop. *b*, a vapour forming at a surface by heat forces would expand the surface in a certain ratio, which is shown by the duplication of the bubble area upon parts of the surface. A liquid could not evidently expand to so great an extent under the cohesive restraint of quiescent evaporation where heat forces were less active. I therefore offer the above purely hypothetical proposition as a means of accounting for the apparent quiescence of a liquid under evaporation, that from certain facts and conditions appear to my mind probable.

b. It is in the first instance probable that evaporation, either quickly or slowly, is a similar *mode of motion*, although the case of extreme evaporation, or boiling, will be a motion more intense in relation to the temperature than that of a quieter evaporation. In boiling we have, as shown in the last proposition, areas of excessive evaporation, where the surface is thrown upwards as a bubble. If we assume in quiescent evaporation that we have also areas of intensity which will be proportionally less, then although these areas may not possess sufficient surface extensile force to entirely throw up the surface in bubbles on any part, yet they may still be active as a molecular compression, which may seek another mode of expansion. This modification, I think, may probably be by a protuberant nipple or a *molecular tumerole* of invisible dimensions.

c. Under the general conditions of evaporation resulting in contiguous compression of the surface of a liquid it appears to me that the principles generally offered for the fluid state will show a nearly sufficient cause, which will be relative to the conditions of the last proposition. This we may conceive in that a fluid is a body the molecules of which will not bear pressure upon one side more than upon the other without moving from the excess of pressure. The force of gravitation will of itself, cause every molecule of a liquid to seek the lowest point, and thereby jamb, as it were, its fellow molecule, so that at the surface, the pressures upon the sides and bottom of every surface molecule will be greater than the pressure upon the top of it, although the extra pressure upon the lower surface of the top molecule may be only equal to the action of gravitation upon the mass of the superimposed molecule itself, leaving the surface molecules in vertical equilibrium (16 prop.). Therefore, as the side pressures will be jambing by their excess all surface molecules under every condition, these will be in the most unstable state of equilibrium of the liquid mass. This constant upper minus pressure and internal jamb will influence a surface molecule to constantly move upwards against the cohesive attractive forces surrounding it as demonstrated for capillarity, 18 prop., particularly as it will in any possible downward movement be strongly resisted by the more solid construction of the liquid immediately beneath it, and that it cannot move to a lower level by its gravitation or cohesion. In this manner also, any small movement of the aerial surface, by which one molecule is made by the most infinitesimal quantity to be more prominent from any cause than another, this molecule will tend to be ejected from the surface by the elasticity of the system and the closing of the lateral compressed circumferential molecules around it, and under it, in their endeavours to restore the equilibrium of surface gravitation. In this case the particular molecule selected may overcome cohesive forces, and even be ejected with a force proportional to the excess of elastic compression over cohesion of the surface molecules, which permits them to close the space beneath it. If we imagine such an ejected molecule in this case as one evaporated; for the further continuity of the evaporation after the surface is broken, it only becomes a question whether the ejected molecule, has or has not, some mode of connective molecular construction with other free molecules, or that it possesses elastic force sufficient to support itself at its temperature above the liquid surface, so as to become a free molecule, in visible vapour such as we see seething in mass over a boiling liquid, or to evaporate to a gas after its ejection.

d. Under any conditions, if the elastic surface force is sufficient to eject any slightly distended liquid molecule, we may observe that as soon as this molecule is free, the entire surface of the molecule will now be exposed to evaporation by the perfect release from the hydrostatic forces that surrounded it formerly when it was a part of the liquid system, so that it will not again interfere with the surface from which it was ejected, unless it is pressed into it.

e. By the above theoretical considerations it appears to me that there may be only a certain number of points under quiescent evaporation in which the liquid evaporates, although this number will be at all times very great, and that these points will, by the principles of surface extension, be equivalent to *molecular bubbles*, or perhaps, *tumeroles.* At the position of such constantly forming bubbles or tumeroles the evaporation may be carried on by a constant ejection of molecules separately. The series of molecules occupying the surface next beneath the extreme surface would in this case fill up the gap formed by the extruded molecule, and by the lateral surrounding pressures these would again be in a condition to eject the newly-intruded molecule. Then, as the molecules beneath will be thrown out of equilibrium, and those on the surface also, the protrusion of one molecule would form a minute bubble or nipple, to which the expansive force of the system would tend constantly to press others, and the nipple being the most free point, it would be the point of ejection of all the near molecules constantly as they arrived. The same principles of motion may be observed by pressing a series of balls in a groove; the balls of the series will be ejected at a certain position only.

f. If such a system be active, as that described above in the formation of vapour or gas, we could also imagine that the reverse would occur under condensation. That is, if the molecular force of the elasticity of the system of aerial matter superimposed, exceeded the resistance of the distention of the molecular system of the liquid beneath, that the molecules of vapour would be pressed into the surface in certain areas, and by intrusion resolved into the liquid system.

g. The principle of evaporation by infinitesimal bubbles, by surface expansion upon this hypothesis, may be roughly shown diagrammatically as follows as a trial hypothesis for perfectly quiescent evaporation.

Let Fig. 33 be a surface under quiescent evaporation. Let the expansive surface force place the whole of it in unstable equilibrium of molecular compression. Let forces, extending to a distant surface represented by the arrows, press upon a molecule above that is assumed from any primitive cause to be one most out of equilibrium of the surface series; then would evaporation by distention of surface constantly occur at the same point, this being the weakest, to resist surface expansion by heat forces.

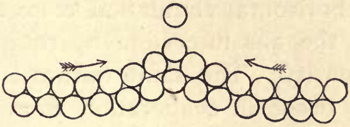

Fig. 33.—Molecular condition of Surface Evaporation.

h. Under the above conditions it will be seen that any force which causes a surface disturbance, and thereby necessarily produces convexity on parts of the liquid surface, that the evaporation will be by this cause rendered much greater.

i. It will also be readily conceived, that having a system of tumeroles established on a surface of sufficient extensile force to produce such tumeroles closely together, that there would be a tendency for one tumerole to break into the other, and thereby form ridges or lines of evaporation. Further, if evaporation be by any cause more intense at one side of an aerial plane than another, this would lower the side of the plane and perfect equilibrium of surface would be destroyed. Under this condition the tumerole or ridge would be deflected on one side as in Fig. 34, above, or overflow the plane entirely as in Fig. 35, and evaporate at the edge of a mole-

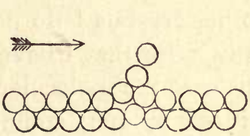

Fig. 34.

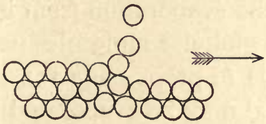

Fig. 35.

cular plane by the constant movement of ridges, that would be thereby formed and take one direction.

The above condition, I have been able to approximately observe by an experiment. Thus, if we intimately mix fine dust, as, for instance, that of lycopodium with a drop of alcohol, and observe it under the microscope, the surface plane will be observed to be constantly drifting as overflowing currents to the evaporating edge of the spirit, where a horizontal circulation keeps up the same system of evaporation on the absolute edge by rotation engendered in local points of intensity. The same outward extending form of motion would also occur in evaporation from capillary tubes, in which the extensile surface of the liquid, 18 prop. *g* would be constantly set free upon the surface of the tube.

j. I can offer little experimental demonstrations for much of this proposition; but there is one class of facts which may give some inferences for the molecular tumeroles proposed, under perfectly quiescent evaporation; this occurs in cases where gases or vapours extrude from resistant surfaces of some semi-fluids, as in the floors of volcanoes, particularly in mud volcanoes, and the same principle may even be observed in the formation of the cones of volcanoes themselves in relation to the general cohesive surface of the earth. The extrusion in these cases from the more resistant surface is by *tumerole.* The same active principles appear to have been present upon the surface of the moon in producing its present surface. Whether such principles can be applied to a perfect liquid surface under evaporation I only suggest, as this mode of motion is necessarily invisible? Further, I offer this proposition as altogether hypothetical.

k. In the evaporation from ice or other crystals I do not assume the large globular molecules necessarily exist that I have proposed for liquids (6 prop., page 15). In this case the molecular planes of the crystal may be evaporated from it one by one; commencing at the most free angles, and continuing along the planes.

CHAPTER III.

MOTIVE FORCES ACTIVE IN FLUID SYSTEMS IN THEIR MOVEMENTS TO AND FROM THE STATE OF EQUILIBRIUM OF REST:—FLOWING FORCES.

29. PROPOSITION: *Every free motion of a fluid is a movement towards equilibrium with surrounding pressures. The fluid in this motion follows the most frictionless course.*

a. It is well known as a physical law that a unit mass of gravitating fluid has its molecules in such a state, when they are at rest, that equal gravitation force presses upon every stratum at the same depth in its system, and that this force is everywhere *per area* as the entire weight of the fluid above the same area exactly.

b. If force be impressed by other means than by the simple effect of gravitation, as by heat expansion, or a pressure in a part of the system, the fluid thus impressed extends this pressure in a certain small space of time with equal force to all sides of its containing vessel, *per area*, as far as the fluid extends, so that every particle, if not further disturbed, rests finally in equilibrium, and the force with which the one part of an inscribed mass was impressed, is now equally impressed *per area* on all other parts of the same fluid plus the effects due to gravitation (*a*).

c. In a liquid issuing from a vent, its movement will be at every instant directed to restore the state of equilibrium which has been disturbed by the opening of the vent, and the movements will be greatest where the pressures vary the most. The liquid will therefore be moving from all parts towards the vent, proportionally to the differences of molecular pressures in the parts of the liquid and the resistances offered by other parts. The minus pressures upon each molecule being in this case situated towards the vent by the minus resistance in this direction, and the greatest pressures, upon the same gravitation plane, will be upon the most distant parts of the system from the vent. The nearest possible condition of motive equilibrium will be established by a system of pressures decreasing in constant ratio from the most distant parts up to the vent where the pressures are entirely released.

d. The exit from a vent embodies many conditions that may be applied in the practice of hydraulics, and the components of forces which govern this efflux have engaged the attention of nearly every writer on hydrodynamics; however, exact laws have not heretofore been discovered, or even such components as may reconcile theory with practice. I will therefore follow this matter in detail for such demonstrations as I may be able to offer.

30. PROPOSITION: *That in the movement of a liquid toward a hole in the bottom of a vessel, the hydrostatic pressures will act directly with gravitation in the parts of the liquid vertically over the hole, and as resistances in the parts of the liquid horizontally contiguous to the hole. The area of efflux of the liquid from the hole will be the mean of the directive impulses of vertical and horizontal pressures; except within the area of the hole where the directive forces will enter into composition.*

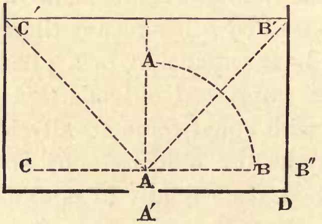

Fig. 36.

a. If we disturb the equilibrium of a body of liquid at rest, as by opening a hole in the bottom of a vessel of water, then the lateral pressure towards this hole will be on every side equal to the vertical pressure over it as was stated in the last proposition (*a*), and the volume and velocity of the water will be regulated by the *mean force* of the *directive influences* and the reaction of the compressed elasticities in the parts of the liquid nearest to the hole.

b. If we take a vessel as represented in the figure above by the outline C′ C B B′ (Fig. 36) containing water in a state of rest to the line C′ B′. Then the lower stratum of the water in the vessel would be pressed at every point equally by the weight above; consequently, if a hole were now opened in the bottom of the vessel so as to remove resistance from this point, as at A to A′, then the water from C to A and from B to A, supposed a thin stratum inside the vessel, and very near the bottom, would be a part of the water that would be first induced to move towards the hole, this being the part of the system most compressed, therefore more especially influenced by the minus pressure at the hole.

c. If we take the hole to be a point for the sake of omitting the conditions of its diameter; then the parts of the stratum of water upon the bottom surface of the vessel would be equally moved in every horizontal direction towards this point at the same time, and with the same radial velocities; that is, they would move towards the hole by pressures varying inversely, for any stratum, as the squares of the distance into the pressure of the head of water above the stratum taken. Under these conditions the movement from C to A would be equal to the movement from B to A, but as these moving forces are opposite in direction, we may imagine that, in so far as the horizontal impulses of the parts of the lower water moving towards the hole would have power to act by their direct momentum, that the water would be jambed by their opposite directions, and thus form a resistance to efflux, by a kind of compressed constructive arched circle of the water round the hole, and the action of these horizontal forces would thereby *contract by their pressures the free area of efflux* in proportion to the forces of impression upon the descending column.

d. That this is the actual condition present may be partly demonstrated experimentally by inducing in the general system of motion of water in a cylindrical vessel a greater than average resistance to the horizontal impulses of the lateral parts of the water, so as to withhold a part of their lateral impulses from the vertical descending column at a position directly over the hole. In this case by my theory the vertical column would be projected through the hole, and the horizontal forces would remain in their arched static position in the liquid above the hole. This effect may be produced experimentally in water flowing from a hole in the bottom of a vessel by giving the water a slight motion of horizontal rotation. The tangential forces of the revolution will then slightly decrease the horizontal pressures which act as resistances, and restrain them from direct impulses upon the vertical column, so that the vertical column will now empty itself out through the hole, and a clear aerial pipe will be formed from the surface of the water to the hole.

e. By the conditions offered in the last proposition *a*, we assume that the hydrostatic forces above the area of the hole are exactly equal to the vertical forces over all other horizontal parts of the bottom of the vessel *per area.* Therefore other parts of the fluid of equal depth will be equal in motive energy to the parts near the hole, or, for instance, to those surrounding it, so that if we were to take a narrow ring of the lower stratum of water surrounding the hole, and conceive that hydrostatic pressures were equal in all directions, the pressures of the lateral horizontal forces upon the column directly over the hole in the vessel, would be exactly equal to the vertical forces of the water in the hole at the exit of the vertical column; and the vertical and horizontal forces would be equal. The vertical forces in this case may be assumed to be arranged proportionally to their freedom from resistance radially *from* the axis of the descending column, and the horizontal pressures arranged as resistances also radially from the circumference of the vessel *towards* this axis. Then it will be clearly seen that the vertical descending forces will act directly with gravitation, and the horizontal pressures directly as resistances. The outflow of water would be necessarily as the mean of these forces. It will be seen by this construction, taking into consideration the perfect resistance of the bottom of the vessel to all hydrostatic forces except those over the hole, that the flowing forces do not form, *a resultant of the composition of horizontal and vertical forces*, but that the entire vertical forces are *direct*, and that the entire horizontal forces which are deflections from the lateral vertical forces act as *resistances,* so that, supposing these are equal about the hole, the outflow would be equal to that of exactly half the area of the hole, except for the gravitation forces in the water directly over this area, where the direct horizontal forces would lose their quality of resistance and enter into composition with the vertical forces.

f. To satisfy the conditions of the horizontal impulses being direct and resistant in the above construction, we must, as before stated, consider the hole as infinitely small, as the resistances of the bottom surface of the vessel, which deflect the gravitation forces from direct to horizontal impulse, will cease to act quite in this manner after passing the edges of the hole. Further, in the direction of movement of the horizontal forces towards the hole here supposed, every lower surface plane would possess a certain amount of *carrying force* towards the hole, so that a particle of liquid descending on the lateral parts of the vessel would not quite reach the horizontal plane of greatest pressure before it was deflected towards the hole. Therefore it would not offer resistance at the hole equal to a horizontal force *exactly*, but rather according to its moment in an oblique direction at its last position, towards and over the edge of the hole. Further, within the space of the hole such oblique directions as are induced within the efflux, would be active beyond the inner edge of the hole, and would therefore carry their impulses directly towards the centre of the issuing column, and this would cause the assumed jamb of the horizontal forces to be outside the surface of the vessel instead of directly over the hole within it, that is, if the hole were made in a material assumed to be very thin. The vertical forces by this cause, would have the equivalent value of a greater fall in the greater height to the same exterior point, which would be the point of actual resistance to the vertical column. Therefore the efflux volume of the liquid would appear to be greater vertically than that due to depth, proportionally to the size of the hole by an exterior quantity, and we may assume, that directive pressures would react by further deflection of the current to increase the aperture of outflow, above half its area as just assumed (*e* above) for components of vertical and horizontal forces only.

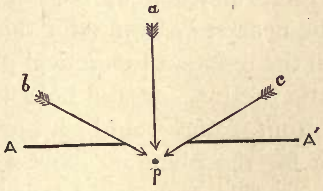

Fig. 37.—Diagram of Direction of Forces.

g. This principle may be shown by the above diagram, where the bottom of the vessel is represented by A A′. The vertical force by the direction of the arrow *a*, and the deflected horizontal pressures by the direction of the arrows *b* and *c* which meet at the point *p*, the theoretical focus of the resultant of the component forces acting at the hole. In this manner, the ascendency of the directive force in the vertical column over the horizontal would render the force of projection hyperbolic, so that the focus of the composition of deflected forces in the hole would be somewhat lower than that derived from the initial directive forces in the vessel. The above conditions relate entirely to conditions of aperture for efflux of the water by composition of directive forces only, and do not in any way touch the conservation of energy of the system, which will be taken presently.

h. In the engraving Fig. 36 the bottom of the vessel is made for illustration as wide as the vessel is high. This is immaterial, as the vertical force gives the ratio to the equal horizontal directive forces for any horizontal plane about the aperture, which will be found nearly equal for all areas of sufficient extent to give directive impulse to the liquid at its efflux.

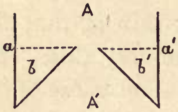

Fig. 38.—Diagram.

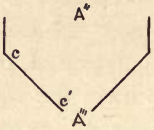

Fig. 39.—Diagram.

i. If the hole were interiorly conic, as in Fig. 38 above; the resistances of the inclosed volumes *b* and *b′* would be equal to the hydrostatic pressures upon them at the bottom of the vessel previously taken, and the area of outflow would be nearly the same as with a square bottom, except that the imaginary static water surface *a a′* would be less frictional than any solid, and this would give a slight excess of horizontal force, and thereby restrict the area of efflux, so that this would be the nearest condition for the efflux to be half the area of the hole, from the resistant theoretical plane of motion in the liquid possessing little friction. If the hole were exteriorly conic, as in Fig. 39, the bottom planes would give less resistance by their inclination, as shown for the plane *c c′*; but as more lateral water would be deflected by this minus resistance, against the direct descending current A″ A‴, and thereby restrict the direct vertical fall, the outflow would not be much greater.

j. If, in the above experiment, we had a vent at B B′ instead of at A A′ in Fig. 36, the hydrostatic force A B would be direct, and take the place of the vertical force in the previous cases, and the force in the direction D B′ would be negative, and take the place of the horizontal force.

k. If, in a hole upon the bottom of a vessel, we diminish the horizontal force by supplementing the vertical force, as by the addition of an exterior short pipe leading from the hole at the bottom; the vertical force becomes the greater component from the excess of hydrostatic pressure due to the excess of height, and therefore the horizontal is no longer equal, and the efflux area becomes greater proportionally as the directive resistance of the horizontal force is less. The same principles will apply to horizontal efflux area by a short pipe; neglecting friction in all cases.

31. PROPOSITION: *That a column of liquid falling by free gravitation, will possess motive energy exactly the same as any other coherent body, or as a solid, which will be for its velocity and mass conjointly as the height of its centre of inertia into the height that it falls.*

a. If we apply the above proposition to the fall of a liquid through a hole in the bottom of a vessel, and make no consideration of the cohesion of the liquid to itself and to the contiguous parts of the vessel surrounding the hole, this will be the *measure of the force of efflux.* We may further assume that in a small liquid system, not complicated by intricate parts, by pipes or otherwise, there will be little force lost by friction from cohesion in direct mass movements, so that the direct fall of a vertical column in an open liquid will measure very nearly correctly the potential energy that the liquid possessed in its static position, by the gravitation velocity engendered by its weight, falling the average distance of every separate molecule in the column; this may be taken as the unit of force of a gravitation system which may be used as a measure of the *entire motive force,* and this *force value* may be divided or subdivided into other quantities, but cannot be exceeded by any means; or be lost except by the friction of the system of forces considered; which are here supposed for a simple system, as for instance, that from a hole in a thin plate, to be very small. Upon these conditions the outflow from a hole in the bottom of a vessel will be as the *full area of the hole, to the gravitation force of the liquid above it simply.* By certain causes the efflux may be a quantity less than this as by friction, and by any excess of velocity caused by the energy of elastic forces, increasing the velocity of outflow to be hereafter considered; but we shall find that this increase of velocity will be proportional to the diminution of efflux area; the excess of velocity being as the conserved elastic forces in the compressed fluid about the hole. In this and in other cases, the entire motive forces will not exceed in energy the entire volume forces due to gravitation of a column standing vertically over the hole of the full diameter of the hole extending upward to the surface of the water.

b. For the demonstration of the above, if we describe a circle upon the bottom of a vessel containing water, and measure the static forces of the water directly impressing this area, they will be found to be exactly proportional to the weight of the water standing vertically above the hole. Now, as the weight is derived from the action of gravitation upon the water, and that this fluid is throughout of equal structure, we may consider its *static force* as the gravity height of the system, which may be expressed by gh for any unit of area, and we may take this to be the exact measure of the gravitation force active as a constant outward pressure upon the circular area we have taken, which can never be *exceeded or diminished*, for the constant forces upon the same space upon the bottom surface of a vessel by the hydrostatic force or weight of the liquid above it simply. Now as the water is assumed in this proposition to be a cohesive body of equal structure throughout, as represented by a column standing over the hole, and that it moves in falling from the surface the *whole space represented by the height*, and from the lowest stratum *for no height distance* whatever, the outflow from the hole for the entire aperture will be equal to the fall of the water of the same diameter as the hole for *half the height of the column of liquid above it*, and the velocity for the whole area of the hole will be equal to that of a body falling by gravitation half the height of the liquid in the vessel. This is the value I will take for gh in the following propositions for the constant force of outflow from a hole in the bottom of a vessel. By certain conditions to be considered of conservation of energy of elastic forces at the *first instant of opening of a hole*, the forces will be greater than this, in proportion to the elastic reaction by pressures present.

32. PROPOSITION: *That a stratum of a static liquid at any depth receives elastic compression per area as the entire weight of the vertical column of matter standing above it. This elastic compression will react in any direction immediately the pressure is removed by which it was formed. Under these conditions the velocity of issue of a liquid from a free vent will be in proportion to the elastic compressions at the vent, the area of efflux, at this velocity, being as the directive velocities towards this, as before proposed* (30 prop.).

a. The elasticity of a liquid is found experimentally to be permanent, so that, after impression of any force that diminishes its volume, it will react immediately, and will return to its original volume when the pressure is entirely removed. In this process the pressure may be impressed in any direction, and removed in any other direction, with equal effect of reaction. So that if we were to take the condition of the pressure upon a stratum of water upon the bottom of a vessel with vertical sides, this pressure would be in every direction proportionally as the entire weight of the water above impressed the elastic forces of the *lowest stratum of it.* The energy of compression would be as the *conserved momentum of the descending forces of the entire liquid by which the compression on the bottom* of *the vessel was formed.*

b. Now as pressures in liquids by our proposition, will be ready immediately to react, the action of gravitation on a permanently elastic mobile fluid will not engender a velocity by the removal of resistance from any part of the lowest stratum, for instance, from a hole in the bottom of a vessel as of a body falling the mean height as it would be in an inelastic system, but as the compressions upon the lowest stratum which would represent in motive energy twice this amount as the motive force in this case cannot be assumed in equations as the mean, but as the lowest stratum which is double the depth of the mean. Therefore if gravitation forces for any vertical column of equally dense fluid be represented for velocity by a body falling from the mean height by gh; then by reaction of elastic pressures upon the lowest stratum, the velocity at issue from this would be as $2gh$, giving pressure and velocity one sign.

c. As I have already shown in the last proposition, that the entire energy of a liquid flowing through a hole in the bottom of a vessel, will for the entire area of this hole, equal the velocity of a body falling half the height of the column above (gh); it will easily be seen that elastic compression, if it occurs, is wholly due to resistance to gravitation; for if we were to consider the weight of a column of water resting on the bottom of a vessel as gh, and the resistance as being equally active in opposition to gh, the compression of an elastic system would be $=2gh$; or if we take the hydrostatic force gh, and resistance of the bottom of the vessel as equal to gh, and we remove the resistance, that is, one gh, the direct impulse of gravitation would be through the elastic system for the instant $=2gh$. Further, as this elastic force would rest upon the whole area of the bottom of a vessel, at the instant of opening a hole in it, this force would react; so that the efflux from such a hole would be for the whole area of the hole, and for the first unit of time that the elastic forces could react, equal to $2gh$ in velocity, that is, equal to the pressure upon the lowest place in the liquid in the vessel; this force being derived from the conservation of energy of the liquid falling to form the compression on the bottom, as shown at the end of § *a* above, and this would continue to act in diminishing intensity during the release of the elastic forces. But as soon as the elasticity caused by the resistances *had reacted*, and that a kind of motive equilibrium was induced by equality of pressures surrounding the hole (30 prop. *b*), then the area of the motive aperture would be partially closed by the directive projections, and a value equal to gh restored for the equation of area into pressure.

d. By the above conditions of the action of resistances in forming elastic compression in a liquid, it becomes clear that the elastic forces due to resistance act *plus* to direct forces. Therefore, we may assume a resistance to gravitation to engender an elastic compression $=2gh$, this compression being equal to twice the direct impulse of gravitation, which I take for velocity or pressure as gh. So that if we have $gh+gh$ engendered either through resistance or by direct force, it will be equally represented by $2gh$, and it will follow that as the same elastic compression in a liquid is caused either by a plus direct force or by a resistance, that the plus direct force may impinge upon the direct force at *any angle*, so that a plus force impressing a liquid at an angle of 90° to its direction of motion acts as an elastic pressure upon the system in the same way as though it was impressed as a direct force or as a resistance. $2gh$ being the resultant of elastic compression in any case due to the force or to the depth at which the liquid is actively impressed.

e. It will now be convenient to consider the conditions of the *constant flow* from a hole in the bottom of a vessel through a hole in a thin plate, in components of elasticity. In this *establishment of constant force* the entire hydrodynamic forces are taken to be equal to gh. If we take a single point in the centre of the hole in the bottom of the vessel previously considered, the *directive forces* moving upon this point (*b* above), will be vertical, $=gh$, and horizontal from each side $=gh$. Therefore, by conservation of energy of the elastic forces of the system (just considered at *c*) at the point of contact of the opposing forces may be represented by $2gh$, but in this case the entire horizontal forces are shown, 30 prop. *e*, to act as resistances of area, so that the fluid will have its *volume restricted in exactly the same proportion as it will have its velocity increased.* Under these conditions we may take the compressed elasticities due to resistance as equal for velocity to $2gh$ for both vertical and horizontal forces; the volume for unit of area of efflux will then stand in equation with the hole as $2gh$ for *vertical*, and $2gh$ horizontal, every particle directed towards the hole acting as resistances, therefore the volume will be for area (a) and pressure in equation $\frac{1}{2}a.(2gh)$, the volume of possible outflow being equal to that of $2gh$ for half the area of the hole, which must, in this case, be considered as infinitely small.

f. For the investigation of the functions of the diameter of the hole we may conceive that at its margin the impulses of horizontal forces will cease to act as resistances to the efflux, but they will leave the impression of their directive moments in elastic compressions upon the vertical column descending by direct gravitation, into which the horizontal forces will now enter in composition. So that the elastic forces engendered by the resistance (b, c) will diminish towards the central area of the hole, and be deflected and leave the vertical forces alone active. So that if we were to enlarge the hole indefinitely the liquid would fall in the central area as a free gravitating body; and as I assume little resistance from the now distant margins of the hole, there would be no elastic reaction in the central part of the efflux; therefore the area of central efflux would be the full area falling as the velocity of the mean height gh as before. Upon this principle, as all holes have dimensions, the velocities of efflux will be compounded of the forces represented by $2gh$ and gh, and the areas of efflux between *half* and the *entire area.* The values of which for holes of moderate dimensions I will now proceed to discuss.

g. I have shown, 30 prop. *b*, that the compression will be greatest upon the bottom stratum of water resting in a vessel, and that in the case of a hole in the bottom, that this stratum would be the first to move by the reaction of the elastic forces, but the motive activity of this compression must be proportional to the nearness to the hole; for although compressions will aid in impressing elastic force for velocity, the water on the distant parts of the lowest stratum cannot reach the hole to displace other parts that are crowding towards it with equal or greater force, therefore the entire compression on the compressed lower stratum of water will not project its force quite horizontally, as the water will receive part of its impulse from the directive momentum of all parts of the water moving towards the minus pressure at the hole, as discussed 30 prop. *f*. We may, therefore, conceive that the *most compressed*, therefore most active, lower stratum of the water, will have, as before proposed, a certain energy of *carrying* force towards the hole upon the strata above. Thus in the general composition of forces of efflux, this directive impulse will act at a certain *angle of inclination* to direct gravitation in pressing towards the hole, so that it will form a *resistance* to the descent of the vertical column of less energy than if it acted at an angle of 90° as previously assumed for the directive influence of the bottom of the vessel; and the impulse at this angle will increase the volume of efflux to a quantity greater than half, as it would be, if the horizontal forces were impressed quite horizontally as before taken, 30 prop. *b*.

h. Now if we take the resistance due to the moments of the efflux of lateral water through a hole in a thin plate, as proposed in the last paragraph, to be as the co-sine of the angle of inclination of direction for hydrodynamic resistance to vertical efflux, then the elastic forces will act directly as this co-sine in the moving water. For in this case the *elastic compressions will vary as the depth of motive directions of all the particles that can move towards the hole.* If in this manner we can by any means, as by the form of the vessel in which the hole is made, conceive resistance to efflux within the water near the bottom of the vessel to be equal to a force acting at a certain angle to direct projection, so that the inclination of lateral forces by this cause is less than 90°, then the elastic forces will decrease inversely as the co-sine of this angle, and the forces of direct gravitation will thereby proportionally increase the area of efflux, so that if the angle of inclination assumed is increased till it reaches 90°, or its direction becomes perpendicular to the horizontal plane, then in this case the co-sine representing elasticity vanishes, and resistance with it, so that again we have the full area of hole at a velocity equivalent to pressure gh. These principles may be shown by the following diagram.

i. Let A A′ represent the surface of water in a vessel of the depth A B; B B′ being the resting plane having a small hole at H′. Let the horizontal line C be the centre of inertia and therefore the centre of gravity of the column of water over the hole H′ so that the gravitation force for the column equals a body falling freely by gravitation the distance C H′, and for which, as previously considered for the full area of the hole, I use the term $g\,h$. Now we can assume that the elasticity of the system may increase the velocity of efflux above C H′ in any proportion to a body falling freely by gravitation from any part of the distance C H, the height C H′ being equal to h as before, and the height H H′ being equal to $2h$ proposed for pressure only. The hydrodynamic force $2gh$ being equally impressed at B and B′. If we consider the carrying force (§ g) to give direction to the particles of falling liquid upon a plane represented in section by the line H′ I, circumscribing the hole at like angle to the horizon;

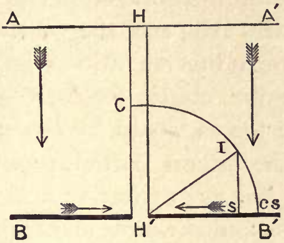

Fig. 40.—Diagram of Directive Angles.

we may then take the lineal values in the inscribed segment H′ I B′ to represent proportional areas to the lines. The above being accepted, then the resistance to gravitation impulse of such particles as fall upon plane H′ I to the efflux through the hole will vary as the co-sine of the angle I H′ S as represented by the space H′ S to the radius H′ B′; this last being taken to represent perfectly horizontal direction of resistance. I have before shown that the resistance may theoretically in extreme cases equal the direct forces, so as to increase compression to double the mean gravitation forces, and decrease the efflux area to half. Now, if the radius H′ B′ which represents horizontal resistance be taken as a quantity $= gh$ as plus elastic force, to a direct gravitation system; the co-sine to the radius of the angle I H′ S, may be represented by $gh.\cos$ I H′ S; the equation for elastic reaction being then $gh + gh.\cos$ I H′ S expressed in lineal values. Further the deflection by the plane H′ I which increases the area of efflux above half, may be represented by a circumscribing area proportional to the lineal versine, or as the space S, C S to H′ B′ and the equation for area may be represented by $\frac{1}{2}a + \frac{1}{2}a.\mathit{ver}$ I H′ S, $\frac{1}{2}a$ being that previously proposed for horizontal impulse exactly, $\frac{1}{2}a + \frac{1}{2}a.\mathit{vers}$ I H′ S being found experimentally to equal ·61 of area of efflux, allowing ·01 for friction of the system.

j. The angle between the carrying forces and resistances taking the resultant at unity, may be found by the formula $\cos\theta = \frac{1-(P^2+Q^2)}{2PQ}$,

θ representing the angle, and P and Q the carrying force and resistance respectively. This I find by calculation to be 42° 29′ 35″ for a hole through a thin plate omitting corrections for diameter of hole.

k. The angle given above must be considered as the equation of a curve formed for values of inclination in parts; but as the velocity of a body down an incline is the same as through a cycloidal arc, which has greater curvature, the areas of efflux would be as the incline into the vertical nearly. The form of curvature would be actually *parabolic,* as water nearly static, or in rotation, as 43 *art. ante* will show, would rest in the corners BB′, and the velocity of impulse of carrying force would increase in inverse ratio as the squares of the distance from the vent where the elastic forces would be released. The compositions of these forces would by the means proposed give a hyperbolic path for the lateral particles and for the directions of pressures to the hole; but as gravitation acts as a constant quantity throughout the process and the movements of this force are directly vertical, a parabolic trajectory would be really induced which would resemble inversely and approximately the path of a projectile thrown nearly vertical from the earth.

l. By the above conditions it is seen that the lateral impulses resist the direct impulses and regulate the area of efflux. It will also be seen that this area will be *constant,* for if we increase the vertical force we must increase the horizontal also; but the velocities will vary as the compressions or as the height of the liquid in the vessel. The compressions will also vary as the curvature of the free path of the particle as these are impressed by depth of the liquid through all parts of the curve, therefore, it will issue with velocity at the angle of compression of the last or lowest place on the curve before it enters the vent. This final velocity angle I will endeavour to discover by experiment presently.

m. If the fluid is very shallow, the directive energy of the lateral parts impresses relatively less force on the column from molecular friction, which we may assume a constant quantity equal in all cases, so that with greater depth, the lateral friction from this cause is relatively somewhat less, and the aperture is a little more restricted in a deep vessel than in a shallow one.

n. As the velocity of issue from a vent, as here proposed, is due to the compression upon the lowest place before issue from the bottom of a vessel, it would appear to be evident that, if we were to increase the area of the vessel at the point of issue so as to release the elastic forces before absolute issue, that we might increase the area of efflux in any proportion until the liquid issued at the full area of the aperture at the velocity gh as proposed. This must evidently be done by increasing the area of aperture constantly in direct ratio, that the compressed elasticities in the water are able to react by the mobility of its system. It will also be very clear that the form of aperture to produce this effect will be that of a frustum of a cone. With such a cone as here proposed it is said that Eytelwein by refinements of experiments of Venturi, but not acting on principles here given, was able to obtain efflux equal to a free area of ·98 of the aperture, ·02 only, being lost by friction, for what I consider the perfect equation of area for velocity equal to gh. This condition of the velocity of issue from such cones I find easily demonstrated by experiment. Thus, if we construct a vessel so that by an aperture we may project a jet upwards through a hole in a thin plate from a supply cistern above, as shown and described on page 108, and measure the height of the jet so projected, we find that it reaches nearly the height of the cistern. If we now take half the height of projection, and construct a frustum of a cone whose area nearest the vertex equals the area of the hole, and at its base twice this area, and place this vertically over the hole, the liquid will then have insufficient projectile force to rise higher than the inverted cone, and will simply overflow smoothly at its mouth. For the friction of water through a cone which causes the small loss of ·02 only of area, I will show the conditions further on.

o. If we consider the efflux area from a vessel to be derived from gravitation and elastic forces conjointly, and if these are proportional for one height, they will be proportional for all heights in like ratio for issues from the same form of aperture and under the same internal conditions, in all cases. Thus, supposing we have a thin vessel pierced with equal holes at equal descending depths, and we measure the efflux area of the first hole in ratio to its gravitation and elastic forces for its volume and velocity, all holes lower will be proportional to the depth of the first. Thus, assume the influences to be due equally to gravitation (g) and elasticity (e), so that half the effect is due to each for velocity and volume of issue;

then for efflux, $\frac{1}{2}g + \frac{1}{2}e =$ height of *first hole*,
$g + e = 2h$ second hole,
$1\frac{1}{2}g + 1\frac{1}{2}e = 3h$ third hole,
$2g + 2e = 4h$ fourth hole,

and so on, or we may say of the fourth hole in relation to the first that the liquid issues with double the volume and double the elastic compression $=4\,h$. If we add a pipe or cone, we may increase gravity area of efflux in any proportion until finally the elasticity disappears, and the fluid issues in volume equal to the aperture, with velocity equal to a body falling not less than half the height of the supply cistern minus the friction of the aperture, as in the cases of cones just discussed.

p. As in the projection of a liquid from a thin plate, the horizontal forces may be assumed to project their impulses beyond the edges of the plate into the hole, and as Eytelwein was able to construct a cone that delivered so much as ·98 of the aperture, and that from a hole such forces being free in a thin plate to enter into composition with the vertical forces; it became difficult, upon my conception of the principles of efflux, to see how the edge of the hole in a thin plate could offer any measurable resistance to the outflow; as the thin plate must offer much less surface friction than a long cone, therefore, a jet projected upwards from such a thin hole should rise very nearly the height indicated by the forces present at its efflux, in so far as the friction was concerned. If the elastic forces at the hole were equal to $2\,gh$, the efflux by reaction of elasticity (32 prop. *a*) should project the water the entire height of the supply reservoir. I therefore devised the following experiment to try this.

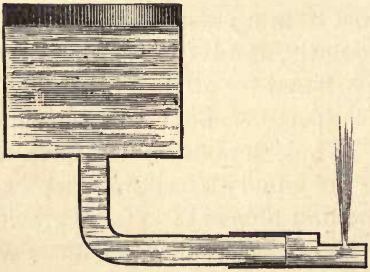
Fig. 41.—Ex. Upward projection of jet.

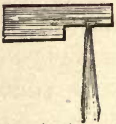
Fig. 42.—Ex. Downward projection.

q. An iron cask was connected with a pipe of 2½ inches in diameter as shown in the engraving above. Upon the end of the pipe another piece of pipe was fitted, so that it could be turned at any angle to the first. After closing the end and cutting exactly half this pipe away lineally near the end, and filling in the cut part with a flat plate (the drawing is incorrect, it shows less); the end piece described could be turned in any direction about its circumference; so that a line drawn along the centre of the upper part of the surface-plate would keep one position in space. Now making a hole central to this line on the plate, and fixing another very thin plate with a smaller hole over this, I could direct a jet upwards or downwards from the same spot. Means being taken by the principle of overflow of keeping the water in the vessel at one height.

r. By fixing two rules vertically, in such a manner that one was at a distance of 6 inches before the hole, and one at the same distance behind it, the eye could, by looking over the edge of the two rules, at once measure the height of an intervening jet by the incidence of the lines which could be seen simultaneously with the jet. With this apparatus, the water in the reservoir standing constantly at 38 inches, a jet was projected, when this projection became moderately steady, 37 inches. The projection (not the force) being $\frac{1}{38}$ less than the entire height. This loss would be due to resistance of air, friction of apparatus in deflection of directive impulses and side contact, and minus velocity from the composition of forces in the vessel and in the diameter of the hole. At this height it was quite clear that all the water from the jet did not reach the apex. It was also clear that it had to support a greater volume of water than that which would equal a constant jet of the size of the issuing column, for the column of water issuing from the ·25 of an inch hole was spread out at the vertex of projection upwards to a cone, whose base was as nearly as I could measure ·65 of an inch in diameter, so that perhaps the extension of the column equalled by resistance, the imperfect projection for height. By comparative measurements I found this cone of similar form to that which in a conical pipe would permit the greatest volume efflux from the vessel.

s. The jet was now inclined at an angle of about 5 degrees to get the greatest elevation, and the water flowed over the edge of the projected cone just described, the path of projection not being in this case parabolic. To ascertain the resistance from interference of direct projection by the returning water, I reversed the movable end of the apparatus by turning the jet downwards, as shown in the second illustration, and calculated the relative outflow from the two positions in time for a given volume of water.

t. For the above experiment I placed a glass tube in the supply cistern, not shown in either illustration, with a bulb blown at one end, this tube being kept upright by a covering tube, in which it could freely slide; a hole was ground in the face of the outer tube, both tubes having index marks. With this apparatus the water was allowed to flow out of the cask until the glass tube sank to a line upon it which corresponded with a line upon the hole in the outer sliding tube, in which it was placed, having a similar line on the inner glass tube for a starting point. With this the time of overflow by a watch, from one line to the other was taken, which time occupied for upward projection 10 minutes 45 seconds. Now, reversing the tube as shown in the second illustration above, I found that the same quantity of water from line to line flowed out downwards in 10 minutes 41 seconds.

u. There being 4 seconds difference of time due to additional resistance from upward over downward projection, showed the projection to be nearly as the force of efflux for the highest part of the jet, the time difference for upward and downward projection being only $\frac{1}{161}$ part of the whole time. Further, in the downward jet there were tractional forces derived from cohesion of the water, conditions of which I have yet to consider, but which I conceive accelerated the downward projection by a quantity as great as may possibly be attributed to resistance of the air in the projection. Therefore this upward jet could not by evidence present be projected by reaction of a pressure $2gh$.

v. If we assume the height of projection H and the height of the water H′, the pressure will be represented by $\frac{H}{H'}$. Now, taking for granted that the angle calculated in paragraph (*h*) to be $= 42° 29' 35''$, which, by the following figure 43, we may represent as c B′ X the difference between these two angles d B′ X and c B′ X may be found by the formula—

$$\text{Sin}\, c\, B'X : \sin d\, B'X :: H : H',$$

which angle will indicate the direction of lateral forces at the instant of issue for the parabolic curve described, and the whole scheme of projection would be represented as in the diagram below for one side of the projection, which we may take by following a particle of water descending from the surface Z to its issue at Z′.

Let A A′ be axis of projection in equilibrium; B B′ hole in the bottom of a vessel; c B′ X angle of equation of incline to the horizon for area of volume $=42^\circ\ 29'\ 35''$ (h); d B′ X final angle of projection of forces within the vessel; giving the velocity of projection from the compressed elasticities of the last motive part within the vessel shown upon the vertical Y. This forms the motive angle of projection

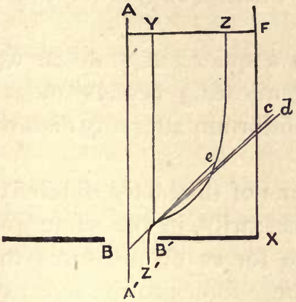

Fig. 43.—Diagram Path of a Falling Particle.

into the boundary of a column of the diameter of the hole. This I find subtends an angle to the horizon of $41^\circ\ 7'\ 34''$ or $=\frac{\sin.\ c\,B'X}{\sin.\ d\,B'X}=\frac{H}{H'}$, $\frac{H}{H'}$; being equal to $\frac{37}{38}$. As there are many difficulties in the experiments, I only assume the above approximate, and most probably c B′ X and d B′ X should be one angle; e B X angle of composition of forces for the area of the hole within the projected column, which gives to the issue a slight minus velocity, from inclination to the horizon $= 65^\circ\ 33'\ 47''$. The angle is drawn in the figure above at about 45°; my first conception of it by equation of the composition of moving and static forces. It should be in equation with the moving forces only between $41^\circ\ 7'\ 34''$ and 90° (vertical).

w. A particle falling from Z will be deflected into the column issuing at A′ in radial position to the semidiameter of the hole A′ B′ at Z′, as Z was to A F at its starting point near the surface in respect to the hydrodynamic forces upon it, that is, neglecting all function of the friction of its motion; the general conditions of which will be taken in the next chapter in more demonstrable phenomena.

33. PROPOSITION: *A unit mass of fluid is a self-contained cohesive system of matter. It will conserve its motive elastic forces if active, or these forces may remain in the equilibrium of rest within its system, although in contact with other motive systems of fluid matter that are different; except in so far as the exterior systems, by friction* *upon, or intrusion within the fluid mass, may impress their motions upon or within it.*

a. It is well known that any number of communicating vessels filled with a gas will communicate the pressure the gas receives in any one part, after a time, equally over the whole surface of the vessels.

A liquid resting in a quiescent state, as water in a vessel, or in any number of communicating vessels under equal pressure, will find its gravitation equilibrium after any disturbance which produces motion in one part.

b. If two fluid systems of distinctly different densities are in contact, they will not necessarily, in the same way as the above, communicate their motive forces or pressures to each other as they would communicate these forces to the same system of matter; but the communication in this case will be conditional to the material cohesive systems of the fluids and their powers to embody the induced motions or pressures through motive intermolecular elasticities from the one fluid to the other.

c. The first proposition very generally offered in hydrodynamics for the equilibrium of fluids is that when a mass of fluid, supposed to be without weight, is subjected to a pressure, that the pressure is so diffused throughout the whole that all its parts are equally pressed in every direction. We have no fluids without weight, so that experiment must follow fluids with weight. The experimental demonstration generally offered for this proposition, is that of a *unit* mass of fluid surrounded by a dense vessel, when exposed to pressures at some one or more open parts of the system, will communicate the same condition of pressure, therefore of static equilibrium, in all other parts by an equal pressure per area on any equal surface; and so far the proposition holds good, for in this case the equilibrium of a single fluid of limited area is distributed to the entire mass, and motions derived from pressure are carried over to the distant parts, which act as resistances to the continuity of the pressure until these parts attain the same pressure per area, with a velocity in ratio to the fluidity or mobility of the system. But these motions or pressures may or may not be communicated to other elastic matter or to another fluid *equally*, as the mobile fluid can only move as an elastic cohesive *system of matter*, according to our proposition. Some of the special conditions of which movement I will hereafter discuss. I wish now only to demonstrate the fact as it practically exists. The proposition infers that, the equilibrium of a fluid may be partially confined to its own elastic system, which will be within the limits of its forces of cohesion, and that motive forces or pressures will not necessarily be entirely communicated to other material systems beyond its own system in certain cases; this the following experiment will demonstrate.

d. Let A A′ be a bent glass pipe of $\frac{3}{4}$ of an inch in diameter, each bend being 6 inches, containing air from B to C, and water at all points below this.

Let S be the surface of water in a reservoir in communication with the glass pipe, at 8 inches above the surface A′ in the pipe. Now if pressures and forces were equally communicated or diffused through contiguous systems of fluid matter, S and A′ should rest in equilibrium at *one height*, minus the difference of height C, A′. This should be so in that any pressure impressed at S, as that of gravity, acting on the mass of water in the reservoir, would be equally active on B. And if the fluids could communicate their motive systems of pressures through these forces as hydrostatic pressures, the pressure on B should be equally impressed at C, and the pressure received from the air at C should be equally impressed at D, through the aqueous system, so that all intermediate parts being of equal hydrostatic pressures, the extremes should be equal, and S and A′ should rise to one height, less only the difference of height between D and A′. But as there are in this case by separation, three distinct systems of fluid, the communication of motion necessary to convey pressure from one to the other, will depend principally upon the *unity of possible motions* in these systems, which in this case, by difference of density, is not great, so that in fact the water in the reservoir at S rests at 8 *inches* above the water at the point A′ in the above diagram, which was drawn to scale. The quantity 8 inches is taken arbitrarily, it will depend on the quantity of air in the tube, and

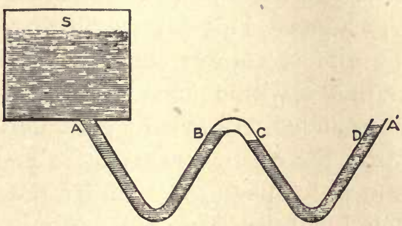

Fig. 44.

may be, I find, 24 inches in the same experiment. This principle is well understood in the practice of plumbing, where a few inches of upward flexure in a pipe will, under certain conditions, withhold the hydrostatic force of a head of water of 20 feet or more.

e. Where densities are approximately equal, and the fluids upon contact have attractive forces between each other, or chemical affinities, the impressed forces will act by continuity with a certain velocity which will be conditional to these forces. But there will always be a certain function of continuity of impressed forces by preference in their own systems, in which the motion is originally induced, as for instance, steam rising in dry air will maintain set forms until it is invisible, and may possibly do so afterwards; but this matter concerns diffusion, which I will hereafter consider.

f. The theory generally offered for the equilibrium of surrounding pressures, demands that the fluid should be without weight, that is, without gravity acting more upon the lower than the higher parts; but for the mobility of a fluid system in its power to distribute pressures, possibly the heavier or more dense the fluid the more perfect the distribution, as we can imagine in a liquid, for instance, there will be less deflection by contained elasticities within the system than in a gas. This I infer, in that the elasticities of the system will be almost entirely contained within its molecular gaseous envelopes, that is, within much less area, assuming the liquid molecule to be surrounded with such a gaseous envelope (8 prop., page 20), also in that the elasticities are more active under greater surface pressures (9 prop., page 27). Under these theoretical conditions a heavy liquid will distribute pressure more perfectly than a gas, particularly a permanent light gas. Experimentally we know that it is not difficult to convey force through water, but it is very difficult to convey force through air. It is said that the celebrated Papin, with compressed air produced by a powerful water-wheel, found that he could not move a very light machine by the air conveyed through a large pipe at three-quarters of a mile distance; that in another similar experiment with air tried by an engineer in Wales, a machine that produced powerfully compressed air at one end of a large pipe a mile long, would not blow a candle out at the other end —that in this experiment the compression of the air had to be continued for ten minutes at one end of the pipe before it could in any way be perceived at the other.[1] It is possible the perfect fluid with

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[1] 7th edition, *Encyclopedia Britannica,* vol. viii. page 109 (Pneumatics).

least density, hydrogen, would be the least active in distribution of forces. This is, I think, inferred by the resistance hydrogen offers to sound motions.

Traction of Fluids.

34. PROPOSITION: *Fluids having parts moved asunder by a force of not great velocity, will draw after them contiguous parts in ratio; that these are not withheld by greater forces than the mass cohesion of the fluid system. The moving parts in like manner will draw forward other parts, until a motive part engenders by the continuity of its motion a motive system of which the original motion only forms a part.*

a. The cohesion being equal throughout the whole system of a single fluid;—if this fluid is pulled asunder by a force it will part in the nearest plane to the motive force, as any greater distance from this nearest part would have to overcome greater inertia (21 prop. *d*). If any part of a liquid contiguous to a motive part is more free to move, than the cohesion or adhesion to solids in the more distant parts can withhold it, the near quiescent parts will necessarily follow the motive parts. This principle is visible in many phenomena, for instance, it is shown in the manner in which a small stream will run down a vertical solid surface in a close column, or upon the under side of an inclined plane. It is also shown in the manner that a wet cloth will empty a pail of water when it is hung over the edge. In this case the capillary force causes the first rise, which the traction of the water continues, the overflow being independent of the functions of the capillary force, which is equally active to uphold both the ascending and descending parts of the liquid in the cloth.

b. By placing a lathe of wood at an angle of about 25° to the horizon and permitting a stream of water to flow upon the upper end of the wood, this stream will flow along the under side to a depth of about ·13 of an inch without dropping. About twenty years ago I extemporized, and afterwards used for many years, a solid wooden rod of about an inch square to carry my photographic and chemical washings, for about 24 feet obliquely downwards across a yard to a sink, and this simple contrivance conveyed the outflow of a half-inch pipe as securely as a continuous pipe would have done, and entirely avoided the inconvenience from the corrosion of the metal pipe, that I had previously used.

c. The same system of cohesion which causes traction of following parts is also active in a certain degree upon all lateral parts, so that a body adhesive to the liquid, which may be conceived to unite the cohesive system of a certain mass of the liquid, when moving straight forward, will engender a tractional system of motion in the lateral moving parts, the same as upon all following parts of the liquid.

d. The values of tractional forces for the above case are well shown in an observation made by Mr. J. Scott Russell; for this I will offer Mr. Russell's own words as I find them given to describe *a wave of translation*, the properties of which I will hereafter more particularly consider. I use the illustration now to show the considerable effects of tractional forces which may drag powerfully forward a large mass of contiguous water horizontally, in partially overcoming the resistance of quiescent surrounding cohesions. Mr. Russell writes:[1] "I was observing the motion of a boat which was rapidly drawn along a narrow canal by a pair of horses, when the boat was suddenly stopped. Not so the mass of water in the channel which it put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation—a rounded, smooth, well-defined mass of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure, some thirty feet long, and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel."

It will be seen in this experimental demonstration that practically, the wave produced by the boat is equivalent to a mass of water dragged forward and projected upon the forward parts of the stream. This equivalence Mr. Russell's experiments further show, in the production of a similar wave by the intrusion of a mass of water equal to the volume of a wave at one end of a long trough. The water in the case of the boat is not pressed forward in front, as the level is not disturbed until the boat in moving advances upon it; therefore, the phenomenon may be conceived to be induced by tractional forces of the moving boat only. The farther conditions of which, as also the motive lines that the liquid necessarily takes in this and like cases, I will consider further on, where it will be

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[1] *British Association Reports*, 1844, page 319.

seen that the tractional forces equal a certain function of near adhesion.

35. PROPOSITION: *In the traction of liquids, or otherwise cohesive bodies, the parts set in motion that are laterally free, will move towards the central axis of the mass to seek the greatest area of contact. If the moving lineal series of parts be of any figure other than cylindrical, the most exterior parts will be drawn most towards the axis, so as to form a cylinder, or to approach this form.*

a. In 11 proposition, page 30, we have the conditions under which *static cohesive forces* will produce globular masses, by the cohesion of every molecule seeking the greatest area of contact. The same principles will apply to this proposition, but the forces will be *motive* and continuous; therefore the area of greatest cohesion of the molecular parts being in direct lines, will arrange the cohesive matter cylindrically, with the central axis in the direction of motion.

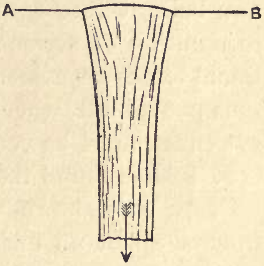

Fig. 45.—Traction of fluids.

b. We may observe the principles of the above proposition in pouring water from a mug. The broad stream which issues over the edge is drawn into a narrow cylindrical stream, and the water stands up in beaded outline upon the edge of the mug, where it is drawn over. In this we may notice that the cohesive force of the water, seeking the greatest area of molecular contact, acts in the same manner as the cohesive forces in homogeneous plastic solids or semi-solids; for, if we use force upon a homogeneous soft solid to stretch its length, as the force of gravitation stretches the stream of water, an exactly similar contraction is caused by this stretching as occurs in the more mobile fluid, and the above illustration, Fig. 45, will represent either the stretched band of water, or of semi-fluid as red-hot glass, or of a metal as cold copper. In which cases the line A to B may represent either the edge of the mug over which the water is drawn, or a mass of solid hot glass, from which a band of glass is drawn, or a vice in which a band of cold metal is held to be drawn by a superior force to that of gravitation.

If this stretching, in the above cases of the fluid or other plastic material were continued, and the material were sufficiently cohesive, the band would constantly decrease in transverse breadth, and a thread would at length be produced. This thread would break asunder at the instant the limit of the cohesive force in the fluidity of the system of matter was exceeded.

c. The disposition of stretched fluids to form threads or cylinders has been attributed to surface tension. I have shown previously that surface forces tensile or extensile would in free matter produce the same globular form, where the molecular forces sought the greatest area of contact (11 prop. *g.*), the motive cylinder by this proposition, being as the sphere of a static system. This proposition being directed to the conditions of masses does not take any recognition of surface forces, for the same forms induced by tractional forces are evident in falling masses of water of many thousand tons per minute, where the surface forces, at most, as components to the effect, must be very insignificant.

d. I have found that the molecular tractional force of water may be very well shown by the following experiment:—Take a glass tube of about an inch in internal diameter and of about a foot in length. Place this vertically in connection with a reservoir, so that water can flow through it with small force. Now, if the supply to the glass pipe be by an aperture or pipe of a quarter of an inch circular diameter, it is certain that a larger stream than this of $\frac{1}{4}$ inch cannot flow from the glass pipe. Therefore, if the pipe be allowed to fill by being stopped at the end, and then released, the water, as it afterwards issues from the bottom of the tube, will be drawn out gradually to the quarter of an inch diameter of stream in about eight or nine inches. In some experiments I have followed, I have found that I can get a denser liquid stream by this means than by any other that I have been able to devise.


Fig. 46.—Ex. Traction of liquids.


Fig. 47.—Cap for Fig. 46.

e. If, in connection with the above apparatus, we place an open cell over the end of the pipe, in which there is a triangular opening as shown in the engraving above (Fig. 47), the stream at issue will be in this case triangular; but it will form a perfect cylinder in about 6 to 8 inches, if the supply force be small. If the stream issue with great velocity from the height of the reservoir, there would be present compressed elastic forces which would cause the stream to distend at issue; the conditions of which I do not now propose to follow.

f. The conditions of contraction of a lineal liquid system under direct strain, would permit the continuity of tractional forces in the centre of the system by lateral supply for an indefinite time, or until the liquid thread so produced assumed invisible fineness, if there were not present surface forces sufficient to overcome the continuity of the system at a certain diameter, as previously discussed. This is possibly the case with glass, which I assume to possess no special surface force, which may therefore, if kept fluid, be drawn to great fineness. But with liquids possessing extensile surface force, which possibly nearly all liquids possess, at a certain point the extensile surface force overcomes the cohesion of the mass, and the thread instantly breaks up into globular beads by principles discussed 23 prop. page 74.

g. Some demonstrations of this proposition, as regards lateral traction, were approximately given by Venturi as principles of lateral communication of motion to fluids. One of the experiments he offers is particularly demonstrative; this is Ex. 1.[1] The horizontal cylindrical pipe A C is introduced into the vessel D E F B, which is

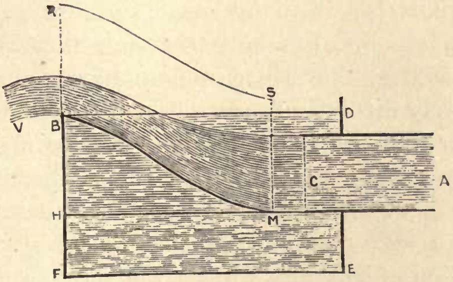

Fig. 48.—Venturi's experiment.

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[1] Lateral Communication of Motion in Fluids, by J. B. Venturi, translated in *Nicholson's Journal*, vol. ii. p. 173.

filled with water as high as D B. Opposite, and at a small interval from the aperture C, commences a small rectangular channel of tin plate, S M B R, which is open at top S R; the inclined bottom M B rests on the edge of the vessel B. It is 24 lines broad; the diameter of the tube A C is 14·5 lines; the extremity A is applied to an aperture at A in Fig. 48. The water of the reservoir being suffered to flow through the tube A C, the jet rises along the small channel M B, and flies out of the vessel in the stream B V. By this means a current is produced in the fluid of the vessel D E F B; this fluid enters into the channel S R, and issues by M B V along with the jet A C, so that in a few seconds the water D B falls to M H. The proposition offered fully meets this case, the water in the vessel is only the *most exterior* part of the system; it will be therefore drawn towards the axis of motion, as in all other cases.

h. The amount and mode of lateral communication of motions in fluids become very important in the consideration of the communication of motions to masses of static water contiguous to ocean currents, which by this proposition are induced to preserve a constant section, or rather to contract the section, and the same principles will be active in moving air also. But these phenomena in nature are complicated generally with other principles of active forces that will be hereafter taken into consideration.

i. The extent of lateral action of cohesive forces as tractional systems in liquids has in many cases been much over-estimated; this, it appears to me, has been the case through fluid motions being investigated by the theory of fluxions and the use of the calculus upon purely theoretical principles. Whereas, by experiment, I am led to infer that the tractional system may more conveniently be assumed to act by the *elastic cohesive* properties of fluids, the motive effects of which mode of motion I will describe in the next chapter. But I may mention now that in the system I shall propose, the traction of any plane will equal the movement of a given depth of liquid upon this plane only, and this will limit the tractional force which induces one direction of motion to a moderate distance, and will not permit unlimited extension, as we have assumed in what is termed the *vis a tergo* system of motion.

j. As the *vis a tergo* motion has been supported by so great an authority as Sir John Herschel, it may be convenient here to briefly discuss it, as it will not be convenient to return to it hereafter. By this system, a moving fluid flowing upon a static one of the same or other density, is assumed at first to move or *drag* with it an infinite small depth of fluid in contact, to which the adjoining stratum is conceived to offer a certain amount of resistance by its inertia, so that it *slips a little from its hold* upon the moving fluid, nevertheless it induces motive velocity upon the *lateral* stratum, by which this second stratum moves with a little less velocity. This stratum of induced motion then moves another contiguous to it in like manner, and this another, until by consecutive strata the inertia of the mass is overcome in diminishing series of velocities by the tractional or *vis a tergo* force; or if the force be continuous at equal velocity, and the lateral mass small, by moving contiguous parts, by which the inertia of resistance is assumed to be overcome, and the entire mass enters the system of the flowing stream as in Venturi's experiment. In this case the value of the cohesive force is estimated in the compound ratio of the flowing surface velocity, of flowing force, of time by continuity, and of *slip*, upon the lateral assumed static fluid. Therefore the lost velocity must constantly be proportionally equal in equal times, in the consecutive strata, to that the quiescent fluid gains, in like proportion, as these velocities on a cohesive system must react equally upon each other. Therefore, if we take the *vis a tergo* force to diminish in the duplicate ratio to the distance, which practically it may do upon these principles in a certain unit of space, the resistance being everywhere equal, the diminished velocity will be as the square of the distance.

k. I have no doubt that the above answers approximately to practical observation in certain cases, as I have proposed 34 prop. for some small distances from the central area of a flowing stream; but I doubt extremely if the principle has any reality whatever, as regards the loss by *slipping* being a function of fluid motion, except that it may represent in a certain rough way an apparent principle of action of cohesive forces. For the motions of fluid *I extremely doubt the possibility of slipping or gliding motions in any form whatever*, for which I shall endeavour hereafter to show reasons. In the present case the *vis a tergo* system, or any other system that may be expressed *by any constant proportional fluxions* that I have met with, will not answer for the lateral communications of motion. This is seen in that *any possible series of fluxions would require local maxima or minima at a certain distance from the moving part, to be true to practice, instead of terminal ones in the most distant part or in the extreme distance, as the theory demands.* For instance, I have not met with any form of fluxion that answers for the motive direction of fluids in distant lateral parts of a stream, where proportional motive functions, if real, should be still active, as in this case the *fluxional* force could not *act negatively:*—Whereas experience shows us that a flowing stream in a river does not move the quiescent side waters in its own direction, or accelerate them, if flowing more slowly in this direction by increments of velocity, in any proportion whatever to the lateral distance traversed; in fact, in certain cases which are common we find that the entire flowing force of a stream in a wide channel induces motion in the lateral water *near* the banks *directly opposite to the flowing force.* This, I will show hereafter, is in principle uniformly and *necessarily the case* through the state of elastic cohesion, always present in the liquid.

l. The whole of the conditions of the above I will hereafter consider on principles as demonstrable as I can make them by experiment. The proposition here given is confined to tractional forces that are *laterally free* from exterior resistances, and which cause fluids to flow uniformly to central areas by attractive forces, as defined 11 prop. So that this proposition, as regards tractional forces, applies to the water above the inlet at D in Venturi's experiment in the last figure, but does not apply to the water under the flowing stream M H.

Some conditions of flowing forces.

36. PROPOSITION: *The flowing force of a fluid moving without resistance is as its mass and velocity conjointly, which produces an equable force in one direction only. Any separate body that moves in a fluid of equal density will have a flowing force of the same value as an equal bulk of the fluid for its entire mass, whatever its internal motions may be. If the separate body be immersed in a liquid, and is of less density than the liquid, the part immersed will act as an equal bulk of the liquid, and in moving with it will possess an equal momentum of flowing force, or of direct energy.*

a. Newton has given the principles of the above proposition, which may be used to define *flowing force.* If any body, fluid or other, move forward in space, in which there is no resistance, such a body will move in direct lines with continuous equal velocity, and will preserve throughout its movement a constant configuration. If the body, in which a mass of any kind moves, is a fluid moving in parallel lines with equal velocity, and the body is of equal density to the fluid, there will be no resistance, and its form will be preserved throughout its projection, representing at the same time a motive force in the fluid equivalent to an equal mass of it. This form of motion may be observed in clouds that have a specific gravity not greatly in excess of the stratum of air in which they move, and in which they preserve their configurations. So that we may infer that all free motive parts will equally preserve their positions and velocities in flowing matter, if there are not present internal motions active upon external resistances.

b. If we take a case in which internal motions are present that do not disturb the external form in a flowing force, these motions will form a system, whose motive forces will be in relation to other parts of this system *only*, not to the general momentum of flowing force; and if the body be carried forward by the flowing force with the same velocity as the flowing fluid, its mass will have the same entire flowing force as an equal mass of the flowing fluid, although the flowing force of its parts may be different. For instance, the hull of a ship, whose submergence represents an equal bulk of water to the submerged bulk below the water line, would not cause any deduction to be made from the flowing force of the stream in which it may be floated, as the force of the bulk of the vessel moving in the fluid is equal to the same mass of moving fluid, neither would any operations within the ship, as the working of engines or moving of cargo, affect the velocity of the ship or its flowing force in the stream, except these motions cause external friction. Therefore motions that affect flowing forces are exterior or *surface forces* of contact on other matter, which cause by adhesion acceleration, resistance, or deflection.

c. Motions that are exterior to a fluid, and that do not act as resistances or accelerations to its flowing force, may engender motions within the flowing system which will henceforth be carried forward in it, and the motions may cause some parts of the flowing system to move more directly or quickly, and other parts to move inversely or more slowly; as, for instance, a cyclone in the air may be carried forward in a current, and part of the retrograde side of the cyclone may appear on the earth's surface to be motionless air, although it is in *relation* to the motive system of the cyclone, as active as the other parts, and the general momentum of the direct flowing force of the cyclone remains the same in the current as an equal volume of the current directed in its stream lines.

37. PROPOSITION: *All lineal parts of a free flowing fluid not affected by exterior resistances, that have a velocity more or less than the average velocity of the current, will be constantly resisted, so that a particle going too quickly will be retarded by other particles in front, and one going too slowly accelerated by others behind, until the final velocity of the parts becomes equal, and the fluid moves in direct continuous mass. The same principles will hold for a cohesive system moving by rotation upon an axis of inertia, where the velocities of the separate circumferential parts will finally become equally active, or as the radius of their motions about the axis.*

a. The principles of this proposition are given by Newton in his third law. They are here discussed to consider certain conditions, where the whole forces within a fluid are assumed to be contained in matter in contact. Any parts of the system that have by any means a plus velocity, will engender compressions upon the forward parts and rarefaction on the backward parts; and after a certain small space of time, the compression will be reflected and return upon the rarefaction, and equilibrium will be restored in equation with the average velocity of the system. By this means also aerial compression and rarefaction, assumed to be engendered by sound, would be obliterated in a short space of time.

b. This proposition does not relate to conditions where the compressions can escape out of the system and take another form of force, as, for instance, horizontal compression upon the surface of water, where the pressure can escape into the air and form a wave, which, as a potential system, can only recover equilibrium in the water by disturbing the general equilibrium of the surrounding system, so that it becomes motive, the conditions of which I will hereafter consider. Neither will the conditions of the proposition hold in any case where there are exterior resistances which may deflect and retard the direct motions of the continuous current in parts, the effects of which will be hereafter considered; so that the proposition relates entirely to forces and resistances *within the flowing fluid system.* The conditions of the proposition may be assumed, for instance, to be active in the central area of a flowing stream, where the lateral resistances are very distant, or exactly equal on each side of the flowing system, otherwise it would scarcely be possible to witness the action of the parts of a free fluid subject only to its own internal resistances. The proposition will therefore represent a *principle* which may be followed under certain conditions that have practical application, as components of fluid motion; in which the most free forward and backward parts of a lineal series of parts in a current are ultimately brought to uniformity of motion in their lineal directions, and the flowing force is continued in equation with the forward and backward momentum of the separate parts.

c. For the conditions of a circular equation, if we admit a general principle of cohesion in the fluid, and impress opposite sides of a fluid mass by any means with different velocities, or retard one side of the projection by any resistance whatever, the tendency will be to cause rotation in the mass, so that it will now in its flow describe an arc, or a complete circle, about the superior resistance, in such proportion that the original impulses of the flowing force may be free to act. In a coherent system, an induced circular motion by separate impulses will, if left to itself, after a short time produce and preserve a uniformity of motion in a circular area under the same conditions as proposed for a direct area, without greater additional friction on the entire motive velocity of the moving parts than is caused by the resistance at its tangential and its resting plane.

d. This proposition will be found in practice to apply to central forces of flowing streams. If there be lateral resistance in any part of the stream the motions will not be direct, as I will hereafter show. The proposition is offered as a principle that ensures constancy of velocity over established lineal directions, direct or circular, which causes a continuity of force in lineal or circular areas, where the motive power is derived from a *local cause*, that does not at any time affect the entire area of the fluid projected, but at first or in part only. The proposition is important in the consideration of oceanic and aerial currents to be hereafter discussed.

38. PROPOSITION: *That flowing fluids will preserve the same motive volumes and velocities by cohesion of their parts, and by equation of internal forces, in so far as they are free from internal strains or external resistances to do so.*

a. Taking a flowing fluid as a mass system in motion, it will flow approximately over equal areas in equal times, preserving, as far as possible, a uniform sectional area. Thus, if a stream flow along a river, and there are bays or inlets in the river, the stream will not be distributed into these bays, unless there is some exterior point of resistance which makes this the least frictional line of motion; but the stream will generally induce such a system of motion in the bay that its equal flowing volume may be maintained in the most free area. Some motive conditions under which this occurs will be hereafter considered. If a stream flow through a lake, there will be established lateral motions which render its path of little friction, and it will maintain an equal course, as is seen in a stream flowing through Lake Constance in Switzerland. The same will occur in a current in the ocean.

b. If an impediment, as a stone, be placed in the bottom of a stream, the water will rise as it approaches this impediment, and the presence of the stone will be clearly indicated by a deflection of the surface of the water. If the stream pass over a hollow, it will fall into it and rise upon the opposite side, so as to continue an approximately uniform motion as a lineal system of fluid. In these cases, the water being raised above gravitation equilibrium, there will be incident waves; but these are complementary phenomena that we need not follow now.

Cumulative elastic forces in liquids.

39. PROPOSITION: *If a free flowing liquid is resisted by a solid which offers by its form some less free opening for the continuity of the flowing force than that of its approach to the obstacle, the flowing force of the liquid will accumulate its momentum in elastic compression within the current, and press forward the flowing liquid through the contracted area, with a hydrodynamic force as the parts most compressed; therefore with greater velocity than the flowing liquid previously possessed in flowing through a more extended area.*

a. The conditions of the above were partly considered in the 26th proposition for hydrostatic forces in close areas. Our present purpose will be to consider open areas and constant forces; in which, if a flowing force be resisted, the resistance of the momentum will accumulate in the liquid as a compression in front of the obstruction; therefore, by the proposition above, the velocity will be increased as the hydrodynamic force of the compression at this point, into any possible free opening.

b. Taking this matter in detail for a constantly flowing current, we can imagine that the first or forward molecule of a free liquid will be brought in actual contact with the static solid, and as its motion will be stopped, it will make a percussion upon the solid, and be reflected. The second or following molecule will immediately impinge upon the first, and be likewise reflected, and a third molecule will impinge upon the second in like manner, and so on to the extreme distance, until the projectile force of the mass is taken up in the percussions and reflections, in which it conserves its elastic force equal to the force of resistance to the moving mass. Under these conditions the elastic force of the flowing mass is conserved as an entire compression in *itself*, and in the fluid in front of the resistance conjointly, so that the forces of reflected resistance are compounded with the direct force of the flowing mass. By this means a flowing stream in a free open area will be compressed and partially retained, so that it will rise against the static resistances, but will have its flowing force *increased* in the deflected current thereby, equal to the conserved elasticity of the compression, by which the flowing force acts cumulatively in increasing the velocity ot a smaller sectional area of the flowing force which appears in the deflected stream, exactly as in the cases of the hole in the bottom of a vessel discussed, in 26, 27, and 28 props.

c. If a large surface of impediment is placed in a running stream, as the pier of a bridge, the water will accumulate in front of the pier, and be pressed up above the average liquid surface for a long distance up the stream, and form a conic area of resistance in the water before the running stream encounters the pier; but the average velocity of a stream beyond the bridge will not be much impeded by the resistances of the piers, as the restrained elasticity of the resistance will react and greatly accelerate the water passing through the arches.

40. PROPOSITION: *Horizontally impressed forces in an open flowing liquid, will accumulate elastic compression at a distance in front of a point of perfect resistance, greater than at the same absolute point.*

a. The active molecules of a flowing stream, first coming to a point of solid resistance, will after contact be the *most reflected.* Other molecules impinging upon these, being more distant from the static resistance, will be less reflected from the less static solidity of their impact, and others less, and so on to the greater distance, until the approaching molecule will have a velocity only an infinitely small quantity greater in the same direction, than the resisting molecule immediately in front of it, or that the flowing force of the mass will be nearly equal in direct momentum in the same direction of motion to the resisting force. *Therefore, although the flowing force will be equal in the entire mass approaching the point of* *resistance, the reflective force will be greatest at the point of absolute resistance;* and as the reflected percussionary force by its direction opposes the approaching flowing force, at a certain point in the system, these elastic forces in opposition will act cumulatively and deflect the current with greater velocity than that of the flowing force into the area of least restraint, or conserve the elasticity of the flowing and reflected force, if there be no free egress.

b. The principles of the above may be shown experimentally, by placing in the central line of a wide open trough an upright peg, and allowing a current of water to flow down the trough past this; we may then observe that the flow of water at its general average height does not quite reach the peg, or that there is no elevation about this obstruction; but that the reflex action of the resistance of the obstruction raises a ridge in the water before the peg is reached, and that the ridge flows outward from the obstruction by a current, so that it entirely avoids the peg. The surface of the water, as it appears in this case, is sketched roughly below.

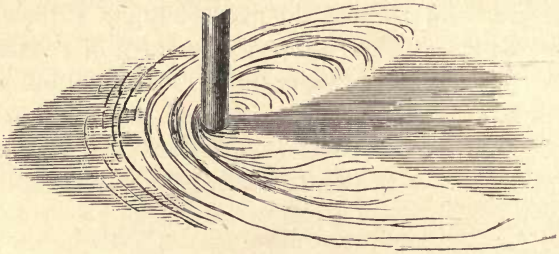

Fig. 49.—Ex. a peg in a stream.

c. The same principles of motion may be observed in horizontal obstructions, as impediments at the bottom of a flowing stream, which may be caused by any stone or projection thereon, but for a more exact definition an experiment may be offered. The apparatus I have used for this purpose of studying the exact conditions is a trough of about 4 feet long and 3 inches wide and deep, with glass sides, and open at both ends. This was placed at an angle of 15 degrees, so that water flowed along it freely. Across the trough near the lower end, a bridge was placed as shown in the engraving; any deflection of a flowing current, caused by the bridge, could be observed through the glass sides of the trough. Now, permitting an even current to flow down the trough by overflow from a vessel of water at a constant height at its surface, so that there was a stratum of water a quarter of an inch or more upon the bottom of the trough, enabled me to estimate clearly any resistance to the flow at the bridge by the difference of depth of water near it. The cumulation of elastic force from the flowing current was observable long before it reached the bridge; but its greatest accumulation was at *a*, as in the figure, which in this case was at about $\frac{1}{2}$ an inch in front of the bridge.

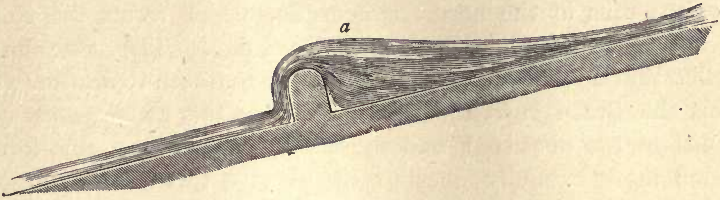

Fig. 50.—Ex. A bridge in a stream.

In this experiment, as in that given (12 prop. *c*), cumulative elasticity of the liquid is shown in expansion of volume. The water appears in this case to pile itself near the top of the bridge to nearly double the height of the current in a small stream of a quarter of an inch in depth. The conserved elasticity becomes released after it passes the obstruction, and there remains the same volume of outflow, at nearly the same velocity from the trough as though there were no obstruction.

d. The cumulative conservation of elastic force in front of a resistance becomes an important element in the motive forces of ocean currents, under many circumstances and cases. The general effect being the elevation of the water at a distance in front of the point of resistance, and augmentation of the velocity of the deflected current, where the flowing force and reflected resistance produce the greatest mass compression. Many instances of this principle of force action, that are visible in surface motions occur in ocean currents. One of the most striking of these is perhaps in the western flow of the main equatorial current in the Southern Atlantic. This enormous current takes its western drift, from reasons that will be hereafter discussed, direct upon the immense eastern promontory of South America, which extends from Cape St. Roque to Pernambuco, having its axis of greatest average projection at about 7 degrees south latitude. Over this solid point, or rather front, of resistance, the Southern Atlantic equatorial current is said to divide at a distance of nearly 300 *miles before it reaches the front of absolute resistance.* In the main equatorial current at 600 miles distance from the resistance, the water is said to flow at a rate of about 20 miles a day, whereas, in the deflected current, at 300 miles distance, under the cumulative elasticity of reflective force, it flows at a much greater velocity; thus in the northern deflection which forms the Guinea current, the rate is at about 40 miles a day. The same form of acceleration of velocity occurs also in the southern deflection which forms the Brazil current. I will return to this matter when other conditions are discussed, and the active principles of this form of motion more carefully investigated. I give this note to illustrate the importance of the principle enunciated in this proposition.

41. PROPOSITION: *If two flowing forces meet at an angle, by which they suffer resistance from each other, the elastic forces of the flowing masses will be conserved, so that if there is only one possible issue of restricted area, this will carry the fluids when they are thrown together with greater velocity than they each separately possessed.*

a. The conditions of the above are nearly the same as those discussed, 30 prop. page 94. It is found that a tributary flowing into a river, where the section is not greater, the current becomes swifter without a greater incline. The union of two pipes or conduits does not require greater area in proportion to volume of flowing water, to conduct the joint streams. In these cases, the resistance is partly a function of extent of resisting surface, and the distance of the flowing parts from this surface; but this would not account for an *acceleration*, which I assume is caused by deflected momentum, which is conserved, as elastic compression or hydrodynamic force, as in the conditions previously given in several propositions.

42. PROPOSITION: *The aerial surface of a free liquid, or that which is impressed with the smallest amount of gravitative force in the superimposed matter, will be also the least resistant of the mass, and to this surface, projectile elastic forces moving horizontally within the mass will be deflected.*

a. In an extensive volume of water, as the water is nearly incompressible, motions of displacement entailing greater or less bulk of water, will necessarily occur at the aerial plane.

b. Taking resistance as a function derived from gravitating mass only in its power to retard direction of motion for the most part, in the area of least possible freedom, the conditions would be as in the following diagram:—

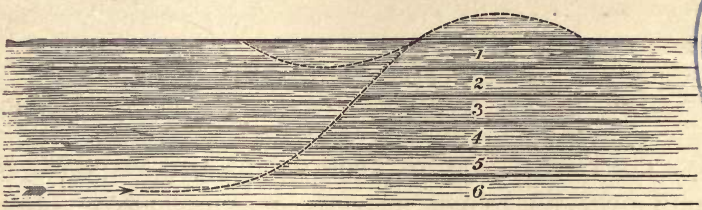

Fig. 51.—Diagram of minus surface resistance.

c. Let the above diagram represent a vertical section of water in a quiescent state. The lower surface being a rigid plane of adhesive resistance. Let the arrow represent a force embodied in fluid matter proceeding in lines parallel to the surface, and to the plane of resistance at any depth. Let the liquid suffer any kind of resistance from adhesion or viscosity greatest at the static lower surface. Now the force, represented by the arrow near the lower plane, finding the greatest resistance at 6, by the laws of fluid motion it will move to the less resistant plane 5. By such movement or deflection it will now possess a directive force upwards from the horizontal plane of its original force, and this directive force, being further deflected by the consecutive planes of decreasing resistance, will consecutively deflect the force more and more, as shown from 6 to 5, 5 to 4, and so on, until it reaches the surface 1, or the plane of least resistance. Now above 1 we have air, which offers considerably less resistance than the water, and we can imagine, following the same line of argument, that the force would be ejected here, and finally exhaust itself, possibly by this ejection against gravity from the surface water. But we are reminded that force does not exist except in accompanying gravitating matter, and that in this case it has manifold resistance by its adhesion as previously shown, so that it will simply swell up under the compression caused by the projectile force and overflow.

d. The above conditions are given for projectile fluids only, and do not relate to frictional movements of solids, that intrude no new mobile matter, the conditions of which will be hereafter taken.

CHAPTER IV.

MOTIVE CONTACT OF FLUIDS UPON SOLIDS AND UPON PARTS OF THEIR OWN OR OTHER SYSTEMS. PRINCIPLES OF ROLLING CONTACT IN FLUIDS MOVING UPON LATERAL RESISTANCES.

43. Principles of Motive Contact of any Material System of Matter.

a. Motions of displacement in constant contact of one mass or system of matter, upon or about another, can take place only upon one of two definite mechanical principles.

1. *One mass, unit, or part of any body may slip or glide upon any other unit or part.* This it may do either in a straight plane, lineal with the direction of motion, or upon any curve of regular radius upon which it is free to move tangentially. In gliding motions the whole of the surfaces of the mobile parts remain in constant contact upon the plane of motion where the gliding occurs.

2. *One part of a material system may be displaced in relation to the other part by rolling upon or about any plane or division where one part of the surfaces in contact is free from rigid fixing. The mobile part of a single system can have only one point of contact, and the contact can be on the same part of the plane for an infinitely short space of time only during motion.*

b. The cases in which any form of slipping or gliding motion in contact can occur in any material system are strictly limited to the conditions of the active parts of the material system being *rigid*, particularly at the surfaces of contact, the surfaces being necessarily held firmly to the mass, for this form of motion to take place, *by strong cohesive force at every point of contact, and by general fixing so that there can be no free local movement of the parts in contact possible near the plane of contact exposed to the friction of the gliding action.*

c. The cases in which rolling contact can occur, are limited to the conditions that one unit of the system, where it moves upon another is free to move upon its *centre of inertia*, or by some mode of *jointing* in a part of the system at a position *normal to the plane of contact.* Or that both units of the system be in like manner free to move on a joint, or free part or parts.

d. The above defined principles of motion are common in mechanics, and are known as motions of *sliding* and of *rolling* contact. It having been clearly demonstrated that the friction of sliding contact, however smooth or perfect the planes of motion may be, will be much more frictional than any system of rolling contact for the same weight of matter displaced; and for this reason also to resist the strain by friction of sliding, all parts of a sliding system, as just stated, must be made perfectly rigid. This rigidity in mechanics is effected by the hardness or perfect cohesion of the matter of which the sliding surfaces are formed, by the smoothness of the parts, and by the perfection of fixing at points in and near the area of contact; as we find that in any material mobile system of two bodies in motive contact, if one of these bodies by defect of internal cohesion and of fixing, is so far free that it can move upon any point not in the plane of contact in all directions, or in one direction only perpendicular to the plane of motion, it will then make under displacement *rolling contact* tangentially to the radius of such axis, joint, or free part of the system. The mode of fixing for the conditions given above may be by the action of gravitation upon the body, by inertia of mass, by cohesion, or by mechanical means.

e. From the conditions of the above mechanical principles, which are clearly applicable to every mobile system, it would appear to be certain that *slipping* or *gliding* motion could in no case be rigidly applied to the motion of contact of a free mobile system of matter of any kind, or of a fluid in motion upon any material body whatever, as a fluid mass must be conceived to be matter possessed of perfect mechanical mobility, which is equal to an *infinite number of joints* or free parts in its system. The only conditions under which the principles of mechanics could possibly permit any elements of gliding contact in a fluid would be in cases in which the rolling contact of the jointed parts were confined to so close an area that the general adhesion of the fluid to a solid, and cohesion to its own matter, would place the fluid under such molecular restraint, that this would be approximately equivalent to a system of mechanical fixing.

f. There is no doubt in my mind, that by taking some very superficial appearances of the facility of separation of the parts of a fluid and entirely neglecting the consideration of a fluid as a cohesive infinitely mobile system of jointed matter, the conditions of which are most palpably evident independent of all theories of fluid matter, that it has been taken for granted that no mechanical friction-saving motion would be necessary to be considered to overcome liquid cohesion. In such light, lateral resistances have been heretofore considered to be actually overcome by separating or gliding motions as they are termed, and formulæ built upon this principle without the introduction of quite arbitrary functions have signally failed.

g. If we consider the perfect structural solidity of water as we witness it under compression, and upon this condition of solidity alone, conceive any possible condition of molecular aggregation that can produce its cohesive system, we must conclude that if a frictional motion of slipping is possible in such a system, there must be present some angular or planic molecular forms of primitive aggregation capable of supporting the rigid planes necessary for this form of motion; and of this we have not the most remote evidence. Otherwise if there be in the construction of the fluid any system of free mobile contact with its separate parts, which the very nature of a fluid appears to indicate that there must be; then we must imagine that slipping would be very improbable, as any form of rolling motion in such a system would be almost infinitely less frictional.

h. We may realize the amount of friction in a sliding or gliding motion in a fluid by measuring the force necessary to separate the adhesion of the fluid from itself, or from other bodies in which it is in contact, as we find conditionally in phenomena of adhesion of wet surfaces. For it is certain, that for any slipping or gliding motion to occur, the force of cohesion of the fluid must be actually torn or broken asunder by any displacement of the separating planes, quite irrespectively of the direction of motion. It is possible, nevertheless, that there may be cases of fracture or of sliding separation in any material system whatever, but in a system of matter that is, practically speaking, infinitely mobile, I do not see how this may occur unless the necessary force is applied in such short interval of time, that the inertia of the smallest parts of the mobile system cannot be overcome in the same time. Certain apparent cases of this kind that may occur I will hereafter discuss, and such I take to be the only possible cases of displacement of a fluid in a plane by any element of sliding motion possible to be applied.

i. Taking the whole of the above in the very narrowest philosophical light, we are bound to imagine a fluid a more mobile system of matter than any machine possible of human construction, and in the machine we find that the mechanic in its construction, to save friction, will avoid sliding surfaces where possible, unless he has much surplus power, and that even when using sliding surfaces where absolutely necessary, he will make use of a fluid that is a mobile system of matter, or *lubricant* as it is termed, between his necessary sliding planes, so that what we absolutely find in mechanics generally, is that sliding planes of motion are reduced to as small an area as possible, as in the wheel-work in machinery. This principle we see followed also in road and railway carriages, where the friction of progression by contact with the earth is thrown upon the less area of the axles of the wheels, which axles are reduced to the smallest possible frictional area consistent with the necessary strength for the work to be performed by them, and this friction is again reduced by the intervention of the most perfectly mobile fluid system or lubricant, possessing the smallest chemical force of adhesion, that can be found capable of intruding itself between the sliding parts.

j. The facility with which rolling contact can occur in a mobile system will depend upon the direction of impulse to the axis joints or mobile parts. Thus if the resistances are directly in front of such axis, they will produce a pressure, so that it will be only in proportion that the material system or fluid can be directed aside to oblique impulse that rolling contact will be possible. On the other hand, resistances that are placed laterally, or that act perpendicularly to the plane of motion, will be those in which a mobile system will make rolling contact with the least possible friction.

k. A great principle *negative* to slipping contact of fluids was discovered by Venturi, in the evidence he produces of the lateral communication of motion to fluids, in his important work before alluded to; although he does not offer any theoretical system of rolling contact, as I intend to do; still he demonstrates in many ways the evident cohesive forces of liquids, which by itself would render slipping contact very frictional, if *possible,* in a mobile system of matter (35 prop. *g*).

l. Perhaps I should notice upon this subject, what is otherwise evident, that motions of rolling and slipping contact may be combined in one system, as, for instance, in certain grinding machines, where the velocity of the periphery of one of the rollers is made by mechanical means greater than another against which it slips and rolls. In such cases the friction is nearly as the amount of slipping motion, so that this may be taken mechanically as a sliding system. Unless very great refinement is required, then its components may be taken separately.

44. Two Cases of Rolling Contact in a Material System.

a. A body may be displaced by rolling contact upon another body in two ways, which are both applicable to fluid motions. 1. The body may itself form the roller and be of globular or cylindrical or annular form. 2. The body displaced may be a plane supported upon a number of such globes or cylinders or rings which may act as *friction-savers* to the motive plane. It will be convenient briefly to consider the principles of these motions before offering discussion upon the principles of rolling contact to be applied to the motive parts of fluid matter.

b. In the first place, for the conditions of a free roller, it may be held mechanically that perfect rolling motions produce very small friction on the planes of contact, as in this motion, the separate parts of the body of the roller are balanced by other parts, so that all parts remain in *constant* equilibrium. The equally distant balanced parts are supposed to fall consecutively upon the contact plane, and remain until they are again lifted in equilibrium by the continuity of momentum of the system. The friction of a free roller is therefore only as the force of adhesion, and the resistance by the depth of elastic depression upon the planes of contact. If the planes of contact of a rolling system in equilibrium are by any means adhesive and perfectly smooth, all force in motions of perfect rolling contact in this case may be conceived to be placed in the momentum of the rotational motion of the roller, which moves upon the plane as a free balanced lever, whose length is the diameter of the roller, and whose fulcrum is an imaginary frictionless point.

c. The mechanical conditions of frictionless rolling contact for the above case may be shown experimentally, or as nearly so as may support theory, by taking a very smooth level plane of glass and placing a perfect globe or cylinder upon it; the glass being now placed at any slight inclination, the friction of motion will be shown to be extremely small, by the small potential necessary to be given to the free roller to produce its motion. For the *perfection* of rolling contact we have only to imagine a level plane and a roller to be infinitely smooth and indeflectible, and the roller to be infinitely true in all its circumferential parts. Then any impressed movement upon the roller, be it a cylinder, globe, or ring, that can overcome the inertia of its mass to cause it to roll, in the first instance, will be theoretically a *constant force*, gravitation being assumed to act equally about the axis, and normal to the plane of motion.

d. For the conditions under which a plane may be displaced by the intervention of such free rollers as are described above, if there are more than two rollers, the mobile plane must be of necessity parallel to the plane on which it rests, and between which the rollers are interposed. Then if the two planes are perfectly smooth and indeflectible, and the rollers perfect cylinders or spheres, the system being without gravitation or in equal potential of gravitation, during the motive displacement of its free plane; and we now impress this motive plane with any force in the direction of its free motion, that is sufficient in the first place to overcome its inertia and that of the rollers, this force, as far as the mechanical conditions are concerned, will be a *constant force.*

e. It is quite certain that we cannot attain the perfect conditions assumed in the above paragraph, with known material and by human skill, but such a system may be experimentally shown, with sufficient freedom to support theory, by a plane of plate-glass supported upon another like plane by three equal metal globes as perfect as can be made in a turning-lathe. With this experimental apparatus it may be shown that when the inertia of the system is overcome a very small force will be sufficient to continue the motion to the extent of the area of the plane, or as far as this is in gravitation equilibrium in the system.

f. As soon as we come to investigate the principles of rolling contact in a fluid, these principles become complicated with functions of lateral resistance, head resistance, and traction. I will, however, endeavour in this chapter to consider the conditions of rolling contact under aspects as simple as I am able for the demonstration of this principle of motion only, to show simply that this form of motion is clearly applicable to fluids. I will take further conditions afterwards, where we may more clearly estimate the influence of direction of motive forces, by deflections under head resistances in the next chapter, and other conditions of circumscribing lateral resistances further on.

Principles of Rolling Contact of Fluids.

45. PROPOSITION: *If a free mass of fluid flow upon or against a plane of resistance, the motive contact will engender a system of rotation in the whole or in the near parts of the fluid mass, which rotation will approximately resemble the motion of a free roller on a smooth plane.*

a. It is assumed that we admit the general principle of adhesion of fluids to solids, and of cohesion of fluids intermolecularly, as previously discussed, and as shown clearly in the modes of lateral communication of motion given by Venturi (35 prop. *g*, page 119), by which a liquid in motion moves near quiescent parts in its own direction. Now if we assume the fluid as being adhesive to a solid, and in itself cohesive, we may conclude that a flowing fluid moving against a solid plane will have less flowing velocity near the solid, where it experiences greater resistance from its adhesion, than at a greater distance from it, where it experiences less influence of resistance, by cohesion continued through the system. This higher velocity of the more distant part of the flowing fluid will necessitate the physical condition that one part of the flowing fluid mass must move past the other part, either by revolution upon the part in contact, or that some general principle of molecular separation must occur throughout every part of the system for unequal parallel velocities to be possible in the separate parts.

b. I have already pointed out the conditions under which a material system can slide, that is, one in which all parts are fixed (43, art. *b*), and I have discussed the possibilities that a fluid is molecular (8 prop.), so that upon these principles a fluid taken to be a system of separate molecules, equivalent to an infinitely jointed system, the joints of which are assumed to be free from fixing, such a system of matter can only *roll*, to be displaced upon any other body whatever. The exact conditions upon which rolling contact of an entire system could occur would necessitate the condition that the fluid must be divided into separate masses to form rotary parts, as may be represented by the rollers, but actually this can scarcely occur as a condition of an entire moving fluid; yet such directions of motion in detached masses may be in certain cases observed as may make the fact evident, and the demonstration of this is all I shall attempt in the present proposition. I may, however, preface this matter with the remark, that my own observations since I first observed evidence of the principles of rolling contact in fluids, amount to many hundreds; indeed, I have found so few instances of fluid contact to occur in which this principle cannot be observed, if tested by making the fluid motion visible, that I have not been able to find any real case of doubt. I will offer a few simple cases.

c. While I am writing there is a chimney at the house opposite (a copper flue possibly) pouring out smoke very profusely, and the smoke-clouds are being carried away slowly by a gentle breeze. There can be no doubt that the smoke, as it feels the resistance of the relatively static air outside its visible column, rolls over in its upward motion upon contact with the surrounding air, as the motive force of the lighter smoke is constantly retarded at its outward parts. It is quite observable that it curls outwards in detached consecutive rolls. In this case the rolls appear to come out at equal intervals of time, the direct motion being most probably retarded by consecutive adhesions at points by irregular contact upon the chimney. The slowness and regularity of the motions permit the principles of rolling contact upon the air to be made clearly manifest.

Fig. 52.—Ex. Rolling Contact of Fumes on Air.

This phenomenon may also be observed in smoke issuing from a funnel of a steam-boat; this I have watched with a telescope as rolling by contact for many miles upon the still air, after issue from the funnel. It may also be seen in smoke or steam issuing from a locomotive boiler on a railway.

d. For another illustration nothing can exhibit this beautiful phenomenon of the nearly frictionless rolling contact of fluids better than a simple observation that we may make any day, of the mode in which a current of hot air and steam rises from a vessel of hot or boiling water into the quiescent air, as it may be seen in any position against the light in cold weather. The vapour as it rises, chills and becomes visible, giving the forms of the vapour motion at the instant of condensation. As we watch the steam rise it rolls over upon the still air, which forms its surface of resistance, maintaining the small velocity of its rising current in its almost frictionless path. It thereby becomes deflected into rolls and scrolls. The effect of the resistance of the air above is observable in its constantly turning the upward current from direct upward continuity. This I will hereafter consider. The same forms of rolls and scrolls are visible in the clouds upon any windy day, or on a still day when there are currents passing above, and it is only in exceptional cases that other forms are noticeable. In warm air charged with invisible vapour as it is rising it encounters the cold strata, projects its rising force against it, intrudes its mass, which becomes visible by loss of heat, and the vapour is set in cloud, in the form that was possible for it to intrude its small projectile force by its less specific gravity, into the quiescent air above. It is possible that the cloud is a less mobile form of matter, also one of greater specific gravity, than the vapour of which it is formed, so that the first visible form is fixed in a certain degree static in gravitation equilibrium, and afterwards retains this form. If the cloud forms which we term a *mackerel sky* are examined by a telescope, they are found to be detached rollers, which possibly either form a projectile plane, or intervene between two planes of motion in air of different temperatures.

e. We may often observe the intrusion of currents of vaporous air rising upwards and drifting along the surfaces of the denser air above by rolling contact. This becomes often particularly observable at sunset, the air being in the lower strata warmer and more charged with vapour than in the upper strata, which is more exposed to radiation, there is then a rising and drifting of the relatively lighter strata upward into the heavier; and as the sun sets strata after strata form cloud lines as the temperature sinks by the sun's force being cut off at the horizon. The rolling vapour clouds show continuous curled forms upon their upper sides, showing that at the instant of condensation they were projected, generally obliquely, against the resistance of the strata above, and that they enter and move upon it by rolling contact. As these rolling forms have always been depicted by artists, we may conclude that they have always been observed, although not correlated by science to causes.

f. In the August of 1873 I was descending Mount Pilatus to Alpnach by Lucerne, in Switzerland, when I passed through a cloud that extended to considerable depth for half a mile or so. The cloud was moving slowly downwards in the direction I was going, so that by stepping out a little I was soon through it. When I was quite clear of it it stood up with majestic, distinct rounded outline to a height of 40 feet or more on the front edge. I had the curiosity to run beyond it and watch its motion in flowing towards me. I could see in this instance, that it made rolling contact upon every plane, and rolled over leisurely as it advanced. Permitting the cloud again to reach me, I could see within the cloud the direction of the motions of its parts clearly visible by the small differences of tint from density, as its parts were rolling in equilibrium of the general mass system, by which the forward parts appeared to fall to the ground. I have repeated the effects of this experiment by pouring smoke on a smooth inclined wet surface.

g. With liquids, the continuity of adhesion of surface, will under the conditions of separate rolling systems of motion, be less apparent and less general. However, the following experimental evidence of the principle may be offered. Having a tank 4 feet in length, 8 inches in depth, and 1½ inch in width, with glass sides. In the centre of each end of this, I placed a half-inch pipe for the admission and egress of the liquids. Having filled the lower part with a clear saturated solution of common salt, and the upper part with clear water, I coloured some water of the same temperature with aniline dye. In this case the water as it was projected along the plane showed the evidence of division into separate rolling systems, which moved over the plane formed by the salt water, establishing particularly, a rotary system near the entrance where the supply of water was constant.

Fig. 53.—Ex. Rolling Contact on Water.

46. PROPOSITION: *If a flowing fluid is held in the lineal direction of its flowing force by its general cohesion, and by gravitation, so that all central areas of the fluid are compelled to flow parallel to any plane*

of resistance. The flowing force will in this case, as in the last proposition, tend to engender a motion of rotation in the lateral parts of the current, so that the central areas in relation to the lateral parts, will move by nearly equivalent mechanical motion to that of a true plane upon free cylindrical rollers, over or past another plane of resistance.

a. If we conceive such a system of motion for fluids, as that offered in 44 art. *e*, the exterior line of a current, assuming rolling contact upon all surfaces, would continue its motion parallel to the surfaces, and maintain the highest velocity in the most free motive part. So far, this would very conveniently represent one of the conditions of rolling contact, but as we know there are no such distinctions in the relative parts, or of their motions in a flowing fluid, as that of planes and rollers, we must conceive that this cannot be more than a principle of motion of which we have to modify the conditions to meet actual cases. Thus, in the first instance, in attempting to apply the above principles to a flowing fluid, it is not necessary in a perfectly detached mobile system of many joints, such as we conceive a fluid to be, to assume that the rolling contact extends entirely by rigid rollers or their equivalents, formed of the entire intermediate motive parts, represented by any exact intervening fluid space, or to extend to such diameter as will reach from the plane of greatest motion to that of perfect rest. We may more reasonably take it for granted, that all the motive parts of the mobile system will make rolling contact upon each other; in fact, this will necessarily be the case if we accept the very probable assumption that the final or definite molecular particle of fluid matter is globular, or, what is equivalent to this, that it is surrounded by forces which are radially equal about each molecule, so that the parts of the fluid are in equilibrium of contact on all molecules, as argued, 8 proposition.

b. Now taking the globular molecule, or its equivalent in radial forces to be the ultimate physical form of the particles of a fluid mass, and that by the equilibrium of surrounding pressures, it is in a large degree motively free. Then any unit of fluid matter moving in a like system, impressed with any force, would be relatively free of other units upon surfaces of contact, so that accommodation for its separate motion could be brought about frequently in small areas of displacement, with greater facility than in larger areas, in instances where greater mass inertia would have to be overcome in the larger area. Thus in considering this conception of rollers theoretically, extending over the distance from the central force to the planes of resistance, we might in certain cases, take it that less friction would be incurred by assuming the motive parts to be affected consecutively in almost infinitely close parallels, which might be represented by intermolecular rolling planes, in which, although the separate molecular friction might be somewhat greater in the near parts, the inertia of the fluid mass, necessary to be overcome for the general motion would be proportionally less. Under these conditions, taking the rolling contact as separately applicable as friction-saving motion between every filament of a running stream, assuming such filaments possible, and extending this filamentous and rolling system to the banks or bottom of any stream, we may on these principles consider the conditional motions that would occur. We will take for illustration the conditions of one parallel plane of water, or a filament which we may consider as composed of globular molecules, placed at one-fourth the width of a stream running between parallel banks, by which the flowing force is resisted. This line of one-fourth that we have selected would be at mid distance between the centre, which we will assume is the most free part of the stream, and the parallel banks which form planes of perfect resistance. Now suppose, as previously discussed, that all parts of the flowing system are influenced by adhesion and cohesion. There would then be on one side of the line we have taken, the more free central force of the current, flowing and carrying forward the near water by lateral cohesion with a certain consecutive velocity, and there would be on the bank side of our selected parallel, the influence of the resistance of the bank, which of itself would have no velocity. This would, therefore, be retarding the motion that the central current would otherwise engender about our imaginary molecular plane. Now as both of these forces would be in activity simultaneously, supposing the principles of cohesion to proceed in water with some equal force for indefinite distance; then any separate mass of the water at the line we have imagined, or any free molecule, would be *constantly moved by a motion of rotation proportional to the difference of velocity of the two contiguous parallel planes near to it*,—in fact, acting as a *roller* between two planes of unequal directive velocity, these planes themselves being assumed to be formed of like rollers.

c. Taking the above construction to be the actual, by this we change the mechanical principles very little from those given in the proposition and in our earlier conceptions in the previous proposition, as we may find that if we theoretically extend the diameter of the rolling parts in the molecular line we have taken, as we may do by continuing the principles of lineal velocities proportionally from any pair of very close parallel planes to planes more distant. We have then, instead of the single molecular series of rollers previously imagined, the interventions of small masses which act equally as rollers placed between these more distant planes, and these rollers will be rotated by the influences of the two lateral unequal velocities that we now conceive the planes to possess. If we further extend this principle of parallel velocities from the centre of the moving stream to the static banks, the same would occur in principle, therefore we might theoretically equally well represent molecules between close planes, or masses, or systems of rollers between more distant planes with like effect. In all cases one of such planes passing over or past the other would give uniformly the centre of the intervening molecular mass, disc, or space that we take for a unit between the planes of motion a moving force equal to half the velocity of the moving central plane; or as the equation of velocity between the motive tangent and the tangent of resistance to the rotated mass. We shall see, however, hereafter that this system will need considerable correction for absolute motion, by the direction of forces induced in the liquid, which react in another manner, although the principle offered is sufficiently clear for a first conception.

d. For the simple experimental observation of this system of motion being in a certain degree evident we may take a flat disc of wood of, say, one foot in diameter, and watch the motion of this as it proceeds down a wide stream. This experiment a friend tried for me in the river Thames at Weybridge, where there is a moderately straight course for a mile or so. The disc used was of inch pine wood painted white, with a distinct arrow across one diameter. This disc being thrown in any part of the running stream the arrow revolved

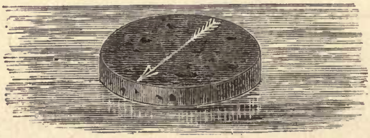

Fig. 54.—Ex. Rolling Contact observable in a Floating Body.

with the current, appearing to constantly roll upon the quiescent water towards the bank of the stream, which reacted upon it, although it was at a distance from the surface of resistance. In this experiment the rolling motion of the disc was not so great as though it rolled upon a solid surface, but the influence of the resistance was sufficiently clear, to be visible by the direction of motion induced; it could clearly in no case produce as great rotational velocity as one of direct contact, unless its centre were the *only axis* or joint of the system. The axes of the liquid system and their numbers of influences being as before shown to be practically infinite.

e. It is very possible that the motion induced in the molecules of lubricants, moving in close planes, in machinery may represent the principles of this proposition more nearly than the motions of any more free fluids, which would be subject to greater interferences, as further propositions will show.

47. PROPOSITION: *That upon principles of rolling contact in a fluid moving upon a plane of resistance the distant separate parts of the fluid will be influenced to move at a velocity proportional, in reference to other parts, as the radius of distance from the resistance.*

a. By the last proposition every motive molecular plane that is more resisted than another, becomes a plane of resistance to any other motive plane endowed with greater velocity, so that by the above conditions in the motion of parts of a fluid, we have tendencies to radial velocities about all lateral places of resistance, whether these are motive or quiescent.

b. For the motion of two planes, between which I imagined a liquid to act as a series of perfect rollers, we may observe, that although these imaginary rollers were assumed necessary for the continuity of motion of one mechanical plane over the other, that at any particular instant of time, only one of the diameters of the rollers would be in active service, that by this, so far as the smallest time of motion was concerned, we should need only one infinitely thin plane through the axis of the roller to be active extending from plane to plane of the parallel planes imagined. In fact that, if instead of our three perfect globes suggested for the continuity of motion of the rolling plane, as a mechanical principle, in the case proposed (44, art. *c*), we had three thin vertical rods of equal diameter, these would act in every way the same, for the movement of the plane horizontally for an infinitely small distance, and space of time.

c. If upon the above principles we might consider the entire fluid between two planes of motion, or of motion and rest, to be represented by radii at every point extending horizontally to the planes imagined. Or if we could extend these principles to the axis of a current passing the resistance of the banks and the bottom of the stream, the central course might be imagined almost frictionless. This is diagrammatically expressed in Fig. 55, where portions of the hypothetical rollers are represented between the current and the bank of the stream.

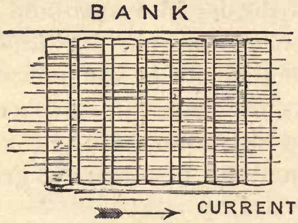

Fig. 55.—Diagram—Rolling Contact of Stream (hypothetical).

d. That certain elements of the principles discussed above are active, may be inferred from the velocities of single waves, to be hereafter considered, and the same principles may be inferred from the velocities of central areas in wide flowing streams, resting upon deep surfaces in which the radial motions need only very small amplitudes of curvature.

e. In the above discussed principles of motion it is not necessary that the radii of rolling contact should possess motive curves at both ends, as the motive plane may be taken to be a continuous lineal system, and the resistance by the distant solid be supposed to consecutively influence this; thus, for instance, an oceanic current flowing over a deep bottom, by the general equation of the motions of rotation in the entire radius from the bottom upwards, the total motion will be only as the differences of displacement of the upper and lower planes, and this we may conceive to be carried through an infinitely jointed system, the resistance of the bottom being assumed absolute, the axis of any possible rotation will then be near this bottom surface only. Now, assuming the radial velocities to increase from the bottom to the surface, the curvature of the arcs will decrease as the radii increase, so that at a great distance from the bottom the motive parts will form practically a level plane. This may be represented roughly in principle as in the diagram below, where the lower curls are put to represent rolling contact upon a resting surface; the radii are shown to increase in segments continuing of equal length of chord of arc to the upper free surface, where they form a motive nearly level plane.

Fig. 56.—Diagram—Rolling Contact under Equal Gravitation.

f. The difficulties that would be most apparent in such a system are that by the continuity of adhesion to the radii of the rollers, these rollers could not act entirely as friction savers, without considerable intermolecular friction of slipping for small distances upon themselves; and this slipping motion our theory of rolling contact will not admit, if there be by any possible means a less frictional motive course. It is therefore necessary that some modification of this principle should exist, for this to be the mode of motion of a free fluid. Nevertheless, the reality of the principle of motion in certain cases, I shall be able hereafter to better demonstrate by observation of the borders of certain frictional currents; in this matter, however, every motion of the kind is complicated with components of head resistances which I have yet to consider.

48. PROPOSITION: *That by the principles of rolling contact of fluids moving upon lateral resistances, free currents will have a tendency to be constantly directed towards the planes of resistance, with velocities which are as the directive forces of possible tangential motions of rolling contact, normal to all the points of resistance in an infinitely jointed molecular system.*

a. Taking the conditions of radial velocities offered in the last proposition as entailing a certain amount of frictional motion, I will in this proposition consider the conditions under which the same principles of motion may be preserved under less frictionless forms, by loss of a certain volume of the direct flowing force.

b. If we were to take, instead of radial lines, as a part of a motive roller, that I have assumed to extend from the motive to the resting part, of a fluid system—a larger portion of the assumed roller, as, for instance, the forward part of its semicircumference, we could then imagine in this case that such a system would be motive in the same area as the radii before considered. The system would be otherwise at the same time less rigid, and as we shall find, more consistent as a motive jointed system. Such a system is shown by the diagram below.

c. In this system we could nevertheless imagine that it would have the defect, that the separate semicylinders proposed would not be constructed of equal parallel quantities of matter, and therefore in supposing the mobility of the part of the fluid to supply this matter, the mass motive forces of the cylinders would be greater in one portion of the perimeter than another, and the mass velocity would be unequal in the parts, which would make the motion frictional. Further, that although these semicylinders would be complete at the plane of resistance they would be constantly running the flowing matter of the central current entirely off to the sides.

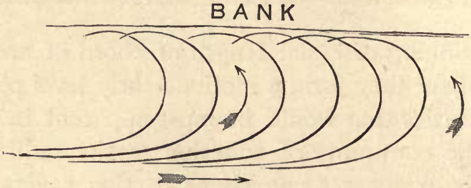

Fig. 57.—Diagram—Proposed Semicylindrical System.

d. If we take another nearly parallel conception to that offered above, and imagine the friction semicylinders engendered by resistance *not* to be formed out of the static quiescent liquid at the borders of the central current, as before taken (*b*, *c*), but to be formed out of a portion of the projected liquid of the central current itself, so that the semicylinders may be assumed to be the products of a deflection of the central current simply. This form of motion would then permit parts of the cylinders to swell out by intrusion of the deflected fluid, and make up the quantityof any matter required to com-

Fig. 58.—Diagram—Proposed System of Inflection.

plete the inequality of mass in parts of the semicylindrical system. In this case the motion may be conceived to be continuous at the expense of a certain disintegration of the volume of flowing fluid, which would, by the conditions given, be deflected into the semicylindrical system of motion, the principle of which is indicated by the diagram above.

e. To demonstrate the above form of motion mechanically we will assume a lineal or filamentous system of stratum for our current, which may be represented by a low pile of smooth paper or ribbons. If we place this pile of paper or ribbons upon a smooth surface, as, for instance, a sheet of glass, and place the glass at a slight inclination, the paper or ribbons would rest perfectly static upon this, as to run down the incline they would have to slide upon each other, or upon the glass. But if we now take the proposed stratum of ribbons, that may be of, say, five to ten separate pieces, and turn down the front ends over towards the lower part of the incline, and bend them under the other forward parts of the stratum by a curve that produces a semicylindrical arc in front, we shall now find that the stratum of smooth paper or ribbons will be rendered motive, and constantly turn in series, separately one by one at the bend, until the stratum is again extended the entire length of the pieces at a position lower down the plane. When this motion is complete, the stratum taken will be left quiescently upon the plane, and move no more, the papers or ribbons being now piled in *reverse order* to their first position. It will be seen that, in this operation we transfer the potential force of gravitation acting upon the upper papers or ribbons in the pile to a lower position of equilibrium, in which each piece will be now held upon the plane surface by the resistance of its adhesion, so that it does not move further by the potential force of the inclination against the amount of friction that would be necessarily present in a slipping motion.

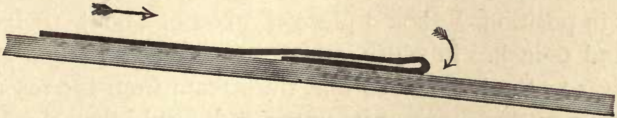

Fig. 59.—Ex. Ribbon on Inclined Plane.

f. We may perform this experiment in a more simple manner by a single smooth band of paper or of satin ribbon upon a smooth incline, as that of a sheet of glass. This band or ribbon if started in a similar manner to that described above for the stratum of ribbons by a bend in front towards the lower end of the incline, will move along the plane and come to rest by turning gradually over at the front edge without great friction upon the surface upon which it afterwards reposes. This arrangement is represented in the engraving above.

g. By this principle of motion, as a rough demonstrative experiment, we may conceive the mode of motion by which streams may suffer disintegration of their volumes and forces, by resistances at the *borders* of their currents only, to insure the nearly frictionless motions of rolling contact for their central masses. Wherein the general cohesive force of the interior mass or main body of the current may be held entire, and move at approximately equal velocity over a wide area without material deflection past lateral resistance, by leaving a portion only to the flowing force static on the plane of resistance, or by inducing within it, a small minus velocity by the momentum of rotation, in which the deflection of the central force, in parts, overcomes the friction of lateral resistance by a small loss of its volume. This principle may be evident in oceanic currents which maintain a certain velocity nearly intact, although the sectional area of the current gradually decreases. It will also be evident in the velocities of transverse parts of streams. I will endeavour to demonstrate the principles now offered by the following experiment, which will be very important for consideration in my further work.

h. A reservoir of water was placed at a small elevation so that a very fine stream was projected from it at small velocity. For this purpose I used an old watering-can that had a very small hole in the bottom, into which I put a finely tapered wooden peg so as to be enabled to let out a fine stream when required. Having this reservoir in position, I took a piece of glass of about 18 inches in length, and 6 inches in width, and fixed this at an angle of about two degrees to the horizon, so that the stream from the reservoir, as it ran out upon this plane at its upper end, would flow slowly down it. Now wetting the glass all over, a sheet of smooth writing paper was made to adhere evenly to it, and this formed a clear white surface upon which I observed the following effects. Permitting the fine jet of water to run from the can for a minute or so, it formed at first a small pool upon the paper. Continuing to watch the pool I found that in a few seconds it was opened out at the lowest point and then ran slowly down the plane in a stream of about equal width, and left a wet line upon the paper. My next object in following this experiment was to discover exactly what portion of the running stream wetted the paper, for it was quite clear that if the paper was wetted by the lower surface of the water only, it must be by a slipping motion. Therefore, before recommencing the experiment I provided myself with an ordinary culinary pepper-box so as to follow, by the floating particles, the surface motions of the stream particularly. Now taking another clean sheet of paper and placing it as before on the wet inclined slip of glass, I again permitted the jet to flow from the can on the upper part of the plane as before, but when the pool was formed and just ready to run down the incline, I instantly dusted it with the pepper, which, being light, rested entirely upon the surface of the pool. Now watching its motion I found that each floating particle as it was carried along the stream ran with greater general velocity at the surface than the average velocity of the entire stream, so that particles arrived consecutively at the front of it. That as soon as a particle arrived at this point it instantly turned the beaded forward edge of the stream, and then lay itself quietly upon the surface of the paper, *so that the pepper that was dusted over the surface of the water in the pool took an entirely reverse position, having moved from the top to the bottom, showing thereby an entire rotation of the current at its head, or most free part.*

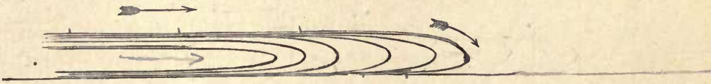

Fig. 60.—Ex. Flowing Stream.

i. In this experiment the pepper on the flowing stream is transferred from the top to the bottom surface by the flowing force, as our strata of paper or ribbon were shown to be. The same principle evidently exists without the dust upon the surface, so that the water alone would leave a wet streak of disintegrated water from the surface of the flowing stream. I presume this to be a visible condition of a principle that is made evident in the above experiment by the adhesion of the liquid to the solid surface.

j. We also find evidence of the same mode of motion in the experiment of the floating disk offered 46 prop. *d,* in a condition not previously mentioned, that the disk will be found not only to rotate as before shown upon its axis of inertia by the differences of velocity of the parts of the stream, but it will be *drifted towards the borders* *of the current.* A friend to whom I mentioned this principle of fluid motion, who is very fond of jack fishing, tells me that he has often been much troubled with this phenomenon just described, having found that the stream not only rotates his float and twists his line, but that it carries his float constantly towards the edge of the running stream, and he tells me he has no doubt that every jack fisher will have noticed this, and perhaps, like himself, have been puzzled to discover the cause.

49. PROPOSITION: *That upon principles of rolling contact, the velocity of a liquid flowing down an inclined plane, will be for a distance equal to the chord of an arc described upon the plane with a radius of the depth of the flowing liquid as the chord of the supplement of the arc from which twice the angle of inclination from its centre is taken.*

a. Let A A′ represent an inclined plane of resistance upon which a liquid flows.

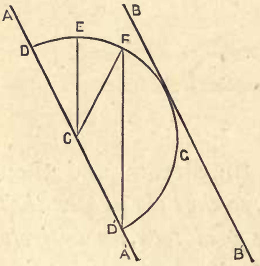

Fig. 61.—Diagram of Rolling Contact Velocity.

Let B B′ be the surface of the liquid flowing down this plane, of a depth equal to the distance between the parallels A A′ and B B′.

Let C be a centre upon the plane taken to be any point of contact of the flowing liquid. Draw a complete semicircular arc with C as a centre, and of the depth of the liquid represented between the parallels A A′ and B B′ as a radius. Draw a vertical line C E above the selected point of resistance, and make the angle E C F equal to E C D upon the opposite side of the vertical line E C. Then will the liquid in the inscribed segment C D F be in gravitation equilibrium upon the point C, and will have no greater gravitation force, as regards this segment, than that which will be perfectly resisted by the point C. So that the chord F D′ of the supplement of the complete semicircular arc will represent by the space F to D′, the force of gravitation upon the system of contact of the complete arc upon the point of resistance C, and the gravitation force upon the liquid flowing upon the inclined plane for the assumed space D to D′ will be that of a free body, falling the space F to D′, and the acceleration by gravitation will be in the same ratio to this space for every space equal to D D′ upon the incline.

b. In the above scheme the liquid is assumed to be under no molecular restraint, but to be taken as a perfectly free molecular or infinitely jointed system as that proposed 6 prop., with no adhesion to the plane of resistance. It is, however, certain that a liquid has an adhesive force upon every plane of resistance, and that this will retard the velocity proposed above in a certain uniform ratio. It is possible that this will form a minus constant quantity equal to the adhesion of the liquid to a free surface, per area of the frictional surface. There will be present also a small component of restraint from molecular friction, that will be inversely proportional to the square of the distance from the surface of contact for all parts.

c. This proposition must be held to be purely theoretical, founded upon principles of rolling contact only as given in previous propositions, but of which I regret that I have had no time for research or experimental investigation for actual velocities under the conditions here proposed.

50. PROPOSITION: *Currents of constantly flowing fluids moving past quiescent lateral free spaces, or in bays in any direction will form rotational systems in the fluid in such spaces or bays, which will move upon the centres of inertia of the lateral masses moved, so that the circumferential movements of the more free lateral parts will approach the flowing velocity of the current by which they are moved tangentially, and this direction of motion will save a large part of the friction of the resistance, to the cohesion of the fluid upon the surfaces against which it presses.*

a. This proposition offers the means by which the friction upon a lateral open fluid space exposed to the action of a current will be much less than the friction of near solid resistance. The proposition is not only important in the consideration of the mode of projection of oceanic currents and the flow of rivers past bays, but is applicable to over currents in deep water also. The mechanical principle of this form of motion may be conceived by the following.

b. If we take again the conditions offered of a perfectly true cylindrical roller between perfectly smooth level planes, proposed 44 art. *c*, for demonstration of a means by which a nearly frictionless continuous motion might be produced in a plane under the equal influence of gravitation, we find that where motion is produced in the plane and the rollers, this will be a theoretically constant force. Therefore, if we take one of these rollers, and if the plane constantly moves over it, with uniform velocity equal to the induced motion in the periphery of the roller, there would be no friction in the plane in passing over this roller. In the present proposition we will consider the central flowing liquid of a current as the motive plane, and the quiescent lateral system as the roller.

c. If we endeavour to take conditions of the liquid roller based upon the frictionless motion which I have assumed to be possible, neglecting for the present all conditions necessary for the formation and establishment of the liquid roller itself, or assuming only that it is a motion that the general infinitely jointed system of the liquid permits by motions of accommodation, then the entire effect of such a roller upon the central current will be to remove the friction of adhesion of the solid lateral plane, by inducing a motion of rotation in the lateral more free part. This matter may be explained by the diagram above.

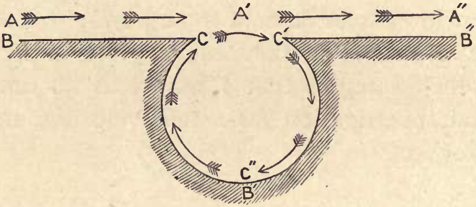

Fig. 62.—Diagram of Lateral Rotation.

Let A A′ A″ be a flowing force in a liquid, moving upon the solid represented in outline by B B′ B″. Then will the friction of adhesion of the liquid to the plane be greatest upon the part of the plane B C C′ B″ from its nearness. It would be equal upon the space C to C′ if the cohesion of the liquid to itself was equal to the adhesion of the liquid to the solid, and the space C C′ C″ would rest immobile. But if we give this space C C′ C″ by the cohesion of the flowing force a rotation in the direction of the arrows, the friction on A′ is removed to the centre of inertia of the rotary system C C′ C″, by which there will be less resistance to the force A A′ A″ in proportion to the distance of the centre of inertia of the lateral mass into the difference of friction of motion in the two systems.

d. It is certain that under the conditions shown in the diagram above that there still remains the friction of adhesion of the surfaces of the inclosed space C C′ C″. But that the rotary lateral system is a less frictional motion we may conclude by observation. For in this case, if the friction of rotation of the lateral mass was *greater* than the friction of one part of the fluid passing the other in the plane B to B′, this space C C′ C″ would not be rotated. But if the friction were less by such rotation it would be rotated, as the tangential action would be equal in each case upon the lateral mass, whether it were rotated or not.

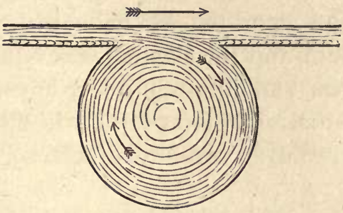

Fig. 63.—Ex. Observation of Lateral Rotation.

e. For evidence of the above being a principle of fluid motion the following experimental evidence may be offered:—If we allow a stream to flow through a uniform channel for a certain distance, and then enlarge the channel upon one side or the other horizontally or vertically, as in the diagram above, we find that as soon as the water is free from the near resistance of the narrow part of the channel, to pass the open part, it rotates the curvilinear part as shown in the diagram. This may be made very palpable by particles of saw-dust, or other light bodies of about the same specific gravity as the water, being placed upon it or in it. The same principle of motion may be noticed in any little pool by a flowing river-course, in the motive direction of particles of scum or other matter that may be floating on the river surface. By careful examination a reverse action on the borders of the current, in the lateral space, may be also clearly seen, active as a friction-saving motion to this rotation; but I shall be able to offer much more certain illustrations of this after I have discussed some further principles. In the illustration above I have simply shown a lateral circle rotated. This will occur in any form of free space for a *part* of the same; therefore it is here used diagrammatically only. For our present purpose for the rotation, we may take it that all slow motions in fluids are less frictional than faster ones relatively to the velocities of the moving parts, so that if motive lines are extended, the system will become less frictional. This will possibly, for one cause, be due to the sum of the elastic forces disturbed, which react to restore equilibrium, but not generally in the direction to increase the velocity of the motive part, or at the instant of impression. In this manner an elastic system resists motion, until the impression of the force is extended in its mass, so that it may overcome the inertia of a larger part of the mass than that impressed, by smaller motions largely distributed, the separate parts being disturbed less from their positions of equilibrium, and the elastic tension being less in any part of the system. Further, an infinitely jointed system, with any polar rigidity whatever in the joints (10 prop.), will be proportionally free from friction as the radius of motion is increased.

51. PROPOSITION: *That a flowing fluid moving upon an open area of quiescent like fluid, will have its exterior flowing lines deflected into the quiescent fluid by lateral cohesion, until the curves of deflection of the flowing fluid circumscribe a certain area, the constantly increasing deflection will describe by this cause spiral lines which will move towards the centre of inertia of a rolling system, formed by the lateral cohesion, until the resistance in the deflected current equals the flowing force at the point of impression. The constant flow after this will pass tangent to the rotary system established.*

a. If a flowing force moves coherent to a lateral infinitely jointed mass or fluid, it will at first move the nearest molecules or infinitely jointed parts, contiguous to the motive parts, upon their centres of inertia; and as these molecules crowd upon each other, the elastic forces impressed will cause the crowded parts to be deflected into the lateral more free parts, to meet resistance by the inertia of a more extended area in the fluid mass. If the flowing force continue the deflection will take larger and continually increasing radius, until nearly the whole mass will be rotated by the constancy of deflection of the flowing force, and a rotary system will be established.

b. It will be convenient before considering the motive effect of the principles of fluid motion here inferred, to demonstrate the general mechanical principles of this motion, which I will define as an *involuting rotary system.*

c. For the demonstration of the above, we may imagine a cylindrical spiral or volute to be in the course of formation by a lateral force, which constantly rolls up a thin sheet of infinitely mobile material of equal thickness upon itself, as we may roll up a slip of paper by longitudinal displacement between the thumb and finger. We will suppose this mobile volute to retain a centre upon which the mobile matter coils. The volute may be either standing alone on this smooth plane or be contiguous to another plane above it. Suppose that the thin sheet of mobile material that forms the volute is brought up by surrounding matter, or is otherwise supported, and that it is constantly in equilibrium at the upper surface of the roller, and is mobile to the impression of any small force, and that the volute only is subject to the force of gravitation; then it would continue to roll for an infinite period without friction if the band of which it was formed was of infinite length, by the action of gravitation only, which would supply the small necessary force upon the forward portion, as the following diagram will show.

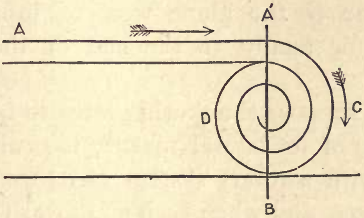

Fig. 64.—Diagram—Spiral Action.

d. Let the lineal plane A A′, Fig. 64, be perfectly mobile and be supported without excess of gravitation, as in a flowing stream, and be without inertia or of such motive inertia that it will not retard the action of the volute. Let B be a perfectly level plane, and the centre of the volute or first roll of the coil that continues to roll upon an axis central to A′ B, and let gravitation act upon this volute only. Then will the predominance of matter on the forward side of the axis A′ B towards C cause the volute to constantly roll up in the direction of the arrow at C, by the greater action of gravitation on the forward side supplying the necessary force.

e. For the condition of fluid projection under tangential action we may omit the force of gravitation, which is here used to demonstrate the smallness of force necessary to produce the continuity of an involuting system. If we arrange the circumstances otherwise, as for instance that a current flowing from A to A′ carries the band A A′ forward towards the volute with any small constant force, so that the flowing force that forms the supply is given constantly to the volute, this would then cause it to continue to rotate and enlarge, if the freedom of area and other circumstances permitted.

f. We may further consider that if the rotary system were not moving upon a frictionless centre of inertia, but upon a plane, that the constant supply of the current A A′ would maintain a frictionless motion infinitely.

g. Further, if there was resistance at C, Fig. 64, to the flowing force, this would only increase its deflection into the volute, and maintain the involuting system static in space. This might occur with very small forces if the system of fluid that formed the volute were in general equilibrium.

h. If, in the above diagram, we were to reverse the volute, supposed perfectly mobile, moving on a perfect plane, and permit its plane to unroll instead of roll up, it would continue to unroll for an infinite time if the plane were of infinite length, as in the first instance, or finally to the end of the coil if of finite length.

i. Now in fluids, suppose the flowing force to be a small constant, then, by small loss of its lateral matter, it could form volutes in quiescent places until a rotary system was formed of the mass in such quiescent spaces, and when such mass was formed the original current of flowing fluid would flow forward by moving tangentially upon this rotary system without great friction, as shown by the last proposition. At the same time, the borders of such involuntary rotary systems would make rolling contact upon the limiting area by the principles of the previous proposition, and remain, if surrounded by resistances, fixed in one position in space.

j. For experimental evidence of the above proposition, we may take the trough described page 141, and fill it as before up to the level of the inlet and outlet, but with clear water only, and now let a stream of coloured water flow over it. The coloured water at first will form a small volute near the inlet, and this will constantly extend in area until a large portion of the mass is in rotary motion. The principle of this motion will be further demonstrated in the following chapters partly by conditions of head resistance.

52. PROPOSITION: *That a flowing or projectile fluid meeting an oblique plane of resistance will overcome the friction of the plane by a motion of involution.*

a. The conditions of the above may be inferred from the last proposition, so that I need not in this case offer more than experimental evidence.

b. A trough that I made for other experiments, twenty feet long and eleven inches wide and of the same depth, was placed horizontally. At one end of this trough a false bottom was fixed, extending for about eight feet, this being of the same width as the trough. The false bottom rested upon the true bottom, one end being elevated and fixed upon the end of the trough, at a height of about six inches. In this manner it formed an inclined plane, as shown in the illustration. Water was poured into this trough to a depth of about five inches on the level part, and the experiment to be followed was to project a certain volume of water up the incline. For this purpose I took a piece of board of the width of the trough, and of sufficient length to hold conveniently in my hands, and placed this at first upright against the vertical end of the trough; now by moving it towards the inclined plane, for a foot or so, a wave was formed, which moved forward over the surface and finally impressed its force upon the plane. This wave, as it met the increased resistance, was gradually increased in height, and it showed by its more rounded form that the rolling principle of contact was becoming outwardly visible. As the wave progressed its protuberance increased, until it reached a certain point up the plane, where the projection above the general surface equalled the depth of water beneath the average surface. At this point, for an instant of time, a complete rolling volute was produced, whose motion was clearly visible as the water broke in its fall over the upper part of the rounded wave.

Fig. 65.—Section of Long Trough.

Fig. 66.—Diagram—Liquid Rolling on Incline Plane.

The water, at the instant of the formation of the perfect roller or volute, will be pushing water in front, which forms sometimes a series of rounded projections or loops, which are not a part of the motion demonstrated by this proposition, but are due to minor rotary systems in which there are present evidences of other motive principles not yet considered.

c. In the above experiment it will be clearly seen that the projection of the involuted system above the surface is wholly due to the perfect resistance of the plane, and is exactly equivalent in principles of motion to the cases previously considered, where the plane is a deflectible liquid (last proposition, *i*).

d. In the inflowing of a tide into a bay or estuary an immense quantity of water enters in a few hours, and the projectile force of such a body must be immense. Yet we see the fine sand on the ground surface very little disturbed. We may attribute one cause of this to the adhesion of the water by continuity deep into the sand. But the principal cause is that the water at no instant either slides upon the surface, or even appears to do so, but as we watch it, it rolls gently over and disturbs the land surface very little, except at the extreme edge, where the involuted forms are broken up by the resistance, and the water makes such irregular contact, that it moves the light sand and stones as though it were by a sliding motion upon them. Otherwise, if the land surface is smooth upon which the current gently advances, the rotary system, I have no doubt, is perfect up to the extreme edge, moving by rolling contact (48 prop. *h*).

53. PROPOSITION: *The amplitudes of motion of rolling contact in a fluid upon planes of resistance will be small if the fluid is held together by weak cohesion or weak polar forces, but it will be greater proportionally as the forces of cohesion or polarity are greater.*

a. If a molecular system is perfectly free in its mode of attachment to the next molecule by the perfection of its equilibrium of position or the perfect equality of radial adhesion without polar force in its first position, such a molecule impressed by a force normal to any series of molecules would move round the molecule to which it was attached, as may be shown by the following diagram, which for demonstration is conceived as a hanging system, the molecular forces being assumed equal throughout the molecular circumference.

b. Let A A′ A″ B be a series of molecules in equilibrium and by equal attraction in *perfectly* mobile attachment, the first molecule A being attached to a plane or being supported by it. Let the mobility of B about A″ be such that the movement of B upon this mobile attachment to A″ will not engender sufficient *friction* to overcome the inertia of A″ upon A′. Then will a force in the direction shown by the arrow upon B move the molecule B about to its second position B′, shown by a dotted circle, or it will move it round the molecule a greater or less space, according to the strength of the impulse upon B in the direction of the horizontal arrow.

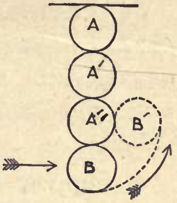

Fig. 67.—Diagram Polar Chain.

c. If there be polar adhesion at the first point of contact of the molecules greater than at other circumferential parts about it, such an adhesion under an impulse upon B as before, would produce a strain upon the whole system, supposed more adhesive through the activity of its polar forces, at all points, and the polar adhesion would move the more distant parts. The same would occur also from adhesion being greater upon the points of contact from any other cause than polarity, as, for instance, by the points of contact being impressed by a weight as that of gravitation to increase adhesion; this matter may be reduced to definite experiment. Thus, if we let a chain hang freely down from a point of support, and then impress a force upon the lowest link, this will by the rigidity of adhesion move consecutive links, and if the force be moderately constant, and not of high velocity, the chain will form a cycloidal arc at the point of greatest motive amplitude by the impressed force at B.

d. If we make a jointed system still more rigid than that of the chain taken above, to represent strong polar forces, the motion, from like impression on the point B, will be entirely upon the most distant joint or point of adhesion A. Thus, if we pile up a stack of single bricks to, say, a yard in height, and impress a moderate force upon the upper brick, sufficient to disturb the general equilibrium, the pile of bricks will fall down by turning upon the axis formed by the *lowest brick*, that is, upon the most distant free joint from the point of impact.

e. By the above conditions we infer that if the cohesive or polar forces of a molecular system of matter are *weak*, that impressed forces will engender motions near the point of impression, but if stronger, to a more distant part, and by the same conditions we may assume that one fluid may take less amplitude of rotation through lateral resistance than another if its system be less rigid under the impression of a force, as oil may possibly take less amplitude in rotation by resistance than water (12 prop.).

54. PROPOSITION: *The amplitudes of motions of rolling contact in the lateral parts of flowing fluids are relative to the velocities of the flowing forces active upon them, but not proportionally so. The minimum amplitudes being those of very small, and of very great flowing velocities.*

a. Suppose a series of molecules as those shown in Fig. 67, to have equal cohesive or polar forces attractive upon each other, but not of sufficient force to destroy the perfect mobility of the system, and suppose a very small force, as before, impressed in the direction of the arrow upon B. Now any mode of communication between the molecules B and A at the extreme end of the series must be through the other molecules A″ and A′. Therefore if B were impressed with an instant force, only just sufficient to overcome its own inertia, it would move on the nearest free joint towards A″ only. Further, as this force must have a commencement of action upon the molecular system, which I have assumed to be represented by a hanging system, it would be relatively to the general inertia of the system in its first impulse *infinite*, or of the nature of a percussion. Therefore by this infinite force the resistance of the cohesive or polar force would be overcome at first upon the nearest joint only, by the smallest impulse capable of moving any part of the system. For experimental evidence of this, suspend a long chain as before, and impress a very small force upon the lowest link insufficient to overcome the inertia of the chain, as by a tap of a very light hammer; the lowest link only will move a very small distance.

b. If we suppose a force of greater amplitude than the above, or one that acts continually, or as a pressure, we have seen that such a force would move more distant molecules or parts upon a cohesive system by demonstrations *c* and *d* of the last proposition, that is, move the system upon the joint A next to the plane of support of such a system as shown in the figure, or would cause a hanging chain to swing bodily, describing with its lowest link a cycloidal arc.

c. If a force be impressed with much greater velocity upon an infinitely jointed system held by weak polar forces as before, this force will act upon the weak resistances of such polar forces as a constant percussion, it will therefore be an infinite force, and will move the mobile system upon its *nearest* axis or free point as in the first case *a*. For experimental evidence of this condition, suppose our hanging chain in the last proposition *c* impressed upon its lowest link with a greater force than that previously proposed, or one moving at very great velocity, then the chain would not be deflected into a cycloid at its extreme curvature, by the force, but the curve produced would be more nearly hyperbolic, the impression being normal to the axis of the hyperbola at its vertex. If we in like manner, impressed a force of great velocity upon the pile of single bricks given at *d*, in the last proposition, upon the top brick, this brick would be projected out of the series, but the pile would not otherwise be disturbed as would be the case with an equal force of less velocity.

d. In our further investigation this proposition becomes important when applied to fluids, as in certain cases both weak and intense forces move small areas of contiguous fluid matter, where moderate forces move larger areas and more distant parts, and moderate motions offer proportionally *more resistance*, than either very slow or very rapid ones.

55. Remarks.—*a.* By the principles discussed in the several propositions of this chapter, it will be seen that by the rolling contact of fluid matter it is not intended that the *roller*, if we distinguish one part of the fluid as such, should be necessarily of any definite form, although the motions induced will *frequently* give this form by isolation to masses of the fluid, subject to tangential or flowing forces upon these isolated masses. But the propositions are directed to demonstrate that the principles of contact will be such, that motive surfaces of fluids that come in contact with each other, or with solids shall maintain *lineal* or *tangential contact without slipping or gliding.* The resultant motion in the fluid being uniformly in principle that of a rolling motion however modified by exterior conditions. It being demonstrated also that this is the most frictionless motion for an infinitely jointed or fluid body, therefore the motive form which a fluid body will always take upon frictional contact.

b. We may also conceive that if the surrounding conditions are such that motions of rolling contact are not easily, or possibly, induced in the fluid system, in movements upon itself, or upon solids, that the laws of hydrostatic equilibrium, as they are termed, will not in this case be followed. This is the case with the syphon experiment given in 33 prop., where at the time (page 113) I could only discuss the principles of cohesion which evidently hold. But for the conditions of motion by which hydrostatic equilibrium could possibly be established, we have in this case, in some way, to induce rolling contact in two fluids which are of such unequal density, that any motion of rolling contact in the liquid could only be very frictionally continued into the aerial system and vice versâ. So that the whole system resists motion by its cohesive forces simply.

c. The same conditions exist for the needle floating on water given (page 47), for although the water resists by its inert cohesive force, this force would practically disappear as a visible function, if motions of rolling contact could be induced near the needle, which its small force cannot effect on an extensive surface under resistance of local inertia. This is altogether independent of the causes previously shown of the general cohesion of the water and conditions of its surface.

d. If the principles of rolling contact were possible in all cases to be fully developed in the actual motion of fluids, so that they suffered no restraint from interference in their active motive forms in a universally jointed system, as that I propose; a running stream of water would be always projected at some intermediate space between the banks transverse to the direction of the current. This we know as an absolute fact does not occur; as in the stream we have the complication of cohesion, resistance, and motive inertia of all parts of the system, so that all the evidence of rolling contact left observable is that the surface system moves at the higher velocity in the central area, stillness reigns near the banks, and in some cases there is a slow retrograde motion at the banks.

e. As a general fact, motions within fluid systems have commonly little power to complete direct lines of impression of forces, so as to become outwardly evident, either singly or by simple resultants, such as we witness generally with solids. This arises principally from the mobility of their parts having power to engender systems of accommodation for the impressed forces impossible in more rigid matter. The loss of completion of motive effects or forces are often, however, made evident through transference of motive force from particle to particle, with small local displacements, which every fluid permits. The same continuity of impressions being also evident in the principles of disintegration of primitive force by resistant surface, which permits by dis-continuity of a small part of the mass small material loss of velocity in a residual part. These principles of motion offer modes of projection of forces, more frictionless for fluids under any movements against resistance, than is possible to be engendered in solid systems moving equal weights. I may further observe, that as rolling motions become restrained, by insufficiency of area to complete their curvilinear paths, the frictional resistances increase, as we find in comparing the resistances of pipes where the fluid motion is cramped, with that of open conduits, where one surface is free. In the one case, the water has no opportunity to move freely upon the resistant surface, or, as it were, to ride upon the motion of rolling contact engendered by the flowing force; in the other case it has this freedom. To this matter I shall return.

f. In this chapter I have only endeavoured to point out principles of motions of rolling contact for fluids upon surfaces of resistance. These may be taken as cases only, for it is quite certain to me that the modes of rolling contact may take place also in other forms than those discussed; but if fluids are considered, as here proposed, as infinitely jointed systems of matter, we may then have every possible mode of motion, and of accommodation for motion in them, and forces impressed will be deflected by every possible directive influence, that offers at any instant of time less motive friction for the volume of matter displaced; some principles of which I will now proceed to consider in relation to head resistances.

CHAPTER V.

PROJECTION OF FLUIDS WITHIN LIKE FLUIDS. PRINCIPLES OF CONIC RESISTANCE. PLANES OF FRACTURE AND OF TENSION IN HOMOGENEOUS SYSTEMS OF MATTER.

56. Projections within fluids and homogeneous systems of matter.

a. Some causes for the motive effects observable in the projection of fluids in, or upon, like fluids have been considered by Newton and others. In these cases with little exception the projectile fluid has been conceived to impress a like area of resisting fluid, the general conditions of which are discussed in Newton's third law; the impressions and the resistances being proportional to the surfaces impressed, to the densities of the fluids, and to the squares of the velocities, conjointly. By the cohesion of fluids the mass active in resistance may be much greater than the area impressed, in the cohesion of the system being supported by all contiguous parts; so that all the various conditions of oblique impulse may be conceived to be active in extending the impression in the area impressed. On the other hand, the general principles of accommodation in fluids, which secures to projectile fluids rolling contact of motive parts, as discussed in the last chapter, may give small units projectile continuity, which can in no way be measured by the resistance of the density-volume of matter directly impressed and the squares of the velocity. The cases which actually occur, wherein the projections affect equal areas to the resistances, appear to me to be necessarily very limited in number, being, as I conceive, possibly confined to close parallel areas only. The conditions under which projection may be resisted by more extensive areas within the fluid, have not, that I am aware of, been considered except under very limited conditions, and these, I conceive, are the general conditions of projections in fluids, so that it may be regarded as quite exceptional, that the areas of impression and resistance are equal. Thus, in the cases of oceanic and aerial projections, the areas of impulsion by which currents are formed, are frequently local, whereas the resistances, acting in an extensive cohesive fluid are wide-spread, the motive effects observable being also consistent with this. The same conditions commonly occur in the practice of hydraulics where outlets and inlets are small compared with the bulk of liquid upon which they act; and the same principle will be active in the projection of solids in extensive liquids, as in the projection of ships, as I will endeavour hereafter to show. Upon these considerations it appears to me, that if we obtain a clear conception of the projection of a unit of fluid matter in an extensive fluid, this will establish the *form of fluid projection* that occurs in nearly all the cases that we can imagine to be actual, even as I find in close areas or in pipes. I will therefore endeavour to follow this matter into all the detail that my powers can suggest.

b. For this research it will be convenient in the present chapter to discuss, at first, the conditions of the directive action of surrounding pressures in a fluid system upon one of its particles in equilibrium. Some conditions of this have been already partly discussed in considering the action of directive pressures towards a hole in the bottom of a vessel, 30 prop. *c*, page 94, wherein the position of the hole was taken to be the point of least resistance of the liquid system. In the following propositions I will endeavour to show that similar conditions hold good for the directive pressures that support the *static equilibrium* of a particle in an open fluid, and in further propositions, to show that similar conditions hold for the forces in *motive equilibrium of the particle* also, although by the motions induced, the lateral directive pressures will be deflected according to other conditions to be hereafter considered.

57. PROPOSITION: *That the pressures of surrounding particles in a fluid about any single particle, that secure its equilibrium, are active upon that particle in inverse proportion to the squares of their distances from it.*

a. Let the dot in the centre of the figure above represent a small portion of a liquid that we will term a particle. Let this particle rest upon a free surface, or upon any horizontal plane below the surface. Now, by known laws of hydrostatic pressures, such a particle will be pressed by direct force on every side equally by the surrounding fluid so as to be on no side free to move. In these pressures, that received by one particle is communicated to the next, and the same pressure to the next, and so on. The pressures may therefore be assumed to extend by sequential action indefinitely, or to the full extent of the fluid, and will, by their united forces taken in the horizontal plane, for instance, secure the horizontal equilibrium of the particle. Now, as this particle receives only the same lateral pressure as any other particle in the fluid, it is quite clear that the pressures of other particles, that is, the entire amount of direct pressure that they are able to communicate to the particle selected, will diminish rapidly with the distance, or inversely, as the number of pressing particles increase; and as all the pressures of other particles, so far as they can affect this special particle taken, will be forces directed towards it; we may conveniently assume that the individual pressures will, as other radial forces, diminish in intensity in proportion of the square of the distance. So that the separate pressures, in relation to the selected particle, may be conveniently represented by the spaces between the radial lines shown in the diagram Fig. 68, above. In this manner the pressures will be, for the entire force impressed upon the particle, conjointly equal at every circumferential distance from it. The *area* of the same total amount of relative pressure increasing in direct ratio to the square of the distance; its *intensity per area* decreasing in like proportion.

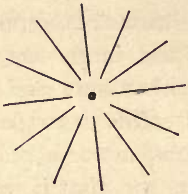

Fig. 68.—Diagram of Horizontal Pressures about a Particle in a Liquid.

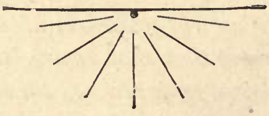

Fig. 69.—Diagram of Vertical Pressures about a Surface Particle in a Liquid.

b. Under the same conditions, if we take a vertical section through a particle in a fluid mass from the surface downwards, supposing no special condition for the surface density; then we find that as the particle is supported at the liquid surface against the force of gravitation acting upon it, upheld by all surrounding pressures in the liquid, that take their directive lines towards it: these pressures may therefore be as before, represented by the spaces between the lines radially directed to the particle, as shown in the illustration, Fig. 69, and the pressures being equal to the weight of the particle will insure its vertical equilibrium in equation with the action of gravitation in the system.

c. By the same principle we may conclude that if the particle selected were beneath the surface, as in the illustration, Fig. 70 below, that every surrounding pressure above the particle would also act radially upon it, for such distances as the radii could be assumed to extend as active forces.

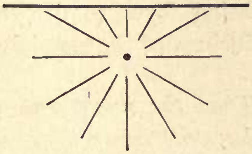

Fig. 70.—Diagram of Pressures about a Particle at a depth in a Liquid.

d. We may also observe that if, instead of taking a particle only of the fluid mass, we had taken a certain unit mass within the fluid, small or large, that the sum of all the surrounding forces or pressures would be equal *per area* to that upon the *surface* of the particle only that we have selected, and that these forces or pressures might effectively be assumed to be directed radially to the unit mass as we previously considered them to be directed to the particle only.

e. We may further observe that these pressures are quantities of force that can be measured exactly by the forces of gravitation, or by the mechanical forces required to resist them; as, for instance, the pressures upon the sides of a ship may be ascertained entire, or those upon any unit of area of its surface below the water line, by well-known hydrostatic laws that are commonly applied to fluid pressures, the conditions of which we need not follow here.

f. In taking areas of surrounding pressures, it might be thought that these pressures could equally well in theory be represented by radial lines directed to the selected particle, whose static conditions we have just considered. This could *not* be; for, if we gave to the radial lines any possible parallel dimensions, they would crowd near the particle, and to reach it must be conceived to be reduced to conic mass, which is the same as the conditions proposed. Further, these radial lines, if extended to any great distance with any possible equal and parallel dimensions, would leave inactive interspaces, the existence of which is contrary to known laws of hydrostatics. Therefore, any pressure communicated to a particle is derived from *all the surrounding pressures* in the medium, and although the total pressure upon the particle is only a unit quantity proportional to its size in the general system of pressures, yet all surrounding fluid matter may be assumed to impress force upon this particle in inverse ratio to the square of its distance, as before proposed. This pressure can only be represented for any single direction, geometrically, by a conic area extending from the particle to the base of a cone at any distance, the pressure at the base being equal in its active pressing force to the resistance at the vertex, which secures the equilibrium of the particle.

58. PROPOSITION: *That the movement of a particle in a fluid against resisting particles, will cause the compression of a conic area of particles in front, the deflection of conic areas of lateral particles, and at the same time it will be supported in its forward movement by the pressures of backward particles.*

a. Assuming the theoretical conditions of fluid equilibrium discussed in the last proposition to be true, we may further conclude, that every particle of a fluid that opposes an impressed force by the resistance of its pressure and inertia, will be supported by equal pressures and inertia of every contiguous particle, in like ratio, as the system is held together by the general forces of inertia, cohesion, density, and elasticity. We may also see that the impression of a force upon concrete homogeneous elastic matter, will be resisted by compression of the matter in front, that is, in the direct line of impression, unless, or until, the forward matter can move aside. It will also be resisted by every *conic area of particles* that makes an angle of less than 90 degrees with the forward direction of the part impressed. This will be found to be so, for if the direct line impressed be deflected by the impression, in ever so small a degree, the force will fall upon all the other forward contiguous parts that take radial direction towards the deflected part. So that in a conic area of resistance, by compensations, the pressures become hydrostatic pressures derived from direct radial ones. Otherwise, if it were possible for force to proceed in concrete matter in direct lines, a small percussion, defined as it was by Galileo as infinite force, would puncture a hole or produce a dent through any solid mass, however dense, that is, if the force impressed also exceeded the molecular elasticity in the direct line of impression.

b. Under the above conditions nevertheless, we need not suppose the movement of a particle in a fluid for a small distance to require a force to move it much greater than its inertia in *equilibrium*, and even for larger movements, the backward pressures, by the conditions of equilibrium, will have, as hydrodynamic pressures, equivalent value to the forward ones. These backward pressures being also theoretically derived from conic areas of surrounding pressures; they will, however, not be free to act instantly beyond the small value of the static elasticity of the fluid. Therefore all forward movements will cause a certain amount of forward *compression*, which the general system of the fluid must accommodate for the particle to move; and all backward parts of the fluid will receive a certain amount of *rarefaction*, which will act as a force of traction on the following parts, to reduce the amount of the following pressures to equilibrium; if the elastic pressures act inversely equal to the tractional forces which cause the relative rarefactions.

c. For the conditions of direct front or conical resistance to projectile fluid confined to a small area we have these conditions considered by Newton, for one case, which he takes negatively only, in the second book of the *Principia*, prop. xli.—"*That a pressure is not propagated in direct lines unless the particles lay in these lines.*" As physically considered, fluids are homogeneous; there are no direct lines except such as may be assumed hypothetically. Therefore the proposition taken in a positive sense, as a law of conic resistance, would apparently meet all cases of fluid projections and forms of resistance within the fluid. Upon the above conditions we may take it for granted that, as the first impact of a moving force in a fluid will engender a *compression* in a direct line in front of the impression, and that in the resisting fluid every side of such direct line of resisting matter will be supported by the entire surrounding matter, so also, we can imagine that the cohesive system in front will at once exert its force through a cone of hydrostatic pressure whose axis is merely a line of superior resistance, *not* the *entire* resistance, and that every circumscribed cone of matter of increasing divergence from this direct line will offer some resistance (or that proportional to the sine of the angle that it takes to the lineal direction of compression). Upon this principle we find that any imaginable base to a conic area of resistance, as it extends outwards from the axis of the direct forward cone, will take a constantly diminishing quantity of resistance with increase of divergence. Therefore if the fluid, or matter, making a direct projection has such a system that it can suffer division, the *divided parts will be less resisted upon the circumference of such a cone than in the direct axial line of it, where the compression, therefore the resistance, is greatest.*

d. If we further consider the impressed force as embodied in matter that will by its innate force, after impression, by the first law of motion, move directly forward over equal spaces in equal times, except as it is resisted or deflected by other forces, we may conclude that as the *resistance* to impression will be greatest in the axis of the forward cone impressed, that the impressed force, if free, would be deflected from this superior resistance to any path of less resistance open to it.

e. Having made out the above conditions for the motive equilibrium of a particle in a fluid, it was clear to me that in any small movement of the particle the area of direct head resistance could not by any means be an area of *deflection,* as all forward parts of this direct head resistance would be in equilibrium of pressure, there being no cause of deflection to one side more than to another; it would therefore clearly be an area of compression simply for the material fluid system. Whereas every *lateral area* would be one of *less resistance* open to the deflection of any force, motive in free matter.

f. If we assume the lateral parts of a unit of projectile fluid to be partly deflected by separation on impression upon a like fluid or more quiescent part of the same fluid, the central part or axis of projection receiving *compression,* as stated above, and the backward parts, if such existed, by general cohesion of the system, producing *tractions,* the hydrostatic forces of the surrounding matter remaining about the moving unit as before its motion; then in this case the surrounding direction of forces caused by the motion of a particle would be approximately represented as in the next diagram, the force impressed being assumed to proceed from B towards A. The space A would then represent a conic area of head resistance; B an area of traction; conic deflection being shown by curved lines in all lateral parts. Thinking that some evidence of this motive form of resistance to direct motion in a homogeneous system might be observed from inertia alone, in equal quantities of matter surrounding a point, I made the following experiment.

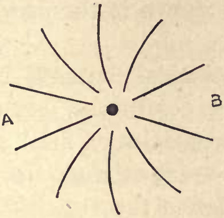

Fig. 71.—Diagram of Resistances to a Moving particle (theoretical).

g. I drew out a length of leaden wire of about ·06 of an inch in diameter, and cut this into equal lengths of about 2 inches. I placed the ends of all the wires together to form a radiating star, and fixed them in this position by solder, upon the end of a brass rod of 3 inches in length and ·36 of an inch in diameter, so that the rod was perpendicular to the plane of the leaden star. Now placing the star on a smooth plane, with the rod vertical above it, I gave the rod a few taps horizontally with a light hammer near its point of union with the star. The inertia of the lead was thus overcome in a manner that it formed a figure as shown above; in which we may observe that the radial directions in front A′, remain in the same positions as before the experiment, but that the lateral parts are all deflected by the resistance of the inertia of the lead to parabolic curvature. In this experiment, by measurement afterwards, I found that the directly forward parts were shortened, indicating active compression, and that the backward parts were equally lengthened, therefore indicating rarefaction, as might have been

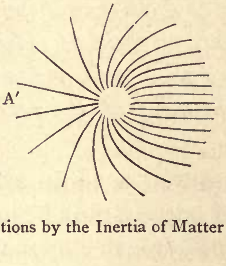

Fig. 72.—Ex. Deflections by the Inertia of Matter about a Moving Point.

anticipated. The lateral parts in this experiment were crowded together; this could not occur in a fluid where every particle would be supported by equal hydrodynamic pressures, but the deflections would be more nearly as shown in the previous theoretical diagram, Fig. 71.

h. Leaving these theoretical matters, I will now endeavour to show how far the motive conditions just considered, may be consistent with the actual properties of fluid projection, particularly in relation to the proposed areas of conic head resistance. In this research, to restrict and define the conditional circumstances of the above as exactly as possible, to make the matter clear, I will consider the motive effects of unit projections first; which will occupy the whole of this chapter. I will afterwards in the next chapter, consider the conditions of continuity of fluid projection. This division will simplify the matter, as the motive effects are much more easily observed in unit projections; and these observations will lead us, I hope, to the correct estimate of forces in fluids moving upon like principles in continuous motions, where some of the links of evidence disappear by a kind of absorption of actual effects. To render demonstrations as clear as possible, I will first consider the impression of unit forces impressed on small areas of extensive resistance in homogeneous solids, availing myself of the conditions proposed, 2 art. *e*, page 5.

59. PROPOSITION: *The impact of a percussion embodied in a unit mass, not large, of solid or of fluid matter moved with sufficient velocity against a homogeneous body or fluid of extensive mass, will Fracture the resistant body forward in a conic plane which will be henceforth the motive plane to the continuity of the force embodied in the matter impelled by the percussion.*

a. By the above proposition a conic area of fracture, diverging radially from the point of impact, may be impressed by a percussion within a mass of resisting homogeneous matter. The conic area impressed will be projected forward and displaced from the homogeneous mass by the effect of the percussionary force. The conditions under which the above may occur will be those in which the percussionary force acts in such short space of time, that the inertia and cohesion in the resisting mass is less partially overcome in time that the elasticity of its entire system can react to support the impressed point.

b. Under the above conditions, the force being assumed sufficient, the cone of fracture derived from a percussion will be perfect, if the area of contact is infinitely small; but as all bodies must have sensible area, actually the point of impact will be of this area, and of so much greater area as the instant deflection of the surface of the body making impact brings other parts contiguous to the area of contact into percussionary action upon the resistant mass.

c. In the case here taken, the active conditions of the homogeneous matter, assumed to surround the point of contact, are those of equal inertia and general cohesion, other forces or pressures being neglected. These conditions may be shown by the following diagram:—

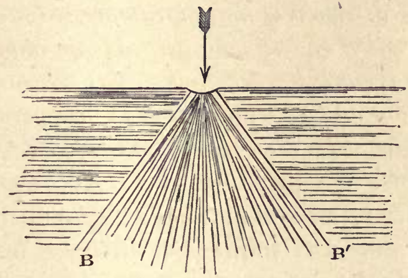

Fig. 73.—Diagram of Conic Fracture.

Let a force be impressed in the direction of the arrow shown above, in the diagram, Fig. 73, acting upon a homogeneous system of matter; then will the impressed force engender a cone of resistance in the homogeneous mass by principles discussed, 58 prop. If this impression be made in a space of time which is too small for the impressed mass to diffuse the force as a general pressure, then, we shall by the above principles, have a cone of matter separated from the general mass in the direction of radial forces within the cone, in such a plane as the cohesion of the system can be overcome. For this separation a *fracture* will be made that will circumscribe the mass of the cone, if the general surrounding homogeneous matter resists by its inertia only, and has insufficient cohesive force to hold its mass together during the impression of the percussion.

d. We may also imagine that if the cohesion of the system is great enough to resist the fracture shown in the case discussed above, or that the fluidity of the system is of so mobile a nature as to be impossible of fracture under the circumstances, that by the same principles of motion a tensile strain in the fluid system will be inflicted in the *same inscribing conic plane* that the fracture would have occurred in a more rigid system. Therefore the line B B′ in the diagram will be either a line of fracture if the system is rigid, or of tensile strain if the system does not permit fracture by want of rigidity. It will be convenient to defer experimental demonstration of this proposition until some other conditions are considered, as all the conditions of a fracture are not met by it.

60. PROPOSITION: *That a fluid or homogeneous system of matter impressed with a force sufficient to separate a conic mass of its matter, resists this separation by the inertia and cohesion of the entire surrounding system, in so far as the resisting parts are in radial direction forward of the lineal direction of the point impressed, at angles inclined to the direction of motion of less than 90°. If the impressed force Fracture a conic mass from the general mass, this fracture occurs where the forces of impression and resistance produce the greatest strain. The resisting part after separation forms an annular system whose internal form is conoidal.*

a. In the previous proposition I have assumed the mass of homogeneous matter that was impressed with the force, at the time, strictly neutral to its influence, or as representing inertia only. We are certain in this matter, by the laws that Newton has so ably demonstrated, that the force which we recognize as a motion is but relative to the motion of another body, and is not, or may not be, absolute. For instance, the whole matter that we observe on the earth is absolutely in motion by the earth's revolution, and moves relative to space of which we know nothing of its absolute movement. Therefore all motions may be considered as relative, or in equation of motion between the two bodies affected by it, in relation to each other, only as equally active forces. So that we are led to assume that the homogeneous matter upon which the supposed impact before considered in the last proposition was made, represents a force equally with the force making impact. And that there is no appreciable physical difference, whether we consider the homogeneous mass at its surface urged against the projectile body which makes the percussion, or the projectile body against this surface. Therefore, for instance, a homogeneous system or a fluid, as water, will resist by a cone of force exactly as though impressed by it upon the projectile body; and it will direct its mobile accommodation for the force equally towards the points of least resistance. The state of the case we are now considering shows that a surface that is impressed, may be assumed to be supported on all sides by a conoid of the surrounding matter, which resists the instant force beyond that part which is impressed by it, and is thus rendered motive, in the same manner that we assume that the cone that received the impressed force, was supported by the resistance of the matter surrounding its axis. This may be rendered clear by the diagram below representing impressed force acting upon a homogeneous system of matter, in section through its mass. I offer at the same time definitions of the parts of the system, to make the motive principles more clear for subsequent consideration in this work.

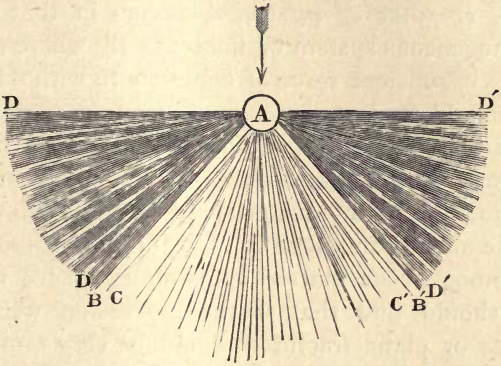

Fig. 74.—Diagram—Section of Conic Resistance and Fracture in a Homogeneous Mass.

Let A represent a body impressing a percussionary force in the direction of the arrow central to the horizontal plane D D′.

A C C′. The conic area impressed by the force upon a mass whose inertia and resistance of cohesion are overcome by it. This part (A C C′) to be hereafter termed *the cone of impression.*

A D D, A D′ D′. Sections of a ring conoid of matter surrounding the cone of impression which retains its position during the impression of the percussionary force, hereafter to be termed *the conoid of persistion.*

A B, A B′. A narrow vacuous plane, which is opened between the cone of impression and the conoid of persistion described above, if the impressed force is sufficient to fracture the system of matter. This plane of *fracture* will be hereafter termed the *plane of infraction,* or *the cone of infraction* of the homogeneous system.

b. I have felt it convenient to coin a word for this, I believe, new observation of the motive effects of impression of forces in homogeneous systems of matter and fluids. I named DAD′D′D originally *conoid of persistence;* but I found, as I progressed in my work, that this term became more and more inconsistent with the motions engendered in experiments, so that I found it convenient to alter it to its present form, as the persistence is not *absolute* but *transitory.* This conoid can only be conceived as a volume of matter of greater residuary persistence than other parts of the system under the first impression of a force. I therefore, to make the termination equivalent to the other parts of the system, term it a *conoid of persistion,* meaning thereby a conoid of imperfect persistence to the continuity of the active impression.

c. The cone or plane of infraction, occurs in that part of the resisting homogeneous system or fluid exactly where the conoid of persistion has insufficient force of cohesion to withhold the resisting matter or fluid against the oblique direction of the active force impressed in the cone of impression; therefore, the fluid or other homogeneous system of matter *breaks* in this plane, and forms the plane of infraction in the material system, as before discussed. Assuming the above conditions to occur in all cases of a unit projection of homogeneous matter upon an extensive mass of like matter, we should find that if the percussion were small, the vacuous space or plane fractured would be very *thin;* that if the force of percussion were too small to fracture the homogeneous matter, in this case it might still be of sufficient force to open a plane of greater strain in the homogeneous matter within the range of the static elasticity of its system. We shall find hereafter evidence of impression of such strain lines in fluids, where the cohesive forces are more readily parted for intrusion of other, or like projectile matter. But I defer experiment until after the following proposition, so that I may clear any experimental evidence that I am able to produce, as much as possible from collateral principles of motion, with which experiments in fluids are so unavoidably complicated.

61. PROPOSITION: *That the impression of a unit Pressure urged with sufficient velocity will open out and continue a plane of infraction in an opposing homogeneous system of matter. Such plane of infraction will be of conoidal form, the angle of infraction becoming constantly greater by divergence from the axis of direction of impression.*

a. By the last proposition a plane of infraction is assumed to be opened by the instant impression of force upon a fluid or homogeneous system of matter. The first impact of such impression would be *percussionary;* but the body making the percussion must have mass which will carry momentum after the first impact upon the surface of impression. This mass would therefore, after the first impact, act by its momentum as a *compression.* Therefore we may imagine that the *percussion* will open out a plane, which the *momentum of compression* of the following parts of the mass will extend, assuming the force sufficient. In this manner the first impact or percussion, and the following momentum of compression would represent the motive energy of the entire projectile mass.

b. Taking another condition of compression:—Every body even in contact must, if it moves, *commence to move;* therefore, its first effort of movement will be infinitely motive to any body at rest that can resist it, so that every compression at the first instant of movement will possess force that is of the character of a percussion, so that every body impressed by a force will at the first instant receive the impression as under percussion, and every body receiving a percussion from any possible body of matter will have this followed by a compression equal to the residuary force necessary to complete the entire momentum of the projectile after its first instant of impact. This form of compression that follows a percussion, I desire now to follow in its effect on a fluid or homogeneous system.

c. Suppose a body to strike normal to a plane of matter. At the instant of contact, percussionary force would be infinite; therefore, if the intensity of the impression exceeded the molecular elasticity of the system of matter, it would strike out a cone of matter radial to its point of contact, of a certain extent and divergence in the mass. Then, as just proposed, the direct momentum of the striking body after its first instant of impact, by its following mass, would continue to press its matter forward into the impression, and the matter in front would be most *compressed* by the force; so that the continuity of the momentum, therefore, of the projectile, if in a mobile system or a fluid, would move forward in the resisting medium in equation with the resistances with diminished velocity to that of first impact, and being more resisted in front than in any other direction; *it would be compelled to take a wider angle of divergence into the lateral matter, where there would be less resistance than in the most compressed matter directly forward in the resisting mass;* or, in other words, the impulse would be deflected from the axis of impression, by composition with the forces of direct momentum into direct compression, or of resistance engendered in the axis, by the direct impression; and the cone of infraction would be extended at its base, or more distant parts from the point of compression, until the resistance in the lateral parts became in equilibrium with the impression by extension of area.

d. The principles of the above may be inferred from the diagram given Fig. 71, page 173, where it might be shown that the first instant of impact might impress the forward cone only, but the continuity of the impression in a direct line by the momentum of a mass would deflect the surface of the cone into the lateral parts; in fact, the forward parts, except those in direct axial line, would become, after a certain projection, the lateral parts, and the continuity of infraction of the cone would produce a conoid of constantly greater divergence. The constant forward momentum flattening out, as it were, the conic area impressed.

62. Some experimental evidence of the last three propositions.

a. I will now endeavour to select cases that may convey some experimental evidence which may render the conditions discussed in the last three propositions more clear. For the experimental demonstration of the principles of conic resistance, I will take the effects produced by impression of a unit force upon a homogeneous solid body, leaving demonstrations as to fluids to follow after other propositions in due course. For this homogeneous body it will be convenient in the first experiment to select a very cohesive substance, as in this case we may apply greater forces, and we need less refinement in experiment to observe the effects produced; besides which, the form of fracture developed, will demonstrate the *plane of infraction*, in such a manner that its form will be firmly set, so as to enable us to examine it at leisure after the experiment.

b. Upon these considerations it will be convenient, for many reasons, to take *glass* as a powerfully cohesive body to show the principles involved, as in this we have a material whose force of resistance to tensile strain is found experimentally to be about 2600 lbs. to the square inch. At the same time we have also in this material a very homogeneous body, not endowed with any conceivable internal molecular mobility of parts, by which compensations could be brought about that would materially alter the direct effects, or cause the angles of the impressed forces to diverge excessively, as might be the case with the cohesive system of some other solid bodies; as for instance of any of the metals, whose general open unequal crystalline structures would permit them to separate more easily in certain directions, or to bend under impressed forces, in a much greater degree than is possible with glass, and thereby ease off the instant effects of the direct impression of the projectile forces upon them; so that the *cone* of impression would become a *conoid.*

c. If we take a disc of stout plate-glass, of say from half an inch to an inch in thickness, and strike this with a light pointed instrument, or, what is more convenient, allow a solid ball of metal to fall upon it with moderate velocity (the circumstances of the velocity of impact may be within wide range), a conoid of the glass will be projected from the back of the disc, and this conoid will give the outline of the *plane of infraction* for *glass,* under the force impressed, as it obviously occurs in this body where the *conoid of impression* breaks off from the *conoid of persistion* that supports the surface, where the cohesion of the particles of the glass are unable to resist the strain.

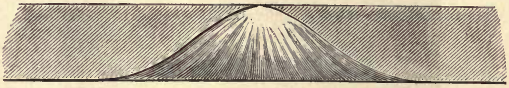

Fig. 75. Conoid projected from a Square of Glass.

d. The above illustration of the experimental effects described above was made by Mr. Collings from a conoid projected from a sheet of plate-glass three-eighths of an inch thick, by dropping a stone marble of half an inch in diameter from an altitude of seven feet upon the plate of glass. The glass was bedded with putty in a wooden frame, as a pane of glass in a window, and was placed horizontally to receive the percussion. The conoid was preserved by pasting a sheet of tissue paper under the surface of the glass before commencing the experiment.

e. In the above experiment the conoid of impression was fractured by the strain into a great many pieces; and although the outline was clear, it would have been more satisfactory to find an experiment in which the conoid could be obtained complete. This I was unable to do. It is also difficult to preserve the outline of the conoid of persistion in thick glass, which would otherwise give a much clearer demonstration of the conoidal form of fracture, as this would be more free from surface deflection than the thinner glass taken above, in which the deflection, by the elasticity of the system, produces greater extension of the base of the cone than the conditions of direct impression would otherwise permit. The best experiment that I could make with thick glass was to fire a rifle bullet close to the edge of a thick plate of glass in such a position that the axis of the cone could be free, and that no secondary fractures could occur from the excessive internal strain. In this case a splinter of glass only was obtained, but it was not split up axially, and its form was a nearly perfect section of the conoid of impression. The illustration below shows a portion of a conoid obtained in this manner drawn from the object.

Fig. 76.—Ex. Conoidal Splinter of Glass. Drawn by Mr. Collings.

f. Many will have often observed this form of conoid of fracture, Fig. 75, clearly cut out in a London shop-window that has been struck by a stone. A similar mode of producing a conic hollow is found in mechanics, in making holes in roofing slates by a sharp instrument termed technically a *saix.* The hollow cone in the slate forms the cavity in which the head of the slating nail afterwards sinks so as to leave no projection.

g. In all cases of fracture by a unit mass of glass the *true cone of infraction,* as defined in 59 and 60 prop., only occurs from the effect of the percussion or first impact of impression at high velocity, and does not extend far into the glass. I have found that it extends generally for a distance not greater than a quarter of an inch from the point of impact; for the distance that it extends for intense forces its plane is perfectly straight from base to vertex—in fact a true cone. After this perfect cone is split out by the effect of the percussion, the deflected conoid follows for the continuity of the force of projection in the projectile by its momentum, causing a compression, as before stated (61 prop.). In these percussion cones, I have found that the angle of infraction was approximately persistent through many experiments, for the same quality and density of glass, under some considerable variations in the projectile force used. The percussion angles to the direction of impulse of the cones at the vertices, being for,—

1 inch plate-glass, ...	33°
¾ ,, ,, ...	24°
⅝ ,, ,, ...	20°

The variations under different circumstances being one or two degrees. As my researches were for qualities of motions, as before stated, not for quantities, I did not extend these experiments farther than a few experiments.

h. The following diagram will represent the theoretical separation of the effects of percussion from those of compression in the fracture plane of a plate of glass of one inch in thickness.

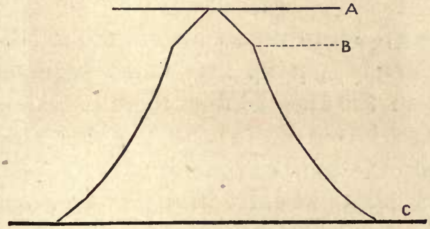

Fig. 77.—Diagram of direction of Percussionary and Compressile Forces.

A to B represents a cone of impression by percussionary force inflicted normal to the plane A on the vertex of the cone (59 prop.). B to C conoid of impression as it would be derived from compression simply (61 prop.), after the percussion plane is opened. This form will be better demonstrated by the infraction planes of free fluids further on.

i. The angle shown at B is theoretical, as is also the outline of the entire conoid of impression B to C. Actually the plane of percussion will have a directive influence for the continuity of impression, so that the one direction will generally merge into the other by compensation, which will make the plane of fracture forming the conoid of impression of a somewhat conchoidal form, particularly when the percussion that opens the infraction plane is very small. The conoid of impression, by continuity of infraction, becomes a volute, as I shall hereafter show, but the cohesion of the glass would of course resist the complete formation of such a form.

j. In any case the compression of the conoid of impression will react as a hydrostatic pressure in extending the angle at the base of the cone if this be possible through the mobility of the material system. In glass, hydrostatic pressure would be difficult to imagine, so that in this body the conditions may be considered under another form, which is only somewhat equivalent. If glass is impressed with a velocity-force sufficient to fracture it either by opening a plane of infraction by a percussion or otherwise, the fracture takes a direction in the mass in a conic plane where the cohesive force of the glass is most strained at the instant. The compression conserves the elastic force, and this conserved elasticity supports the cohesion about the axis of impression and drives the fracture outwards into the contiguous mass, where the matter is not compressed. This compels the fracture to take a more extended area, and tends to open the conoid into a bell-mouthed shape of conchoidal outline.

k. By the internal compression also in the axial line, the cone of impression in glass is split up into radial segments extending in direct planes from the axis, so that there is evidently radial strain from the compressed axis of the cone, together with circumferential tensile strain upon the cone or conoid.

63. Secondary plane of infraction for homogeneous matter. I have found that in using stout glass, a secondary plane of infraction may be formed within the compression area of the ejected cone. This occurs only when the projectile impinges against the glass in the same manner as in the previous experiment, but with greater velocity. For instance if a ball of iron one inch in diameter fall twenty feet upon a piece of glass one inch thick; in this case the ejected cone of impression produced by the impact will not be split up radially to its axis, but a secondary plane of fracture will be formed *within the conoid of impression*, which will prevent the radial lines of compressed matter reaching from the fractured circumference up to the axis of the cone. This secondary plane of fracture is caused by the elasticity of the glass being insufficiently active to bend or crowd its particles together under the impression of the direct force, or to do so with sufficient velocity forward of the point of impact to permit radial fractures to proceed up to the central line; therefore the impressed force opens out a secondary plane where the strain of central deflection is greatest upon the cone of impression. This only so far concerns us, in relation to fluid motions, as showing the strain lines in a rigid homogeneous system under percussion. But the experiment is very interesting in itself, and the phenomenon is curious. The form of the secondary plane has a very beautiful outline, appearing as in the illustration Fig. 78, as a classical vase. This form, perfect or imperfect, is produced under many nearly similar conditions, and is always of the like flowing lines. The experiment also clearly distinguishes the percussionary from the compressile plane. The illustration is taken from the original object by Mr. Collings. The dotted lines represent continuity of the plane of infraction. Further demonstrations of the last propositions applied to fluids, will be found in others forward.

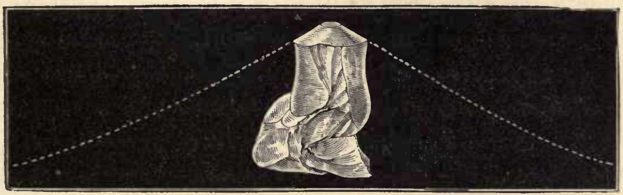

Fig. 78.—Ex.—Secondary Plane of Infraction in Glass.

Conic Resistance in Fluids.

64. PROPOSITION: *If a small unit mass of fluid matter be projected upon a large mass of static fluid with great velocity, the impact of the small mass will Fracture the static fluid in a conoidal plane, as shown by the previous experiments for homogeneous matter. The projected fluid by the continuity of the momentum of compression of its mass, on the resistance of the cone of impression, will by its momentum, intrude itself into the fracture opened by the percussion.*

a. Suppose it to be possible to make a fracture in a fluid, of exactly the same kind as that demonstrated for glass, in the previous experiments, and that this fracture were made by the velocity-force of the small mass of a body possessing fluid properties; then if the percussion had opened out a plane of infraction, the further momentum in the projectile fluid mass would suffer little resistance to oppose its entering in the open space thus formed. Further, as upon the *cone of impression* (A, C, C′, Fig. 74, page 177) great compression from the momentum of the projected mass would exist for the instant, there would be, as before shown, great resistance to oppose the continuity of the projectile motion in a direct line. Therefore we may conclude that if we had impressed the unit force with a liquid body, as the parts of this could easily separate, that the projected liquid would open out at the point of percussion and enter the space of the cone of infraction in the resisting liquid. For this condition we may further imagine that the plane of infraction need not be one absolutely opened as one of *no matter*, or vacuous space, and yet it would in all cases be the line of least resistance to the momentum of the projectile, by the tensile strain or minus resistance that the strain upon its cohesion would engender in this plane.

b. The demonstration of absolute fracture of a fluid is no doubt difficult, from the fact that in a very short space of time the surrounding pressures and conservations of elastic forces near the compressed parts, would, if any possible fracture were produced, fill up this fracture instantly, or in less time than the eye could detect movement. The demonstration of this principle of fracture is nevertheless very important in order to establish the past propositions, and for our future investigation, and it may be taken under several conditions, some of which will be followed for smaller forces in subsequent propositions, the aggregate of which, by experiments, I hope may possibly give theoretical certainty to this principle of fluid motion, as I have observed it in many more experiments than I can possibly give without great extension of the subject.

c. Taking for experimental demonstration for the conditions of this proposition, a force impressed by a small unit of liquid or homogeneous matter, with very great velocity upon the surface of a mass of liquid; we may in this case assume that if the velocity of the projectile were such that any *movements of parts of the liquid among themselves* could not occur during the time of the impression of the force, that the liquid would resist the *impression with exactly the same relative force as a solid of equal mass and of equal incompressibility to itself;* that if the momentum of the force, compressed the extensive liquid to less volume, directly in front of the impression, and we assume no time for movement of the parts exterior to the compression, a *conic fracture must occur* about the compressed parts in the extensive liquid; and the projected unit of fluid matter would enter this fracture by continuity of its momentum after the fracture in the plane of infraction, that is over the conoid of impression formed by the projection or compression within the liquid mass.

d. For experimental evidence of the above principles, the best experimental means I could devise was to take a bulk of liquid for the resisting body, and a plastic homogeneous body that would retain its impressed form for the projectile fluid body; so that the projectile should maintain the set form of the plane of infraction into which it was intruded. Considering the density, and incompressibility of water, as proved experimentally, I determined to take this for the resistant liquid; and to try *cold lead* as the plastic fluid for the projection (2 art. *e*), as I conceived lead would possess sufficient fluidity, that is, mobility of parts, and yet when the impressed force ceased to act, through resistance, sufficiently to stop its flow, it would still maintain its set form for subsequent examination.

e. For this experiment I found it necessary to construct a complete apparatus which I think sufficiently important to explain in detail. The apparatus consisted of a trough that was made about eight feet in length and of about nine inches in width and depth. A circular hole was cut in one of the ends of the trough at a little below its centre, of one and a half inches in diameter. A wooden ring three-quarters of an inch thick was turned in a lathe to the same internal diameter as the hole, and of about three and a half inches external diameter; this was made flat on both sides, six small holes were made round the ring into which wood screws were inserted, and the ring was then screwed to the inside of the trough over the hole. Removing the ring after its position was secured, a piece of parchment was stretched over the inner surface of the ring and attached to its outer edge so as to form a drum. An ordinary flat india-rubber band, such as is used for holding papers, was placed over the hole in the end of the trough inside, and the wooden ring was replaced by the screws with the drum side next the end of the trough. In this manner the parchment drum formed a small part of the end of the trough. The trough was now placed quite horizontally and nearly filled with water ready for the experiment.

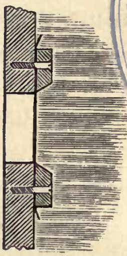

Fig. 79.—Section of Parchment Diaphragm.

f. Having procured an Enfield rifle loaded with a standard cartridge, which included ball and gunpowder in one inclosure, I had this fired at three yards' distance directly at the centre of the parchment drum in the end of the trough, and I afterwards examined the leaden ball to see how far it had followed in the plane of infraction, that I assumed the high velocity of the bullet would open out in the water; also to discover how far interfering causes might have affected my theoretical deductions; but I found the lead so clearly injected into the plane of infraction, that I feel justified in giving the whole circumstances in connection with this experiment.

g. I may premise that in the experiment the hole in the end of the trough had inadvertently been made rather too low to get quite fair resistance from the mass of water, and the trough was made of rather too thin wood, three-quarters of an inch only, so that it gave way under the side compression upon the water to the force of the bullet; from which I estimate that the resistance was rather less than would have occurred, under the conditions and dimensions given above, if the experiment had been conducted with a little more care. It nevertheless gave a very correct idea of the form of resistance, argued from my previous conception of the subject, so that I did not feel it necessary to repeat the experiment, which would have required a new trough, and waste of my time. I think, however, that the matter is important as showing a principle of resistance for liquids, such as would not be generally anticipated except upon the theories of fluid matter I have offered.

h. For the details of this experiment. First as regards the leaden bullet, which represents our projectile fluid; a full-size section of this is shown in the engraving Fig. 80. This form happened to be extremely well adapted to exhibit the effect of fluid percussion and resistance in several particulars. Its form being nearly parabolic, it makes its percussion upon the resisting body, in this case, the parchment drum backed by the water, within a small area of contact. The point of impact being at the same time well supported by its parabolic form. It is also adapted by its section to permit the resistant *cone of impression* to be well developed in the water, as it is not so solid at the apex as to be entirely indeflectable and thereby to cause a splitting of the water in a direct line, as I afterwards discovered would be the case with a hardened steel

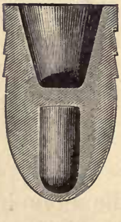

Fig. 80.—Section of Enfield Rifle Bullet.

bullet; there being an air chamber at the back of the point of the leaden bullet, that permits the possibility of some deflection, which we may take to be equivalent to certain functions of fluid mobility, within the projectile, which will be important for future consideration.

i. We may imagine that the bullet as it strikes the water is affected in the following manner:—Upon first contact the apex of the parabolic shot opens out a plane of infraction in the forward water.

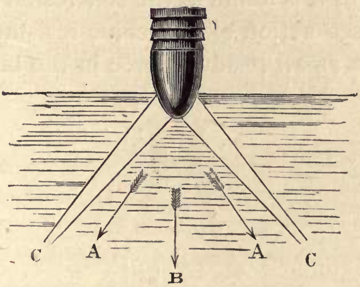

Fig. 81.—Diagram—Effect of Contact of Bullet on Water.

The form of this infraction is shown diagrammatically in the engraving above at C, C. The whole metal of the shot after this instant is embodied in a compression upon principles discussed, 61 prop., and is urged forward with its initial mass velocity. The percussion being of infinite force to the water at rest, the metal of the bullet is compelled to open out its mass and follow in the plane of infraction as that of least resistance. Therefore the lead *flows down the surface of the cone of impression in the resistant water as a simple plastic liquid would;* the point of the shot being stretched out to cover the base of the cone, as far as is possible for the cohesive qualities of the lead to do so, and the lead retains the form of the cone of infraction into which it is injected; this process being no doubt facilitated by the softening of the lead by heat engendered by the impact.

j. The lead as it is at first projected in the plane of infraction may be assumed to cover the cone of impression only to its vertex, but as the force derived from the momentum of the bullet in this projection is greater than the resistance of the cohesive force of the lead of which it is formed, it leaves the vertex of the cone and flows down some distance towards the open infracted base. At this point in the experiment, the lead having become very thin, it separates generally into five or six segments over the cone of impression, and in this segmental condition, its form remains fixed for future examination. In repeating this experiment, it will be found convenient to colour the point of the bullet with a little vermilion in shellac varnish, and allow it to dry, for the purpose of putting the segments together in proper order to form the complete mould of the plane of infraction after the experiment. The point of the bullet which is coloured red, of course forms the outer circumference of the conoid, as it is found at the conclusion of the experiment.

k. After the direct action of the conic resistance is complete in the water, there is a small residual force in the bullet, and a certain time for fluid accommodation has now been given to the water, so that the bullet follows a parabolic course to the bottom of the trough, and remains at from six to nine inches from the parchment drum, where the segments may be found.

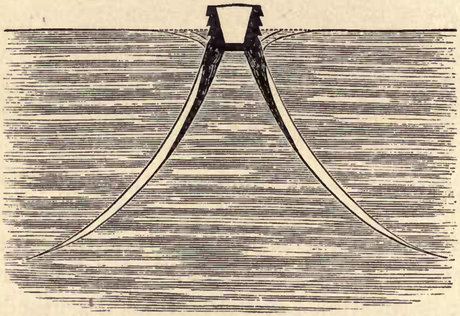

Fig. 82.—Diagram—Section of Bullet entering Surface of Water.

l. The section of the bullet as it would be at the time it is resisted by the perfect cone of impression would be as shown half size by the dark section in Fig. 82; the plane of infraction being shown open in front for its further projection.

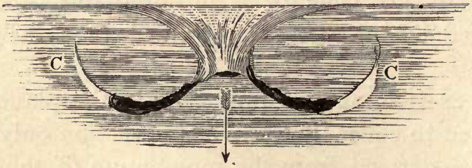

Fig. 83.—Diagram—Section of Bullet, Second Stage of Projection.

m. The momentum of the backward parts of the shot would in a mobile system suffer less resistance than the point of contact; therefore, the continuity of this momentum would further compress the conoid, so that in the next instant the projection would be as in the dark part of the engraving, Fig. 83.

n. By further continuity of the same mode of force, the outward deflection of the conoid of impression by the great momentum of the

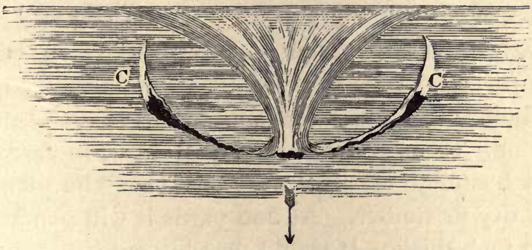

Fig. 84.—Diagram—Section of Projection of Bullet, Final Projection.

projectile after the first instant would react and attain a descending force, and the projectile would be finally set in the form represented in section, Fig. 84, above; the bullet not having sufficient elastic extensibility to continue further in the deflected plane of infraction.

o. The exact appearance of the identical bullet here described with its segments carefully laid together in their set forms, was drawn

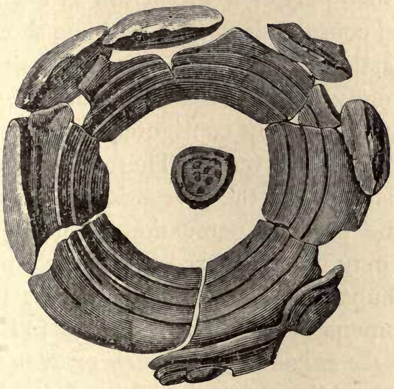

Fig. 85.—Ex.—Lead Bullet after Projection in Water, drawn from Object, Full Size.

on wood, full size from the object, by Mr. Collings the able artist, who has illustrated many of these pages, the section being as represented in Fig. 84. The central object in Fig. 85 is the diaphragm of the shot which is more perfectly resisted, so that it does not materially change its form. The *outer edge* of the shot in the diagram, that is, the part that previously formed the *pointed end* in front of the shot, has the lead rolled up to a certain extent, the reasons for which I shall more clearly show by the motions of absolute fluids, under somewhat equivalent conditions, further on.

p. For comparison of resistance of water with some other bodies to the impact of an Enfield rifle bullet, it may be well to mention some facts given in our volunteers' musketry instructions which have been furnished to me by a friend. The initial velocity of the Snider rifle bullet used for this experiment is said to be about 1260 feet per second. Its extreme trajectory about 3000 yards in air. At 100 yards it has force to penetrate 7 inches of elm plank, or about 10 inches of dry fir timber. At 200 yards it will penetrate an iron plate ¼ of an inch thick. It would possibly enter not more than an inch of water at a distance of 10 yards.

q. In the above experimental results the phenomena described are supposed for our purposes to cease at the instant that the bullet making the percussion has intruded itself into the resisting mass and attained a set form, that is, at the instant of complete projection, and before any phenomena of reflex actions occur. Therefore at this instant we consider the water left with all the compressions of force upon and within it. This has been done as a simplification of the results of such part of a projection as I wish to be observed by isolation. The compressions that we have shown engendered in the water have elastic reactions after the percussion is complete, by conservation of energy, equal to the whole force of the bullet making the percussion, and these by contiguous reactions are distributed through a large mass of the water. Therefore when the elastic force is set free immediately after the percussion, a large mound of water springs up over the point of percussion. The equivalent forms of this reflex action may be considered in other simpler phenomena.

It may be perhaps also well to observe that the impact of the shot in the phenomena described is supposed to be impressed *quite perpendicularly to the surface of the water or plane of resistance.* If it were impressed obliquely, principles of fluid accommodation under this mode of contact would occur, which would produce tangential action upon the mass of liquid and deflections; so that like projected forces, of high velocity, would enter and be resisted in a totally different manner.

65. PROPOSITION: *That in the projection of a small fluid mass at small velocity into an extensive like fluid; the impression of the small force will engender a conic plane of tension in the extensive fluid, by means of which, the projectile fluid will enter by its direct momentum; this plane of tension being to the projectile fluid the path of least resistance. The conic plane of tension produced by this means will resemble the plane of infraction produced by more active forces.*

a. If we accept the theoretical conclusions arrived at in the last four propositions, that a force impressed with any velocity upon a large mass will open out a plane of infraction into which the projected fluid will be intruded. Then in the case of smaller velocities, to which this proposition points, the impression of a small mass of fluid may be taken to be insufficient to absolutely open such a plane, except at the point of impact, as before mentioned. But in this case if the momentum of the small mass is only sufficient after this first opening to continue the projection upon the cone of impression, there will be engendered about this cone a plane of tension where a superior force would have fractured the fluid, and this plane will be that of least resistance to the direct momentum of the projectile force in the small unit of fluid projected; so that this *plane of tension* will offer the same conditions for the continuity of projection to the small force that the *plane of infraction*, shown experimentally Fig. 75, page 181, offered to a larger force in more resistant matter.

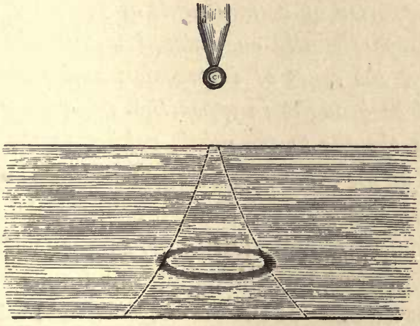

Fig. 86.—Ex.—Projection of a Drop of Coloured Water.

b. The demonstration of this proposition may be obtained by the experiment of dropping, slowly, drops of a soluble coloured liquid upon a smooth surface of water in a shallow dish or other vessel. The coloured liquid may be ink, milk, carmine, or other. A single drop may be projected from a dropping tube, or a point as that of a quill pen. When the drop falls upon the liquid surface it is observed to separate as it enters and to form a ring. The appearance being exactly as though a solid conoid of resistant matter existed under the liquid surface upon the vertex of which the drop falls. The vertex of the invisible conoid appears to pierce the drop so that henceforth it continues to glide down the surface of the cone to the bottom of the vessel. Upon further refinement we find that other conditions occur in the motion of the ring besides its direct projection, but the above gives the first appearance.

c. The explanation of these phenomena is generally given by philosophers as active upon entirely different principles to those now offered. In assuming properties of axial forces which are described as *vortex* motions. I do not object in any way to the term *vortex*, but I shall find it convenient in these pages to avoid it, in that, this term has sometimes been applied to mystical and unmechanical theoretical systems of motion, from Descartes downwards. Whereas all mechanical motions of matter are to my mind simple and direct, and in the cases in which I shall discuss these *vortex motions* they are very clearly only deflections, by resistance, after the manner of ordinary elastic reflections, following known, and with care, easily demonstrable physical laws.

66. PROPOSITION: *That the principles of motion engendered in a mass of fluid impressed by a small unit of projected fluid, will be also relatively and proportionally engendered within the smaller projected fluid unit. That the cones of impression and infraction in the two fluid bodies will be brought in opposition upon contact, and act in an inverse direction the one to the other, subject to the conditions of the relative freedom of the masses from surrounding forces.*

a. That the projectile unit in the last experiment should be subject to the same principle of action of motive forces within it, as a larger mass, might be inferred, but the conditions found in its motive parts render several particulars of the forms, that the motive parts assume different from those observed in the projection in the larger masses. The general condition of the above proposition will be best discussed upon experiment.

b. If we let a drop of water coloured with soluble colouring matter fall for six inches, as in the last experiment, upon a still surface of water contained in a vessel two or three inches deep, the drop after contact upon the surface will be partially reflected and will partially enter the liquid. The part entering will divide upon the conoid of impression incipiently formed by principles already discussed, and by this division form a ring of the coloured water, which will glide slowly down the cone of impression, being at the same time resisted by the conoid of persistion. The principles of the above may be shown most clearly by a diagram as follows.

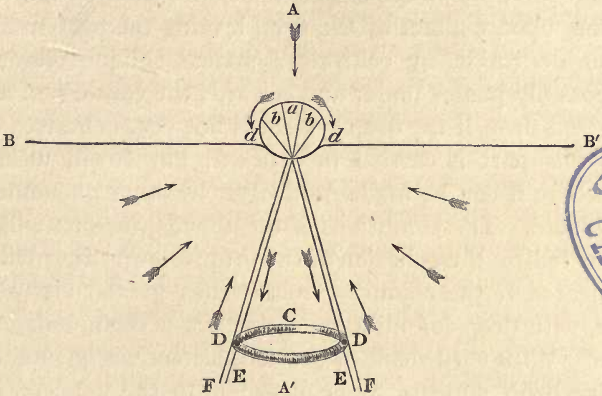

Fig. 87.—Diagram—Parts of the Whirl-ring System.

Let the following parts of Fig. 87, represent the drop system completely.

A to A′. The line of projection of the force, in this case engendered by the attraction of gravitation simply.

B B′. The liquid surface or plane of resistance to first impact.

C. A projectile ring.

D D. Conoid of persistion extending from F B and F B′ on either side to the axis A A′.

E E. Cone of impression extending to the point of impact of the drop under *a*.

F F. Cone of infraction, the plane of greatest elastic tension in which the projectile drop moves with the least resistance, shown by parallel spaces F E on both sides of the diagram.

a—Inverted cone of impression within the drop, the part which is reflected by the release of the instant compression after contact, by reaction of elasticity, at the point of impression of its own cone, with that of the vertex of the cone of impression, directly under it.

b b—The part of the drop whose momentum is maintained after the fracture in the plane of infraction in the drop. This part enters the water in the plane of least resistance (cone of infraction) as a ring which is shown as an after phenomenon projected down the cone of impression at C.

d d—It will be seen that this part of the drop does not make percussion upon the water surface direct enough to enter the cone. I anticipate that this part generally adheres to the water surface, and that the parts of the drop *b b* are emptied out, as it were, by the jerk of the percussion, upon contact of the drop leaving the portion *d d*, which has, after deflection by central resistance, oblique momentum, so that it possibly passes under and towards the centre and adheres to the reflected drop, if the drop system is not very cohesive; or it may be that this part is carried in with *b b*; but I will return to this matter where it can be made more clear by other phenomena yet to be considered. The conditions of the above principles will be somewhat affected by the cohesion of the drop, a larger ring being generally projected if the colouring matter has greater viscosity, as for instance, with drops of milk, or of ink from the mucilage this last contains. With very weak projections also of viscous matter nearly the entire drop appears to be drawn into the cone of infraction. The reflection of the meeting of the two cones of impression which are instantly formed on contact is best observed by permitting the drop to fall from a greater height, say, for instance, 30 inches. In this case the ring is jerked off the drop system and the reaction of the cones of impression reflect parts of the drop for an inch or so above the liquid surface. In this reflection the surrounding surface water is also drawn up by adhesion, and it again opens an infraction plane sufficiently to permit a secondary imperfect ring to be projected from it. I will consider the conditions of this hereafter, my object at present being to follow the principles of conic fluid resistance under conditions as simple as I can make them.

67. PROPOSITION: *In the projection of a small unit of fluid in an extensive fluid, the surface that circumscribes the cone of impression will be an area of acceleration within the fluid to the projection of the entering fluid. The corresponding surface of the conoid of persistion will be the plane of greatest resistance. Therefore the entering fluid will maintain its projection without resistance upon the cone of impression, and with great resistance upon the conoid of persistion, and be*

induced to rotate upon itself consistent with the forces of acceleration on one side and the resistances it encounters on the other.

a. The projection of a unit of fluid has been shown to engender a cone of compressed fluid, in a like resisting extensive fluid by previous demonstrations. To effect this compression in the cone impressed, a certain motion in the direction of the projection of the unit is communicated to the cone, so that, as the cone of impression does not perfectly resist the compression, it becomes a *motive compression,* and the cone of impression becomes exteriorly thereby a certain active force, moving downwards from the vertex of the cone.

b. Now as the projected unit, making percussion or compression upon the conic area of resistance, is deflected from its direct course, this deflection by pressure upon the conic area of resistance accumulates its direct force upon the plane of this cone, and *accelerates* by hydrostatic pressure the deflected fluid upon it; as fully demonstrated in the conditions of deflection by resistance shown 40 prop. by experience *b* and *d.*

c. The conoid of persistion is altogether less disturbed by the impression of a unit projection than the cone of impression, which is broken off, as it were, from the active part of the system at the instant of impact, so that the conoid of persistion remains during the projection down the cone of impression in a rigid state, therefore the projected fluid of the drop or other unit enters the plane of infraction at nearly its initial velocity, and being deflected by the resistance of the cone of impression, cumulates its elastic force (by the conditions before alluded to in 40 prop.), and is accelerated upon the plane of the cone of impression, and retarded upon the more rigid conoid of persistion; so that the injected fluid, as before stated, is rotated by tangential action upon its axes of inertia normal to these planes, in proportion to the conservation of elastic forces, from the deflected projection on one side and resistance on the other.

d. Upon these principles the following diagram will represent the section of such a system which by its rotational forces takes the form of a rotating ring. The short arrows showing the direction of motive forces in the system. The exterior long arrows representing the inertia of the conoid of persistion as active but not in this case as motive. The interior long arrows representing the forces of acceleration by cumulative elasticity by deflection of resistance upon the cone of impression. The velocity of projection of the ring is at first, as the velocity of motion of rolling contact upon the conoid of persistion; but as the ring continues its projection the surface of this conoid is induced to take the same direction of motion as the ring in contact, and the surface resistance to rotation of the ring becomes less. At the same time the direction of motion induced by the ring upon the plane of the conoid of persistion causes this plane to become less static, and the rotational force of the ring to act less thereby as a projectile force by rolling contact, proportionally as it maintains its rotation upon the axis of inertia of every radial segment. So that its projection is more limited in time than its induced rotation. The rotary system of a drop is somewhat in excess of the causes here given, from the fact that the drop, before contact upon the liquid surface, is necessarily a rotary system, the conditions of which will be hereafter considered.

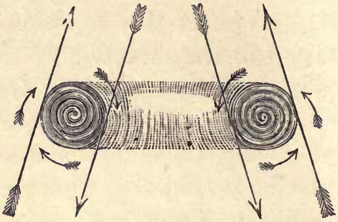

Fig. 88.—Diagram—Theoretical Section of a Whirl-ring.

e. Experiments of projecting units of coloured fluids to form rings have been discussed for a long period, the earliest I have met with are those of the projection of phosphuretted hydrogen gas both in air and in water. The most elegant methods we possess were those devised for air, by Professor Tait, and for water, by Professor O. Reynolds. These experiments I will briefly describe.

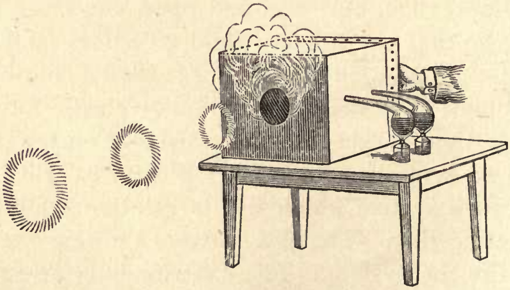

Fig. 89.—Ex.—Prof. Tait's Smoke-ring Apparatus.

f. Prof. Tait's experiment, which has priority, but the original description of which I have not been fortunate enough to discover, consists of an apparatus constructed out of an empty tea-chest without a lid, with a circular hole of three or four inches diameter cut in the centre of the bottom. The open top of the chest is covered over with a piece of canvas which is stretched as a drum-head by neatly tacking it round with tin tacks over the edges. The tea-chest is then turned down sideways. Two holes being made at a short distance apart in one of the sides of the chest and the beaks of two small retorts inserted in the holes. The retorts are supported on stands with a small Bunsen burner or spirit-lamp under each; hydrochloric acid being placed in one of the retorts, and liquid ammonia in the other. As soon as the vapours arise from these fluids and enter the chest they form a white cloud of ammonia-chloride. When the chest is charged with cloud, if the canvas drum be struck with the hand a beautiful ring will be ejected into the air, which will proceed for ten feet or more; or the canvas may be struck so softly that the ring will move very slowly for a short distance and its motion of involution may be very clearly observed. The experiment is altogether a very beautiful one.

g. In the above experiment the vertex of the cone of impression is of course *within the chest,* and near to the canvas where the direct impulse of percussion is made. The projection of a ring in this manner from a large aperture in the chest is much greater than it would be in an open fluid, from the perfect support the conoid of persistion receives by the end of the chest in which the hole is made, as discussed in principle paragraph *d* above.

h. In Professor O. Reynolds' experiment[1] a long tank about 8 feet by 2 feet by 2 feet, with glass sides, solid ends and bottom, is constructed. In one of the ends a circular hole is made about 1½ inches in diameter, in which is inserted the neck of a large tin funnel, and the joint made water-tight. A piece of sheet india-rubber is tied tightly over the outward larger end of the funnel. A small pipe leads into the funnel to conduct colouring matter to the interior. The apparatus thus prepared is filled with water, and the liquid colouring matter introduced into the funnel. If the funnel be now struck with the hand, a ring of coloured water will dart from the hole along

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[1] I am indebted to Mr. J. Cottrell of the Royal Institution for this description. Prof. O. Reynolds' paper, see *Proceedings Royal Institution*, vol. viii., part iii., No. 66, page 272.

the entire length of the trough, enlarging as it goes, with very beautiful effect.

i. The above experiments bear relation to the drop experiment discussed, but the cone of impression having less resistant base, the angle of the cone deviates less from the direction of its axis.

j. In air and water every ring takes the direction of the impulse which forms it. Thus for instance, a ring may be directed in the water either to the top, bottom, or sides of the tank.

k. The expansion of the ring becomes greater as the resistance of the cone of impression is greater, relatively to the velocity of impression, as seen in the expansion of drop rings; thus it is possible to project one ring by a slight impulse slowly in air or water so that it expands and comes nearly to rest at a short distance, and then to project another ring with greater velocity, therefore less divergence, through the first one.

68. PROPOSITION: *If a liquid of unit mass be projected with a certain velocity within an extensive liquid of the same kind near the aerial surface, a ring will be formed as in the last proposition which will be deflected to the surface by the minus resistance upon its top side. Such ring for the part that protrudes above the surface will be cut off by the aerial plane and have this part of its force dissipated in surface motions. But the parts that remain in the liquid will have their forces conserved and will develop two spirals of the projectile fluid upon the surface that will demonstrate by the motive directions the internal forces present in the perfect ring system.*

a. Not having heard of Prof. O. Reynolds' experiments at the time I was experimenting myself, I made an apparatus to project liquids which gave very inferior results to those given in the last proposition *h*; but as I had not handled an apparatus of the kind described and had the engraving made (Fig. 90) of my own apparatus before I had heard of it, I therefore offer a brief description for the sake of describing other experiments that I made with it.

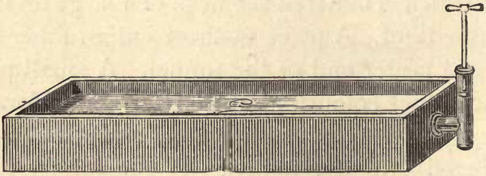

Fig. 90.—Trough for Surface Projectile Experiments.

b. Taking a trough made of inch deal, lined with zinc, such as is made cheaply by any packing-case maker for the export of goods, of the following dimensions, 9 feet in length, 18 inches in width, and 14 inches depth; into one of the ends of this trough at about the centre, I inserted a syringe which was made of a piece of $\frac{3}{4}$-inch brass pipe. The syringe was placed vertically and a pipe of the same size led horizontally into the trough; the syringe had a valve at the bottom by which it could be filled upon raising the plunger. The engraving, Fig. 90, will sufficiently illustrate the apparatus. During the filling of the syringe with coloured water a cork was temporarily placed in the hole from the inside of the trough; after the cork was removed, by pressing the plunger a ring could be projected for two or three feet. I made the apparatus at first to investigate the mode of motion for the projection of rings in water, which I hoped to be able to follow more visibly than in Prof. Tait's beautiful experiment just described, as I had previously found that experiments of the projection in gases, could generally be reproduced in liquids, and *vice versa.*

The best means that I could find experimentally to discover the principle of action of motive forces in projectile rings, was by devising means for making *sections* of such rings visible. This I effected in the following manner by the above-described apparatus. After stopping the hole in the plunging apparatus leading into the trough, with a cork, the trough was placed horizontally and partly filled with clear water to a height of about an inch to an inch and a half above the hole. The syringe described was then filled with coloured water. In the experiment, the cork being removed, and a sharp percussion made by the palm of the hand on the plunger of the syringe, a coloured ring was projected in the water. As this was projected in this experiment intentionally near the surface, the resistance to the projection of the ring being thus less on the upper side, the ring broke through the surface and its upper part was dissipated in surface motions, but the submerged portion continued by the continuity of resistances about it. At the line cut by the surface plane of the water, the internal motions of the projected liquid ring could be very fairly made out.

c. In the above experiment the ring, as it is projected, will show in its section evidence of whirling motion, in the same direction as that given in Fig. 88, page 198, as also in the axis of the ring where it is cut through in two opposite sections, at the aerial plane there are formed two small whirlpools, or deep depressions in the water. The surface is raised round these and contains the injected coloured water, which is apparently circulating spirally towards the centre of the whirlpools. These whirlpools remain persistent upon the surface of the water for several minutes, gradually diverging from each other by the now small effort of the conic resistance that they experience. The following diagram will show the direction of motion sufficiently.

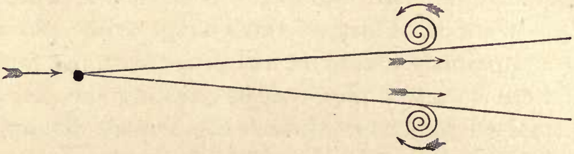

Fig. 91.—Diagram—Theoretical Continuity of Projection within a Cone of Infraction.

69. Definition of Parts of Motions Engendered by Conic Resistance.

As I shall constantly have to recur to the phenomena now discussed, in other experiments, it will be convenient to name the ring projected by any of the means described as a *whirl-ring*. I do this as it is seen to be by the last experiment directly connected with the phenomenon of a *whirlpool*. The small *whirlpools* cut in section described above, I will denominate *whirl-dimples*, to disconnect them from the well-known natural phenomenon of a whirlpool. Of the spiral incurvature of the projectile fluid I shall be able to give better demonstration in the next chapter in considering constant forces.

70. PROPOSITION: *That in the impression of a unit fluid force on a quiescent fluid mass, the cone of impression formed, will possess an axis of repose or of hydrostatic pressure simply; this axis will not be affected by motive forces in the projectile otherwise than as by a direct compression.*

a. This principle I have before discussed (57 prop. *f*), but I reintroduce it now for demonstration. In the cone of impression produced by resistance to a small unit of fluid projection, the axis as in all other cases, would be the most rigid part of the system. Therefore, all more oblique planes extending from the vertex of the cone would distend to areas of less resistance to the direct impulse of the force by which the cone of impression was formed

(58 prop.). In this case the cone of impression might not inappropriately be represented by a cone of tapering poles standing vertically upon a smooth plane. With these it is certain that any directly vertical pressure upon the vertex of the cone where all the poles are supposed to meet would only engender a compression in the central vertical pole, but that in other poles a pressure on the end would engender less compression as it would be impressed obliquely on the plane. Therefore, at a less angle to the resistance these oblique poles would be more unstable, and where the angle was much less than 90° they would have a tendency to slip outwards to an extended base. Nevertheless, a certain limited quantity of central well-supported poles would offer immediately a considerable resistance. Upon the above principles of action, in the manner proposed, it will be clear also that the axis of any conic system of resistance, fluid or other, might be replaced by an immovable solid or static body, without any change in the outward phenomena, except in the minus elasticity of a solid system.

b. It is important that the conditions given above should be made clear to our conception, inasmuch as the axis of motive resistance has been assumed heretofore to be the line of motive force in all *vortex* systems that I have read of in philosophical discussions. Upon the principles I have endeavoured to demonstrate, such a *motive centre* would be entirely fatal to the, simply mechanical, theory of unit fluid projection I offer. But this matter will be better cleared up by the experiment shown in the engraving above.

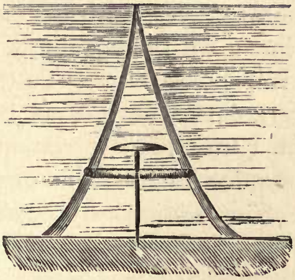

Fig 92.—Ex.—Whirl-rings projected over a Solid Object.

Take a piece of sheet lead of the size of a penny piece and cover this with white paper. Drive into the centre of the lead on the covered side a pin with a round or flat head. The paper is used only to reflect light, to be better able to observe the experiment.

Place the piece of lead with the vertical pin upwards in the centre of a pie dish, and fill the dish with water. Allow the water to become perfectly still. If we now let fall drops of coloured water directly over the centre of the pin, upon the surface of the water, these drops as they fall will glide down the cone of impression, past the edge of the pin without any deflection or deformation, as though the pin were not there, except that from the greater rigidity the pin offers, the cone will have a somewhat more extended base.

c. If in Prof. Tait's experiment (Fig. 89), we place a billiard cue directly in the axis of the cone of impression, the ammonia-chloride rings will be threaded and follow each other down the cue. In this experiment it is necessary to project the rings quite axially to the direction of the cue, as the interiors of the rings subtend a very small angle, about 1½ to 2 degrees for the entire length of cue, and less of course for the axis of compression, therefore any obliquity would interfere with the desired result.

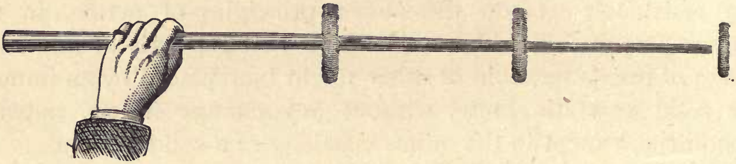

Fig. 93.—Ex.—Smoke-rings threaded on a Billiard Cue.

d. A similar experiment with the cue may also be made in Prof. O. Reynolds' experiment under water. To try this, I have used for the purpose a finely pointed fir rod, as being better than the billiard cue with which I tried the experiment at first.

e. The best demonstration I have found of the perfect quietude of the axis of the cone of impression may be shown by the following experiment.

Fix a dropping tube in connection with a vessel containing very soluble aniline dye. The dye I have used is that called tropæolum; it has a deep orange colour, but no doubt any other will answer as well. The drops may fall at the rate of about fifteen or twenty per minute or less. Place a white dish containing water about 1½ inches deep, so that the surface of the water is an inch or so below the dropping tube. Allow no drops to fall on the water until the surface is quite still. Then if the drops be observed as they fall they will be seen to pass uniformly down the same plane within the water. If this experiment be kept in action for a quarter of an hour, it may be observed, that throughout the whole time the cone of impression will remain quite clear transparent water, even until the

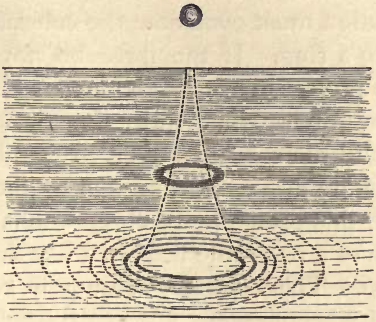

Fig. 94.—Ex.—Consecutive Drops of Aniline Solution in a shallow dish.

whole contents of the vessel is tinted by the soluble dye except this cone. The spot directly *under* the cone will also remain perfectly colourless, showing that there is *no circulation* in the axial part of the system even by solution. In this experiment drop will follow drop and each arrive as a ring at the extended base of the *conoid* of impression. Every drop will remain separated from the following one and be pushed out further by it, so that after a time ten or twelve beautiful thread-like rings will expand round the base of the cone and have a very pretty appearance (Fig. 94).

f. In the above experiment it is not inferred that one cone of impression remains throughout the experiment. This is of course re-formed by the projection of every drop, but the perfect freedom from motion that exists in the axis and immediately surrounding conic parts, re-forms the cone at every impression, out of the *same water as the cone was at first formed.* When the dropping ceases, the cone of impression slowly moves downwards, and falls to equilibrium with the mass, leaving a clearer spot than that in other parts of the water as before stated.

71. PROPOSITION: *The projection of a small unit or drop of liquid in a like liquid will not intrude itself into a plane of infraction unless the velocity of projection is sufficient to separate the cohesion of the projected unit or drop on the vertex of the cone of impression.*

a. This proposition is intended to show the limits of conic separation discussed in previous propositions in slow projections.

b. Every liquid must possess a certain force of cohesion, otherwise, a free drop would not be globular (11 prop., page 30), and if the resisting fluid resists by a cone of impression, as here assumed, the force of projection must overcome the cohesion of the projectile to separate it into a ring. If the force be not sufficient for this effect, and yet the projectile has by any means a force of continuity, as by the action of gravitation, then by the continuity of impression on the resisting fluid the cohesion of the projectile fluid may finally give way and separate into a whirl system by the force of the conic resistance. Upon this principle we observe that there will be conditions of cohesion of a projectile fluid in a like fluid, and a condition of conic separation. This cohesion holding in two cases: 1. When the projectile is highly cohesive. 2. When the velocity of impression does not exceed the force necessary to open out the projectile by conic impression upon it. In the last case the condition will only occur for very small forces, and will be only possible in the projection when the resistance is in very fluid matter; this case I wish now to follow.

c. The conditions of the above (case 2) may be observed in the action of a drop of writing ink placed upon the surface of still water. At first this ink will entirely float, therefore it will possess no velocity due to gravitation. After a short space of time the ink will accumulate near the edges of the expanded drop. These edges will then separate in parts, in very fine streams, in which the ink will descend by its superior gravitation. At first these small descending columns will be cylindrical, the velocity of projection due to gravitation being insufficient to overcome the cohesion of the ink in the descending column. But the constant efforts of conic resistance upon the head of this column will, under the acceleration due to gravitation, finally divide the small fluid force over the cone of impression which is formed in front of the projection at every instant, and an imperfect ring will be formed at the head of the column. This will afterwards break up into others, in like manner.

d. In a case like the foregoing it may be assumed that at the very small velocity previous to separation, that the centre of the column moves by rolling contact in a channel formed by like surface molecular movements within the radius of mobile elasticity of the system. That this is in a certain degree the case, I shall be better able to show in the investigations of pipes and channels further on. The proposition shows that a certain impulse is always necessary to project a fluid to form a cone of infraction, which is all I wish here to demonstrate.

72. Remarks.

a. I have thought that possibly there may be for every fluid in a free space a plane of easy conic fracture under the impression of a unit force. In this plane the cone of infraction will be most readily formed. The angle which such a cone will take may depend upon the cohesive force and general physical state of the fluid in which it is formed, and will only vary according to the velocity of impression of the force in proportion to the forces necessary to restrain the free mobility of the fluid.

Assuming that fluids break under strain, the cohesion of the fluid will extend the strain, so that the conoid of impression will take an angle proportional to the cohesive force of the fluid or other homogeneous matter, as previously discussed.

b. The angle of divergence for easy fracture for free air appears to be by rough measurement of the cone of impression in the projection in Professor Tait's experiment, Fig. 89, page 198, about 1½ degrees. This angle appears to be maintained in the cone of impression under the projection of vapour rings at somewhat varying velocities. As, for instance, that of a gentle projection by a small pressure, and that of a smart blow upon the canvas drum described. The smoke-rings of course constantly enlarge upon the cone of impression, and the exterior divergence may thus vary in any degree; so that this relates to the interior angle only.

C. The angle of divergence of the cone of impression in water by the projection of a unit in free space appears to be approximately 2½ degrees in Professor O. Reynold's experiments 67 prop. *g*. When I tried it myself in the small trough shown Fig. 90, it was about 2¼ degrees, but I found the influence of the sides in supporting the conoid of persistion very material, even when the sides were distant, so that in a tank 3 feet square the cone was about 2½ degrees. A unit of coloured water projected in a tank about 7 feet square and 4 feet deep, subtended as nearly as I could measure, a somewhat greater angle, but in this case there was head resistance to the projection in the nearness of the bottom of the trough.

c. In Venturi's experiments, which I will hereafter consider, the greatest efflux volume of a fluid is through a conical tube of 3 degrees. I take from these and other experiments 2° 30′ of arc as approximate, but there is considerable difficulty in measurements of this kind, and I have not followed the matter closely or experimented exactly, as I had determined from the commencement of this treatise to make my investigations qualitative only, not quantitative, as before stated. And I now only suggest that the angle of easy conic fracture appears to be nearly constant in some fluids at equal temperature and pressure.

d. I may note for anyone who may be interested in this matter that measurements of conic fracture should be made upon the cone of impression, as the surface of this cone is the only fairly invariable plane in the system. The contact of an involuting whirl ring carries with it in some cases, perhaps in all, part of the conoid of persistion into its system, and thereby enlarges its angle of divergence as will be hereafter shown.

e. For planes of very weak conic tension when the projectile force of a projected unit of fluid is nearly expended, so that the cumulative resistance deflects the flowing force outwards by the entire release of compression in the cone of impression, and the movement becomes so slow that every mode of accommodation may occur in the surrounding fluid; then the angle of divergence of the cone of impression becomes rapidly greater from the relatively superior head resistance to the small forces of projection, so that the ring is now projected no further, but involves in one position in space until its rotary forces are dissipated into the resistances. Certain conditions of the above will be considered hereafter for weak constant forces.

CHAPTER VI.

PRINCIPLES OF RESISTANCE TO THE PROJECTION OF FLUIDS WITHIN STATIC OR OTHERWISE RESISTING FLUIDS, CONSIDERED PARTICULARLY IN RELATION TO THE IMPRESSION OF FLOWING FORCES AGAINST HEAD RESISTANCES.

Conic areas of resistance to flowing fluids.

73. PROPOSITION: *That projectile fluids moving within like fluids will expand outwards in area of projection, until the area of resistance equals in force of elastic reaction the force-value of the area of direct impression.*

a. By *force-value* in the above proposition, I intend the same total amount of force irrespectively of the area over which it is distributed; as for instance two square feet of metal plate weighing five pounds to the foot, will have the same gravitation force-value as one square foot of plate weighing ten pounds, and these plates will balance each other if placed in a pair of scales, the force of gravitation being only distributed more, in the one case, than in the other.

b. An elastic system of fluid matter will resist movement by its inertia, and by its elasticity which acts as a repulsive force between every molecule of matter in the direct line of impression. It also resists movement in every line of convergence which can react by the cohesive forces of the system as previously considered 58 prop. *b.* Therefore forces impressed will constantly extend in area of impression, and come to equilibrium by elastic resistance at a plane where the elasticity, inertia, and cohesion of the resisting mass together equal the force of projection. In fluids we may for convenience assume these forces to extend in conic areas, as before proposed, so that the plane of resistance, becomes at a certain distance of extension of conic base, in equilibrium with the forces impressed at the vertex of a radial cone; the projectile force on arriving at this position of equilibrium will have no excess momentum for further projection.

c. In the cases considered 64, 65 props. in the last chapter, where the projectile fluid enters the plane of infraction covering a cone of impression and where the plane of infraction is assumed to be of equal *thickness throughout*, that is of equal resistance per area upon the surface of the cone; the resistance then increases and the velocity decreases in proportion to the extent of the *circumference* of projection towards the base of the cone and not proportionally to the *area* of its base as proposed above. From this cause we may conceive, particularly after taking some of the following propositions into account, how it is that *unit projections* of fluids are *motive* as projectile whirl-rings, 67 prop. *f.* Whereas *constant projections* of equal velocity are static upon a certain close area of conic resistance, the condition of which, I will now endeavour to show.

d. If a jet of steam blow into the air from a pipe or orifice, it will cut a clear outline in the air and will not apparently mix with the air throughout the conic area which circumscribes its projection. In the interior of this cone, the steam will be projected apparently with nearly equal force in all direct lines from the orifice which forms the vertex of the cone. To analyse the principles of the projection of this *steam-cone*, we may from our previous deductions conceive it to be projected in the following manner, when traced from the first instant of projection. We will suppose for this observation that a hole is suddenly opened by any means in a vessel containing steam, under a pressure greater than the pressure of the exterior air. At the first instant of the opening of the hole, the resistant air outside would be resting static against the sides of the vessel, therefore the steam on first exit would engender a cone of impression and open out a plane of infraction, as in Prof. Tait's experiment 67 prop. *f.* This we may show experimentally; for if we open the hole for an instant and close it again, a whirl-ring will be projected in the plane of infraction, the same as we may occasionally observe in the puffs of a locomotive. Now if the projectile force instead of ceasing, continues, so that this plane of infraction remains open as the *plane of least resistance*, and the flowing force continues to pass down this plane; then the resistance possible in the cone of impression which was shown to be the active part (67 prop. *d*, page 198) being only just sufficient in the first instant to withstand the impulse of projection, will in no way be able to withstand the continuous friction to the projection of the jet, to constantly deflect the direct momentum of the current in the same thin plane of infraction. Therefore the cone of impression will be, as it were, quickly washed away by friction, and be replaced entirely by the radially directed projectile fluid. Nevertheless, the exterior surrounding pressures upon the conoid of persistion, not acting directly as resistances to the impulse, will remain and circumscribe the cone of projection, except for such part of this conoid as may be absorbed by rolling contact, in friction by the projection; the action of which I shall be able hereafter to give some demonstrations.

e. Upon the principles discussed above, the radial exit of the steam will form a *cone of resistance* to the forward pressures which will impress the orifice of exit from the base of this cone so as to communicate the areal external pressure as a resistance upon the internal pressure in the vessel to equilibrium according to the conditions of the proposition. The mode of exit of fluid matter into similarly elastic fluid, but at a lower pressure, may be shown by the following diagram

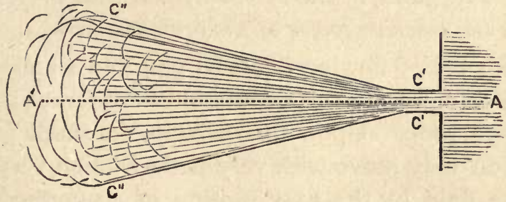

Fig. 95.—Ex.—Projection of a free jet of steam.

Let A A′ represent the axis of projection and C′ C′ C″ C″ a cone of impression somewhat enlarged by exterior friction. The interior of the cone is evidently projected forward by the constant efforts of the flowing force. Insipient cones are formed in the interior of this cone, as may be observed by the exterior rounding impressions upon the resisting air toward A′. The whole of this cone will be termed the *Cone of Resistance* in future, to distinguish it from the *Cone of Impression* defined 66 prop. *b*, page 195.

f. In the cone of impression previously discussed, the whole interior is active as a *hydrostatic pressure*, expansive in all directions. In the *cone of resistance* now proposed, the projectile force is active only radially about the axis of exit extending to its base.

g. Fig 95 represents the efflux of steam at a pressure of two atmospheres from an orifice of half an inch diameter in a thin plate. It will be observed that there is a neck shown at C′ C′ parallel to the axis, so that the cone proposed appears imperfect. (The angles of this neck should be shown eased off somewhat.) If we extend C″ C′, C″ C′ to the axis A A′ the vertex of the cone will be in or very near the plane of the orifice, so that the reaction at the base of the cone is carried to the plane of efflux by continuity of pressure. With cohesive fluids allowance should be made for the cohesion of the jet, but in this case of steam at high pressure, no allowance need be made for cohesion, as this is so nearly overcome by the elastic reaction; but if the pressure in the jet only slightly exceeded the external pressure of the surrounding atmosphere, the cohesive force of the issuing fluid would become a palpable function, and the neck at C′ C′ become longer. This I will consider in the next proposition.

74. PROPOSITION: *A continuous flowing fluid urged by any very small force will proceed slowly in a like fluid in direct lines, unless or until, the head resistance to its forward projection expands the forward part of the flowing fluid, by elastic reaction, so that the conic resistance may overcome the cohesive forces of the projectile fluid.*

a. The tendency of flowing fluids to preserve a uniform sectional area by cohesion against a certain amount of external resistance was discussed in 38 prop. Upon this principle a small jet of fluid of uniform section may move with very little friction very slowly in a like extensive fluid by the easy motion of accommodation of the extensive fluid making room, as it were, for the slow projection of the jet by engendering such close motions of rolling contact within the jet, as to be impalpable. The possible smallness of these motions will be hereafter discussed in considering liquid jets further on. The principles given in the above proposition have been offered for unit projections 71 prop., and the same will be here considered for flowing forces. Experimental evidence of this proposition may be adduced by causing a jet to issue directly into a medium of like density, with such small velocity that the cohesion of the fluid system of the jet will not be broken at first in its axis by the conic resistance of the fluid in front, or until the cumulative tension upon the cone of impression is sufficient to overcome the cohesive force with which the jet is united as a projectile system of fluid matter.

b. The previous proposition shows by the experiment illustrated, Fig. 95, that the conic resistance acts almost instantly upon jets projected at high velocity. At low velocities we may conceive that the reaction upon the cone of resistance may be insufficient to break up the cohesion of a jet, although it is evident that in slow projections the resistant fluid must constantly impress by reaction, its force upon the forward part of the central axis of the issuing fluid; and that this force of resistance will practically extend as a cone, as previously demonstrated, radially in the quiescent fluid from the point impressed. Therefore it will support the axis of a cone formed by the radial system of resistance in the surrounding fluid, whose axis will directly impinge upon the axis of the issuing stream. In this case the cohesive system of the stream being impressed with the greatest force in its axis, will be constantly expanded by the vertex of the conic pressure of the resisting matter. So that it must, if not very cohesive, finally give way in a very limited distance by the cumulative action of constant impression; and the outward conical form of resistance will finally become actively developed, exactly in the same manner as with greater forces of projection.

c. Experimental evidence of the above conditions may be, perhaps, better demonstrated by an experiment of the celebrated Thomas Young, with the importance of which he appears to be impressed as a mode of fluid motion. I give his original description, as this is the only observation I have met with of conic resistance to fluid projection in any case.

[1] "One circumstance was observed in these experiments which is extremely difficult to explain, and which yet leads to very important consequences. It may be made sufficiently perceptible to the eye by forcing a current of smoke very gently through a fine tube. When the velocity is as small as possible the stream proceeds for many inches without observable dilation. It then diverges at a

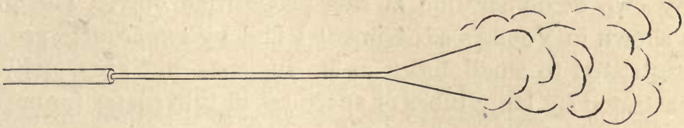

Fig. 96.—Young's Experiment of Slow Projection of Fluid.

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[1] *Light and Sound.* Dr. Thomas Young, Phil. Trans., Jan. 1800.

considerable angle into a cone as in the last figure, and at the point of divergence there is an audible and even visible vibration. The blow-pipe also affords a method of observing this phenomenon as far as can be judged from the motion of the flame; the current seems to make something like an evolution on the surface of the cone, but this motion is too rapid to be distinctly discerned. When the pressure is increased the vertex of the cone approaches nearer to the orifice of the tube, as in the figures below, but no degree of pressure appears to alter the divergency."

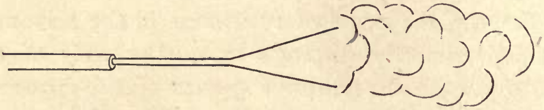

Fig. 97.—Young's Ex.—Moderate Projection.

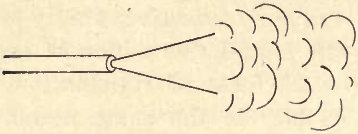

Fig. 98.—Young's Ex.—Rapid Projection.

There are some particulars given in the above which I cannot now follow, as, for instance, the phenomenon of vibration, which is important in relation to sound motions, as Young suggests—this will be hereafter considered in more demonstrable cases.[1] Young does not attempt to explain any part of the phenomenon, but simply offers some similitude to the action of spiders throwing their webs, the value of which I cannot discuss, since I take the experiment for my own demonstration of this proposition only. The phenomena shown in Young's experiment I find by experiment are in no way restricted to small tubes as he suggests, but are equally well demonstrated by large tubes or apertures in thin plates for any slow projections whatever.

d. The same *cumulative* action by continuity of reaction of conic resistance to separate the flow of a jet may be observed in some cases with unlike fluids, as for instance, in the projection of water in air through a thin plate; in this case, if the velocity of the jet of water be small, it will flow for a considerable distance in united mass, being held together by its cohesive force, which remains for a time

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[1] Now deferred for future publication.

superior to the separating force of the conic resistance that its projection encounters. But if it be projected with very great velocity the jet separates at once into spray, or *spirts*, as it is termed.

e. In a very cohesive liquid *impelled by its gravity* in falling through a liquid of nearly equal specific gravity, the velocity may be so arranged as only slightly to exceed the constancy of resistance to the lateral parts, so that the jet may remain entirely continuous by its cohesion. This may be shown experimentally. The cohesive liquid I have tried is mastic varnish. If this be placed in a tall glass jar (18 inches high in my experiment), and a little of the same varnish that has been exposed to evaporation in the air be poured into a glass funnel above the surface of the varnish in the jar, a clear bright column of the denser varnish will slowly descend to the bottom of the jar, without separation, and will continue to flow as long as it is supplied. This column may be observed very clearly in sunshine by the small difference of refraction of the two liquids, or if desirable, the denser liquid may be coloured with a little saffron to make it more perceptible. In this projection at small velocity the column will feel the conic resistance only when near the bottom of the vessel, but even at this point, the resistance will not be sufficient to overcome the strong cohesion of the varnish. The column will therefore generally coil itself up in a conical form that will resemble a coiled rope, upon the bottom of the jar; apparently avoiding in its convolutions the axis of direct resistance to its projection near the bottom of the jar only.

75. PROPOSITION: *That the resistance to division of a current by conic resistance at any time after the first instant is equal to the force required to separate the cohesion of a mass composed of a few molecules of the fluid only at the vertex of the cone of resistance.*

a. The force capable of opening a cone of infraction for a flowing fluid has been shown to be very small by the experiments given in the last proposition; the force necessary for the continuity of the conic resistance will be found to be immeasurably so, as the following principles will demonstrate:—

Let a force as that of compression into the resistance near A, Fig. 99, open a flowing fluid directed from B to A. Let this force, acting in a direct line, through deflection by conic resistances in nearing the plane A, separate the fluid in the planes C B and C′ B. Then assume that the flowing system will be at this instant of time as represented in the diagram below, where the conic force is shown active in separating the cohesion of the two planes C B and C′ B where crossed by the dotted line X. Now, by the continuity of projection in the direction A to B, these planes will separate at the next instant of time at the dotted line Y, and at the next instant at the line Z, so that at any instant of time, the separation of the cohesive force by conic resistance will be in one *infinitely close area only*, and the force that will separate one small unit mass of molecules will, if constant, continue the separation of equal continuous lineal parts indefinitely.

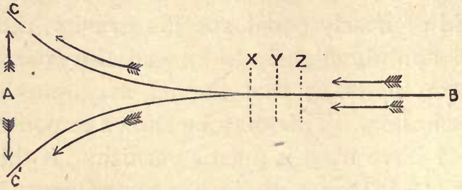

Fig. 99.—Diagram—Conic Resistance—Splitting Action.

b. In the continuity of projection of a jet at very small velocity in a less cohesive liquid than that taken in the last proposition, § *c*, the head, or conic resistance may be overcome so slowly that the principle of the separating action, here proposed, may be observed as in the following experiment.

Take a tall jar of water, and point a piece of glass rod as a style. Dip the style in a solution of carmine in dilute ammonia; place the point of the style just within the surface of the water, and hold it there by any fixing. The carmine liquid will now flow down the style into the water, and, being slightly heavier than the water, a jet will descend by gravitation. This jet at a certain descending point will separate by the conic resistance it experiences contra to its general cohesion, under similar conditions to Young's jets, last prop. *c*, but the central column will continue to flow downwards. The conic resistance will form a ring of the deflected liquid, which will have no projectile momentum beyond its gravitation force in its new position, as the direction of projection will be now deflected nearly to a horizontal plane; so that the ring or extreme edge of the deflected liquid will descend slowly

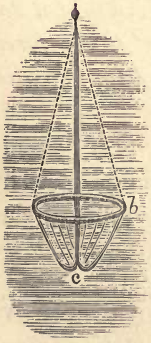

Fig. 100.—Ex.—Slow Projections in Liquid.

in the path indicated by the dotted line as shown in the margin, Fig. 100; whereas the central jet, having accumulated gravitation impulse, will continuously open lower and lower in the liquid as the small amount of conic resistance is consecutively overcome at the descending point *c*. By continuity of the same form of motion the ring will be left far behind, united to the central column as an inverted bell, as shown in the illustration, *b c*, and the coherent jet will constantly descend until it reaches the bottom of the vessel, if this be not too deep. Many other cases could be offered, but the proposition is possibly sufficiently demonstrated. The general principles of lateral resistance to jets for continuous forces will be considered further on.

c. This proposition becomes very important as we proceed, for we find conic areas maintained by forces that are immeasurably small. The action of the cone as herein demonstrated is to split open the current at the plane of cohesion as a wedge splits a solid body, the cohesion being consecutively overcome by the splitting action of the wedge. In this manner the parts of the flowing current move directly asunder at the assumed edge of the wedge. There is therefore, in this case, no other resistance as regards the direction of the current, than the separation of the cohesion of a small mass of molecules at the vertex of the cone only, for any extent or continuity of plane, and no slipping motion to cause friction in the system.

76. PROPOSITION: *If a flowing fluid of cylindrical mass be projected longitudinally into an extensive fluid wherein there is at a certain distance a solid directly in front of the projection, so that the flowing fluid suffer greater head resistance than in a free fluid, a cone of impression will be maintained by the solid resistance, and the conoid of persistion will be disintegrated by the flowing force.*

a. The principles of this proposition have been discussed for flowing streams, 40 prop. *b*, but at that time other conditions, since offered, could not be taken which particularly concern the exterior lateral resistances (cone of persistion). In 73 prop. *e* we have a cone of persistion maintained throughout the projection in direct conical outline. This case is assumed to be possible only where the flowing projectile force is great enough to project forward the cone of impression, to form a *cone of resistance*. In the case of the present proposition the cone of impression will be supported at its base by solid matter, and by this support the conoid of persistion defined page 177, will be more deflected by the conserved elasticity of the flowing force impinging against the cone of impression, so that the conoid of persistion will become of bell-mouthed outline, as it was approximately shown to be in the case of projection of constant units of fluid, Fig. 94, page 205. Further, by continuity of flowing forces, this conoid of persistion will be disintegrated by the impression of the flowing fluid upon it, and be in the same manner carried forward by the flowing force.

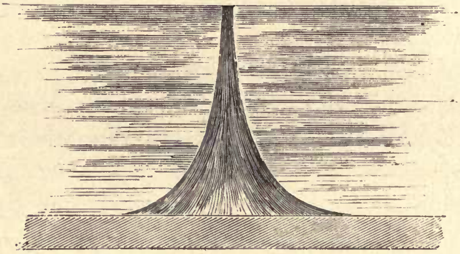

Fig. 101.—Ex.—Static Cone of Resistance to Flowing Force.

b. If we project a stream of coloured water from a small syringe upon a dish containing clear water of a few inches in depth, for an instant during the projection, a cone of coloured water will appear to stand upon the bottom surface of the dish. But upon our ceasing to project the water from the syringe the coloured cone will instantly flow down and disappear. On the spot where the cone of impression was formed in the dish the water will afterwards present a clear space, which may be even quite colourless, showing that the cone of impression formed by the first effort of continuous projection remained throughout the flow of the colour down its plane, as it was demonstrated to do by constant drops, Fig. 94, page 205. At the same time, by this continuous projection of the liquid in the plane of infraction, the flowing fluid will move upon the cone of impression as unit projections were shown to do in 65 prop.; and by the friction of the system the surface of the conoid of persistion will be found to have been rendered motive in the direction of the flowing force. The truth of this proposition will be better demonstrated by experiments in the next, where the continuity of the flowing lines will be better shown.

77. PROPOSITION: *That a constantly flowing projectile fluid in a conic system of resistance, moving tangentially to the plane of the conoid of persistion, will rotate this conoid upon such centres of inertia, as may be conceived to be contained in every radial segment of matter in this conoid, if the cone of impression is rigidly supported at its base.*

a. The object of the present proposition is to endeavour to discover the mode of continuity of the flowing force beyond the supported cone of impression shown in the last proposition § *b.* For it is quite clear that if the flowing force be continuous that its backward parts will press its forward parts, so that by this means, and by the conservation of directive energy in the whole system, the active forces will tend to induce a continuous motion in the projected fluid, which will now move, as in all other cases, in the least frictional course in composition with the forces of its direct momentum and resistance.

b. The continuity of the projectile force in a jet of flowing fluid upon leaving the base of the cone, or conoid of impression, may have insufficient momentum to carry the projected fluid beyond the base, as we find conditionally in the experiment of continuous drops mentioned in 70 prop. *e*, page 205, where the force was very small, and the projection was exhausted at the time the drops reached the base of the cone of impression, where the further projection of succeeding drops constantly, upon arrival at the base, pushed the previous ones outwards, so that their movements become perpendicular to the line of original projection; in this case we may assume that we follow the cone of impression to its *final limits* as an active area of deflection.

c. Now in the present proposition the forces in the injected fluid will be assumed to be continuous beyond any base that we may conceive for a cone of impression or resistance, and on this assumption there can be no doubt that the further continuity of the motive directions of the injected fluid will act upon the same principles as are involved in the first deflections under resistance by the cone of impression. This is demonstrated, in that, by the very nature of the directive forces and the conic resistance encountered, by which the flowing fluid is constantly deflected from the cone of impression by the surface of resistance where there is the greatest accumulation of elastic force (40 prop. *b*), that this deflection will be continuous, so that at a certain point of resistance, as at the bottom of the vessel, the flowing force will be at first turned at right angles to the direction of projection, and then by continuity of the projection it will continue to diverge further and further from the direction of original impulse, until the fluid so urged forward must, by the continuity of like action of resistance and conserved elastic force, take a circuit and meet upon the plane of original projection, that is, it will finally *revolve about the centre of inertia of every radial segment directed from the axis of the projectile system, such centres circumscribing and forming an annular axis to the motive system of the conoid of persistion.* In this manner, by the constancy of deflection of resistance in taking a constantly increasing angle, an annular cylindrical roll would be produced, the radial section of which would be a complete spiral or volute. I had some difficulty at first in demonstrating that such a mode of motion would occur from the entry of a flowing current in a like fluid, as in my first experiments I found that in the confusion caused of injecting a fluid, little could be made out, and one is easily deceived by complicated phenomena; however, in the following experiment I devised means to increase the resistance in equal proportion, *per area* of projection, laterally upon all parts of the system, so as to produce a slower motion, and thereby to permit this motive form of projection to be observable. At the same time I was able to reduce the cone of impression to a thin section so that all parts of the motive system could be clearly seen. This principle of fluid motion becomes very important in our future investigations; I therefore offer experimental evidence in detail, as without some care the realization of the desired effects are not easily assured; although sufficient effects may under many conditions be demonstrable, to ensure the certainty of the principles offered.

d. A narrow trough was constructed by placing two sheets of glass of about 15 by 12 inches in a wooden frame, so that the two sheets were about half an inch apart. This frame was supported vertically by two cross feet screwed upon it by wood screws; the frame being about 8 inches higher than the width of the glass. The sides were held firmly by a horizontal cross piece to support the apparatus under the pressure of the water it was arranged it should contain. The glass trough thus prepared was placed in front of a window so that the light passed freely through it, and in this position it was nearly filled with clear water.

e. Having the above apparatus ready, I took a small glass syringe, the same as used in a previous experiment, filled this with a strong solution of colouring matter, for which I found common writing ink answer perfectly well. Now placing the point of the syringe just under the surface of the water vertically in the centre of the trough so as to disturb the surface as little as possible, and then steadily and forcibly injecting the contents of the syringe, the following effects were observed.

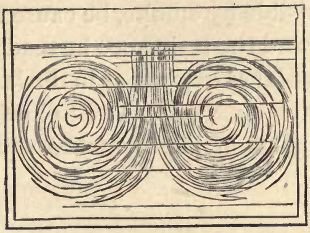

Fig. 102.—Ex.—Projection under solid head resistance.

As the colour descended it formed a conoid of extended base; but the projection continuing and being more than sufficient for this first effect, the stream lines continued to constantly diverge from their original course; they therefore curled over upon themselves until complete volutes were formed on either side outwards from the axis of the projection. As our vessel resisted by the surfaces of the glass the projectile force of the motion on two sides only; the injected liquid in order to escape this resistance was compelled to follow its course on the two free exterior sides of the projection. The volutes therefore contained the whole of the injected matter within a plane of the width of the trough, that is, half an inch in thickness, they consequently came out for observation very distinctly.

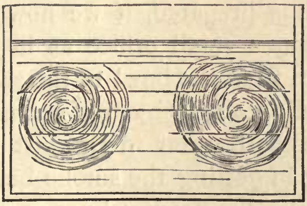

Fig. 103.—Ex.—Final Revolution of the conoid of persistion.

f. If the force of first injection of the coloured fluid was made as great as it could be conveniently by pressure of the hand on the syringe, the continuity of the projectile force entirely removed the coloured water from near the conoid of impression, and this appeared as a quite clear space at the end of the experiment. The curls or volutes continuing to revolve under favourable circumstances until the colour nearly reached the central space as shown in the diagram Fig. 103 last page.

g. The following experiment (Fig. 104) shows the same principles of motion as that given in the last experiment, but for air instead of water. If air, made visible by smoke, be caused to issue from a small tube at low velocity, and the resistance be made greater by causing it to move between two planes of glass as in the last experiment, similar forms of motion may be observed. The following shows the process. Take two pieces of glass of say 6 inches square, and place them together with an interval of about $\frac{1}{16}$ of an inch. This may be done by fixing a piece of stout card or a thin slip of wood down two opposite edges, or upon the four corners of one of the plates, and then putting the other plate over this, securing the contact of the whole by two india-rubber bands placed round the plates. Having the apparatus thus prepared, if we now slowly inject a thin stream of smoke from a small orifice as in Young's experiment (tobacco smoke blown from the bowl to the stem of a tobacco-pipe answers perfectly well) we may by this means produce nearly the same motive forms as with water in the last experiment. I have found great difficulty in getting the smoke between the aperture in the plate in such continuous flowing lines as was readily done with coloured water, in the previous experiment; but the detached parts of the whirls, their curvature, and the direction that they take, may be followed by the eye with quite sufficient precision to assure us that the active principles of the phenomena are identical, except

Fig. 104.—Ex.—Smoke-whirls.

that we have also present certain functions of vibratory motion as observed by Young (74 prop. *c*), which I will consider elsewhere in treating of the principles of sound motion. The illustration (Fig. 104) was made by Mr. Collings at my request from his own observation. It is not quite the theoretical form I should have given it in a diagram, but I have no doubt it is represented as he actually observed it, and as such it will sufficiently well support my theory.

h. The principles offered above suggest that the mode in which the impressions upon the air are made, that we recognize as sound, spread immediately in the surrounding air; so that at the back of a player upon a wind-instrument we hear nearly as clearly as in any forward or lateral direction. That this is the case we have also evidence in an experiment made by Dr. Tyndall, that the sound of a gun was proved to be equally penetrating, whether the gun was directed to the hearer or the reverse.

i. Although in the whole of the experiments given in this and the previous chapter I have endeavoured to make it clear that a projectile fluid *does* enter the plane of infraction, it may not be quite clear *why* the continuity of projection, as shown in this proposition, remains in this plane until complete volutes are involved in the conoid of persistion, as the least frictional mode of continuity of projection of the fluid. We must nevertheless assume, I think, that there is no doubt this is the case. It appears to me that the reason is clear that the continuity of projection in whirl forms is the least frictional mode of projection, in that the *projectile fluid is moulded to a forward very thin edge by its intrusion into the plane of infraction*, and that this thin edge is easily pressed forward by the backward parts of the fluid, opening out an infraction plane in front by splitting open the fluid upon principles given (75 prop.), which splitting, is deflected constantly from the greatest resistance, and thereby permits the intrusion of the projectile fluid in whirls.

j. The conditions of this proposition should not be confused with whirl-ring projections given, 66 prop., although the motive forms are similar; as in the case of a whirl-ring it is entirely projected within the plane of infraction, although it may carry with its projection a part of the conoid of persistion into its involuting system; whereas in the present proposition, for constant forces, the projection acts *tangentially* upon the conoid of persistion, so that this is *wholly absorbed* into the involuting system.

78. Definitions of parts of the whirl system of the above.

a. In further experiments I shall have reason constantly to return to the principle of the previous two experiments, in which the phenomena are much more demonstrable in liquids than in gases, and to shorten references, it will be now most convenient to have the parts of the outward forms of motions engendered, clearly defined.

b. I have already named the *ring* projected by a fluid in a like fluid a *whirl-ring*, and its plane section upon a surface, a *whirl-dimple*, from the certainty of the principles of these motions being like those of the familiar natural phenomenon, a *whirlpool.* I will further use these simple terms, and distinguish the entire principle of motion which produces the curls or volutes described, *the conic-whirl-system*, or *whirl-system* simply. I will denote each of the curls or volutes *whirls.* I will call the natural form of motion caused by head resistance upon a surface or plane just demonstrated (77 prop. *e*) a *biwhirl*, as the shortest graphic expression I can find, the prefix being common in scientific language to denote duplication of similar objects or parts. I will term the directive forces derived from the causes discussed, which produce *whirls*, *whirl-force;* and will use *the evolution of whirl-force*, as an inclusive term to denote conic resistance, infraction, deflection in conserved elasticity, rolling contact, and other principles of motion entailed in the production of whirl forms in fluids known or unknown.

Biwhirl systems.

79. PROPOSITION: *If a flowing fluid of cylindrical mass be projected longitudinally in a plane of like fluid of infinite extent, a biwhirl system will be engendered to take the place of the conoid of persistion of previous propositions. The whirls of such biwhirl, once formed, will constantly enlarge by the deflection of the flowing fluid, tangentially into their rotary systems, and the flowing fluid will continue to flow by tangential contact upon the whirls, with little friction.*

a. Some conditions of this proposition were given 51 prop. page 156, for lateral resistances which may act upon a flowing force. In a similar manner if we project a stream of water into an extensive basin at moderate velocity, we may soon observe, by the presence of floating particles, that a *biwhirl* is formed upon the surface of the stream lateral to the point of projection. As the stream continues, the lateral whirls will constantly enlarge, and the current will project further and further into the basin, showing that the whirls reduce, by their enlargement, the resistance to direct projection. If the current has great force the centres of the whirls will form deep depressions by their centrifugal force, and the centre of the stream will be elevated, so that the current will take a restricted sectional area of projection to reduce its lateral surface of contact as much as the accommodation of the system will permit. This I have observed in many cases. In water flowing through the locks at Boulogne, in emptying the backwater of the river Liane, the central current rises about 30 inches above the lateral whirls. The following diagram will illustrate the principle.

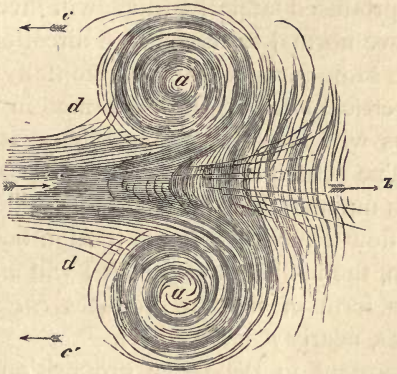

Fig. 105.—Ex.—Surface biwhirl.

b. Let the above diagram, Fig. 105, represent a part of a surface of water in an extensive basin, into which there flows a current in the direction shown by the arrows, Y to Z. In this case lateral whirls will form at *a* and *a'*, and if the current be rapid, it will remove the side waters *d d,* and send forward wave-like deflections in front which will pass into other whirls if sufficiently free from further resistance. In this case also deflected currents will be thrown off at *c* and *c'*. By continuity of projection the entire system will become one of biwhirl rotation, and the water will flow constantly through the central area moving tangentially to the whirls. The centres of the whirls *a a'* will be natural whirlpools by the tractional effects of the surrounding parts moving tangentially to them.

80. PROPOSITION: *That whirl systems are developed normal to the planes of resistance in a flowing stream, and may vary in form from a circle to a very long ellipsoid. Natural whirl forms from local resistances being most generally ellipsoidal, although the least frictional forms are circular.*

a. If we let a jet of water fall vertically into a deep vessel of water, the projection after entering for a certain depth will be deflected on all sides equally, and return towards the point of entry, as will be clearly indicated by the inward direction of the surface water towards the jet. This, from the equal lateral freedom, will produce for every radial section a circular system.

b. If we project a band of great width, as for instance that of the width of vessel into which it is projected downwards, two cylindrical whirls will be produced laterally, which will involve in a similar manner to the above normal to the surface direction of the current. If we project the same form of band horizontally below the surface of the water, a vertical *biwhirl* will be formed in which the upper and lower portions will flow in the opposite direction to the central projection. The principle of this form of projection is important, as it is in this manner that undercurrents and midcurrents are projected, which flow without excessive friction both in the atmosphere and the ocean, the instances of which I will endeavour hereafter to discuss. This form of motion produces *ellipsoidal*, or if in very extensive areas, nearly *planic systems.*

c. It is important in the above proposition to observe that the whirl system developed by a planic current will at all times tend to divide the system of the flowing fluid, so that continuous streams will decrease in volume by the amount deflected from them by the development of lateral whirls of rolling contact. It is also clear that such whirl systems would after deflection form for themselves detached systems which may never again coalesce, to form a single system—as in circular systems.

Unstable equilibrium of a biwhirl system of projectile fluid.

81. PROPOSITION: *If a biwhirl system of fluid motion, as shown in the last proposition, suffer any unequal resistance on the one side or the other; the tangential action on the most free whirl of the two will enlarge this whirl to greater area by absorption of the flowing force as it will also diminish the more resisted whirl. But if the flowing*

force would suffer greater resistance by this change, the more resisted whirl will be enlarged. Further if a motion be induced which tends to enlarge either one whirl or the other of the system, this will continue persistent, although it may be the more frictional of the two to the flowing force.

a. If we take the apparatus described 77 prop. *d*, and perforate a hole midway in each of the sides of about ⅜ of an inch internal diameter as shown in the engraving, and insert a pipe in one of these holes A leading to a reservoir placed at about a foot above its position, a current may be projected from this hole. If we add a short length of pipe to the other hole of equal diameter as shown at B; then the quantity of water the one hole supplies the other hole will let off so that we may have a current passing through the vessel from the one hole to the other.

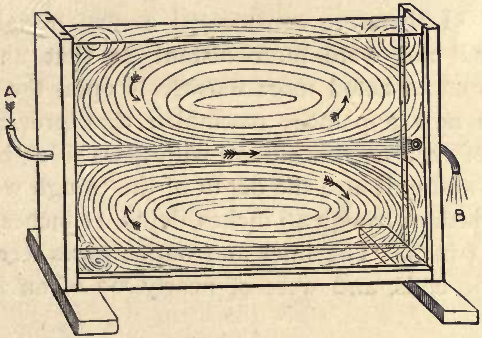

Fig. 106.—Central biwhirl projection.

b. Let the apparatus illustrated in Fig. 106 be filled with clear water and the reservoir that supplies the pipe A contain particles of matter of about equal specific gravity with the water. Then by projection of a current from A its passage through the apparatus may be clearly observed, which will be as follows:—

Upon first projection of the current, allowing a few seconds, in which motions of conic resistance will be developed, a biwhirl will be established, the whirls of which will proceed upwards and downwards, and the axis will at first take a strait direction to the opposite hole, but after a time it will be evident that this system is in unstable equilibrium, and the direction of the central current will be deflected slightly upwards or downwards. With the small apparatus described 77 prop. *d*, the directive force of the entering current will approximately maintain the axial line, and the direction of the whirls will be nearly as represented in the preceding engraving, Fig. 106.

c. To trace the equilibrium of the whirl system induced more delicately, as it was clear to me that the fluid was projected in the small apparatus I first used, which was 15 inches only in width, which I conceived was too near the opposite point of efflux for sensitive equilibrium, or to show visible signs of unstability, the conditions of which I desired to investigate by differences of lateral resistances. I therefore constructed a new trough in which the entrance and exit were more distant, so that any inequality of lateral resistances acted more sensitively upon the axis of projection. This new apparatus resembled that previously described, being, as before, a narrow trough with glass sides, but the distance of entrance and exit also the depth of the trough were increased as follows:—width of trough 46 inches, depth 21 inches, width between glass sides 1 inch. The inlet and outlet pipes were placed in the centre of the ends, and were of nearly ·62 of an inch in internal diameter.

In this apparatus, from the distance of the entrance and exit, the horizontal current was made to flow through the central area with greater difficulty, the projected stream being evidently very sensitive to small differences of lateral resistances. At the first entrance of the stream, however, the biwhirl formed, flowed upwards and downwards in fair equilibrium as before, and appeared as in the engraving Fig. 106; but as the water continued to flow the lower whirls were constantly enlarged, and the flowing current became arched upwards over it by the increase of the lower whirl to nearly a circular area. The tangential force in this case now acting more directly upon the larger whirl as a free system, and the liquid flowing upon the whole with less frictional resistance. This form of motion is shown in the above engraving, Fig. 107.

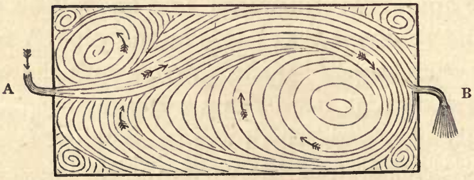

Fig. 107.—Ex.—Enlargement of Friction Whirls downwards.

d. Now obstructing the flowing fluid so as to direct the current to flow for a short space of time upon the lower surface of the vessel, the largest whirl was formed upwards as in the engraving below, Fig. 108, as it had not sufficient force to change already induced motions by passing through the phase of a more frictional system.

Fig. 108.—Ex.—Enlargement of Friction Whirls upwards.

e. The flowing force for the larger whirl of the biwhirl taking its circuit downwards, in this last case, although this was a *more frictional* mode of motion than the first, is very important in some operations of nature, as it shows a certain persistence of motion in some cases, as in ocean currents, when the causes by which they were produced have possibly entirely disappeared. To this I may again revert.

82. PROPOSITION: *If the area of a flowing stream be large and be projected in a fluid of like density of extensive free area, biwhirl systems will be formed; the extent of the whirls of which, if free from unequal lateral resistance, will be finally, nearly equal to that of the greatest circumference of a circle that can be described in the free area. The deflected flowing fluid moving tangentially past such areas of rotation will suffer less resistance inversely proportional to the radius of the greatest circle of motion nearly.*

a. Certain conditions of the above have been taken under principles of rolling contact 50 prop. *e,* but at the time I had not proposed any function for head resistance, so that the matter could not be cleared up, except as regards lateral resistance. This proposition shows the deflection of the direct flowing systems, formed first by conic head resistance (*biwhirl system*).

b. The above proposition will be subject to the conditions of the last at § *c,* namely, that there are not present already induced motions, so that the quiescent area in which a current is projected, is assumed to be equally free on all sides. If there be more or less resistance upon one part of the biwhirl system, this part may be deflected from its direct course or circular projection, but under this deflection there will be a kind of elastic resistance inducing a circular motion, which will cause the whirls to complete their forms over other portions of the free area wherever possible. This proposition is important, in that, it offers a theory that accounts for currents in bays and quiescent spaces in the ocean and rivers, that are often entirely due to the tangential movement of some distant deflected flowing force.

c. This proposition may be, for exact conditions, confined to the projection of large currents in free areas, as I have found that in restricted areas the immediate near pressures upon the whirl, that in this case take the place of the conoid of persistion, leave the rotary system small momentum and great tangential resistance, so that the system is easily deflected and deformed, although there will be present a certain amount of resistance to the deflection by tangential forces, which may be made evident in many experiments; as, for instance, in Professor Tait's projectile whirl-rings (67 prop. *e*), that appear from this cause to possess a certain amount of elasticity if they meet an object, or another ring, by which they suffer surface deflection of the rotary system. However, in any case, if the tangential force be too much strained by the deflection, other whirl centres will be formed.

d. To show experimentally the disposition of fluids to move in circular areas, the same apparatus may be used as described 81 prop. *b.* This may be placed as before in any position to be visibly transparent, and a pipe attached for supply of water, the motions of which it is desired to make apparent by the presence of particles of sawdust or other matter in suspension. Take a piece of wood of the length of the trough, and of the same thickness as the distance between the plates of glass, and cut one edge of this out to any irregular form, so as to remove the resistance of its surface a greater or less distance in certain parts from the axial line, assumed to extend directly from inlet to outlet pipe, to throw the axis out of equilibrium. Take for instance the form shown in the shaded lower portions of the illustrations on the next page, we may then observe the following effects.

e. In the apparatus represented by Fig. 109, if we project a current from the pipe A, we shall find that the resistance of the lower plane will deflect the biwhirl formed, upwards, so that the greatest circular areas of revolution will occupy positions near the ends of the trough, that is, in positions where the greatest circle may be described. If we now take out the shaped piece of wood, turn it end for end, and replace it in the bottom of the trough as shown (Fig. 110). It will now be impossible by the position of inflow that the whirls of the biwhirl shall occupy the greatest areas of the vessel. In this case a biwhirl will be formed at the entrance, and a second further on in the most free area, so that, in this case, the three whirls united will take the greatest possible circular areas in the containing vessel. In either case the current to reach the orifice B in the one instance, or B′ in the other, will pass round a large portion of the diameter of the whirls as the least frictional course. These experiments may be varied with very curious results, but as they are only modifications of the same principles of motion, it is unnecessary to waste time in giving descriptions of these variations, except for a few important cases.

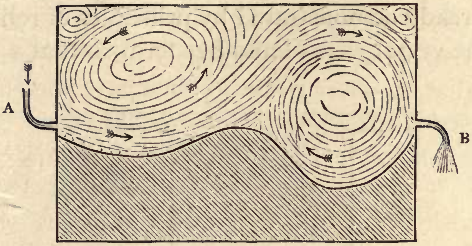

Fig. 109.—Ex.—Unequal Resistance Plane.

Fig. 110.—Ex.—Same as 109 reversed.

f. By the conditions of this proposition a biwhirl cannot be fully developed if the area be such as to permit one circle only to fill the resistant space, unless the current crosses the central area, as any division of the circular space would restrain the force of the greatest unit of revolution, according to the conditions offered. In this case the current would if possible pass tangentially to the unit system. This has been already demonstrated by principles of rolling contact, 50 prop. *d.* If the vessel were of another form so that a circle could be described meeting approximately equal resistances in opposite parts, as that for instance shown in the engraving, Fig. 111, of a square, then the rotation would be formed in the greatest circle that could be inscribed in the space, but in this case there would be also lesser spaces at the corners inactive in the rotational system, therefore we find that to complete the square, smaller rotational systems are set up in these corners. These small systems show experimentally by their direction of rotation that they are *biwhirls* thrown off by the free parts of the tangential system. They can be seen very well by the apparatus shown, Fig. 111 above, which is made six inches square, formed of pieces of wood a quarter of an inch in thickness cut out to form the shaped cavity, and then fixed between two pieces of glass. Water containing visible particles is projected along one side of the square A to B.

Fig. 111.—Ex.—Circular Rotational Systems.

g. In the above experiment the diameter of the inscribed whirl is shown to be nearly that of the greatest circle that can be inscribed in the space between the limiting walls exposed to the action of the flowing stream. By the action of the constant tangential force upon this inscribed rotary mass, when the circular area of projection is complete, other smaller whirls, as those in the corners, will be formed with less friction than the larger whirl would suffer, if it were possible for it to be deflected to the contour of the vessel, to pass the angles by gliding contact.

h. This principle becomes important in considering the mode in which a fluid necessarily has to turn the angle of a pipe. It would be quite clear from principles discussed that if it had to turn the angle by stream lines continuously parallel to the surface of the pipe, as generally assumed, the friction of so rigid a system as water for instance, would be immense; but by means of a whirl in the inner angle, rolling contact is insured, and the greater part of the friction entirely avoided. This may be demonstrated by the same experimental method as previously given, of cutting the angles to form the pipe out in flat sections in two pieces of thin wood and cementing these between two plates of glass to the form shown Fig. 112. The channel thus left for the water may be of square or oblong section. The water being made visible by particles is found to form a whirl in the inner angle, which keeps in constant rapid rotation which deflects the current into its course to follow the outline of the pipe. A train of smaller whirls form on each side of the exterior angle to the projectile current within the pipe. These whirls are all under restraint, and act as friction-savers, in proportion to their possible motive freedom under the circumstances, subject to the already induced motions of the water in the pipe, the conditions of which I have yet to consider.

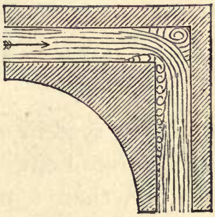

Fig. 112.—Ex.—Whirls at Angles of Pipes.

Elasticity of Whirl Systems.

83. PROPOSITION: *In a fluid system moving under the restraint of lateral resistances, rotary forces will be persistent, and they will avoid these resistances, but the rotary systems may suffer such deformations as may not be more frictional than division into separate whirl systems in frictional contact with each other.*

a. The most certain law of fluidity is that the fluid will move in the least frictional course to continue its motion. This, I hope, I have clearly shown will be necessarily so by means of rolling contact; but the system of rolling contact will be modified to the conditions present, and may not always be an evident form of motion.

b. If a fluid commence to flow over another fluid at rest, it will do so by division of the current into separate rotational systems, as shown 46 prop. *g.* If such rolling motions of contact continue, they will induce a general surface motion, and this being induced in a part of a liquid mass, this will react upon the inertia of the general molecular system and induce equivalent motions in distant parts.

Therefore, if the supporting whirls of an overflowing system cannot complete their forms without division, for which there may be present no weaker plane of resistance than that to direct projection,

Fig. 113.—Ex.—Whirl Flattened by Restraint.

the rotary system will then continue under restraint by the tangential forces being deflected by the resistances to a greater or less curvature, than a purely rotational system, proportionally to the resistance encountered.

c. Thus in the diagram above, which represents the motive forms after the motions are induced in a vessel similar to Fig. 53, page 141. In this case there would be space for two circles under the conditions of the last proposition, but it is quite clear that the re-entrant curves of a pair of whirls in such a system, coming in opposition at their meeting plane, would be more frictional than the restraint of a more direct, although deflected system, as that shown in Fig. 113 by the direction of the arrows.

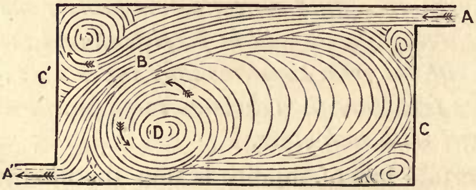

Fig. 114.—Ex.—Whirl Deflected by Restraint.

d. Further, a fluid system may carry a direct impulse that will overcome a certain amount of lateral resistance, and as whirls are formed by resistance only, they will not be visibly formed until such resistance is experienced. Thus in the section above, constructed upon the same principles as those given in the last proposition, the fluid is projected with force at A, and whirls are formed by the resistance towards A′. In this case the lower whirl D has its axis deflected much in advance of the centre of the system. In free areas such deflections are not seen, so that these instances are purely systems under restraint.

Motions in Solid Conic Areas.

84. PROPOSITION: *If the conoid of persistion formed by the inertia of surrounding fluid be supported by near solids, or that it be formed of solid matter, such conoid will offer less resistance to a flowing force proceeding from the vertex of the conoid than any otherwise disposed system of resistant matter.*

a. In 32 prop. *n*, page 107, it was mentioned that Eytelwein, by refinements of an experiment of Venturi, constructed a cone which offered friction to the efflux of water, only by a small fraction of the whole, or ·02, greater than a perfectly frictionless outlet according to the theory I offered in the same proposition. I have shown that in such conic forms of outlet the elastic forces of the issuing fluid are released, but as elastic reactions, they impinge upon the surface of the solid cone, and the motion becomes very frictional, unless we can otherwise imagine some system of rolling contact induced; but assuming such rotation once induced, the conic form of aperture would then present conditions somewhat similar to projection down an inclined plane to the freedom of the whirl system of contact induced upon the conic surface. This whirl direction would tend to draw forward the current and increase the efflux over that of a simple or cylindrical aperture. The restrained elastic forces giving impulse to the rotational system.

b. In 77 prop. we considered the motive effects that would be brought about by supporting the cone of impression; in which we found that the flowing force was thereby deflected into the conoid of persistion, to which it imparted its momentum tangentially, until this conoid became a rotary system of motion, the effect of which was a saving of much of the friction upon the issue of the jet. In the present proposition, the conoid of persistion is assumed to be supported, and in this case, the deflected fluid is restrained in the completion of free extensive whirls, so that the friction whirls developed, are of smaller size and to a certain degree insipient, so that they rotate in many small systems upon the conoid of persistion.

c. By making the conoid of persistion a rigid system to a flowing fluid, we produce for this, the same conditions as were observed for percussionary unit forces, 64, 65 props., for a free projection, wherein a whirl-ring was formed. For in this case the conoid of persistion was demonstrated to be fractured in the fluid system from the area of the cone of impression, so that it remained the most rigid part of the system. Therefore by equivalence, as argued, 73 prop. *c*, the projectile rotary system of the whirl-ring of a unit projection upon the conoid of persistion should be also equivalently represented for flowing forces, when such rigidity of exterior form is given to the fluid system, as may be represented by a solid conoid of persistion. The difference in this case will, however, be that as the flowing force beyond such a conoid of persistion is constant, the equivalent of the whirl-ring should, therefore, be formed at every part of the surface of the conoid of persistion upon which the flowing force makes contact. From the above we may conclude, that as such projectile rotary systems as are shown to be produced in unit projection in whirl-rings, by direct impulse and resistance upon the conoid of persistion, are rendered instantly rigid by the percussionary action of the first impulse; that we may also imagine that by making the conoid of persistion equally rigid throughout by any other means, and giving a greater impulse, as by that of a constant flowing fluid, that the same motions of rotation will be induced everywhere upon the surface of this rigid conoid as in a projectile whirl; that these rotary motions will also be so disposed, by their deflected directions, as not only to save friction against the solid conoid, but also by the principles discussed, in 39 and 67 props., of conservation of energy in the deflected system, to use the flowing force so diverted to pull forward the current itself—exactly as we find the projected whirl-ring overcomes the resistance in front by the same principles of conservation of energy under like deflections.

d. For the conditions which represent a rigid conoid of persistion to a flowing fluid, we may again refer to one of the celebrated experiments of Venturi.[1] In this experiment he constructed a conical, or rather trumpet-shaped, outlet to a pipe which was supplied by a vessel of water 33 inches above, and found that this conic outlet considerably increased the outflow of water above the quantity obtained from a simple pipe, without this addition; the comparative efflux being in the ratio of 5 to 3. That the conic outlet was the cause of the acceleration he clearly proved by the fact that if the cone were *added* to the length of the pipe the efflux of the water would be considerably augmented. He also showed that in making

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[1] Venturi Recherches expérimentales sur le principe de la communication latérale du mouvements dans les fluides. Translated in *Nicholson's Journal*, vol. ii. 1798, p. 176.

the whole pipe conical direct from the vessel of water, giving to the cone the proportions of nine times the length of the diameter at the aperture from the vessel, and making the cone in the ratio of 18 at the outward end to 10 at the aperture of the vessel, the discharge of water was more than double that of a simple pipe of the same internal diameter, that is, the water from the conic pipe flowed out in the proportion of 24 to 10 from the cylindrical pipe.

e. In this experiment the angles of entry to the pipe were eased off so as to produce an easy flow, into the form given by the internal part of the *vena contracta.*

f. Venturi rightly conceived that the air resistance was the principal cause of the acceleration, but in what manner this acted, he was not able to make out satisfactorily to science. His demonstration that it was *so,* is important, which was that if water flow out of a vessel through a hole in a thin plate under an exhausted cylinder of an air-pump, that it flows at the same rate as in the open air. But that if a conic pipe be added to the vessel in air, the outflow is increased.

g. In Venturi's experiment, important as it was as a matter of fact, he had in no way removed the difficulties that were felt by earnest philosophers at the time, in showing how the pressure of the air might be a cause of acceleration of the outflow from a conical tube, as the area of aerial resistance was clearly greater, it appeared rather that, from this cause, the outflow should have been *less.* But the greatest difficulties that were felt, were that Venturi's experiment demonstrated a fact that appeared to be contrary to all principles of frictional resistance of fluids by solids, which were conceived to be gliding motions, in that in his experiment, by *increasing* the area of surface of resistance to the flowing stream, which was evidently done by *adding* a conical pipe to the outflow, the total resistance, as measured by the increased outflow of water from the pipe, was *decreased.* Whereas the resistance ought, by principles of plus surface adhesion and friction, from the larger surface exposed to gliding motions of the liquid moving in the stream lines, which Venturi otherwise fully accepted, as his discussion of lateral communication of motion show, to have caused the resistance to be materially increased, and the outflow to be thereby proportionally less.

h. To return to our proposition. It will, I presume, be easily understood from previous demonstrations of principles of whirl motions, that with a conical pipe, that would represent, the form of a natural plane of infraction of a liquid, or more correctly that of the conoid of persistion, that if the divergence of this conoid were such that the whirl systems induced and impelled by released elasticity of a previously cramped system, moving upon the plane conoid of persistion, made an exact half-revolution on themselves upon this rigid plane, that perfect rolling contact would be assured, and the momentum of the flowing force and released elasticity would be conserved in the rotational system. That this is so, I will now endeavour to prove by experiment.

i. For the investigation of the motive principles of Venturi's cone I followed the same construction of shaped pieces of wood cemented between plates of glass, as previously described, which may be considered, as before, to isolate a longitudinal diametrical section of a current under greater although equally distributed lateral resistances. For this I constructed a flat tube that at the entrance was a quarter of an inch square, the two opposite sides of which were made, by the form of the slips of wood used, to give the internal cavity a convergence outwards so as to produce a channel of a straight trumpet shape for the one section only; the other section being, by the equality of thickness of the slips of wood used, of uniform depth throughout. In this first experiment the internal motion of the water, as seen by the floating particles present in it, was through the velocity of outflow too confused, when the force of the water was made sufficient to fill the cone. The efflux volume, however, was slightly in excess of a uniform square parallel pipe, in the ratio of about 9 to 8.

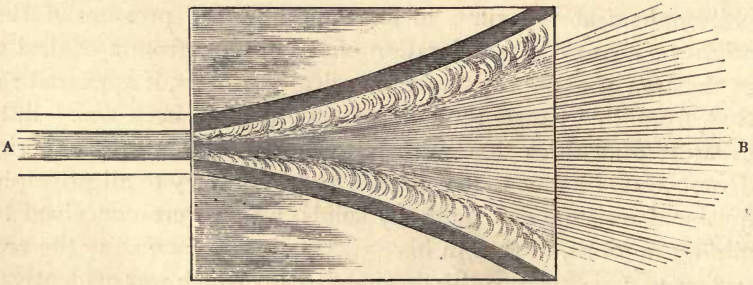

Fig. 115.—Ex.—Rotational Systems bordering Conic Areas.

i. To reduce the rapidity of motion in the above experiment I altered the construction of the cone so as to still further increase the forward resistance, and to render the motion slower, following the same principles as before discussed. For this object I reduced the space of outlet at the outward edge of the conoid towards B by making it one-eighth of an inch only in depth, retaining the quarter of inch square inlet at A, as before — the plan of the apparatus being as that shown, Fig. 115.

k. With this apparatus, having first stirred some coffee-grounds in a supply-cistern, and permitted the mixture to flow through the cone, the motions of the particles as they passed out in the current could be clearly followed, the general directions of which were as follows:—The central stream flowed outwards to constantly increasing area, following fairly a general direction intermediate between that of the axis and the outline of the sides of the cone; whereas the nearer side water was more and more deflected, until at near the borders of the current, perfect whirl-systems were developed, and the particles of coffee were seen, as shown in the engraving, whirling with great velocity, in the direction calculated to pull forward the central current.

l. The above experimental contracted cone, from the resistance of the adhesion of the water to the flat sides of the glass, must not be observed for production of an absolute acceleration of the entire stream, but for increasing local resistance to render the actual motion which occurs in a cone visible at the most free surfaces, and to show the mode in which acceleration is attained when the motive forces are more free from restraint, as in a complete cone of circular section.

m. I have before observed (72 Remarks, *c*, p. 200) that theoretically the cone of Venturi should exactly follow, or be only in slight excess of divergence from, the outline of the plane of infraction in a free fluid of equal density to the fluid projected. The angle of divergence should also bear some proportion to the velocity of the current, and possibly to the plane of easy fracture of the fluid. I have not had time to follow this matter as I at first intended, but I make out from experiments of Venturi and Eytelwein that this is at least nearly the case. In the exact form of exit given by the plane of infraction, which is largely dependent upon elastic reaction expansions, the loss of volume at exit, from a properly formed cone, should not much exceed the theoretical discharge of free projection at half the pressure due to the height of column, in the supply reservoir, as given in 32 prop. *u*, page 106.

n. The direction of the efflux from a cone being thrown by the whirl force *transversely* to the line of motion, the velocity of efflux is by this cause, as well as the reaction of elastic compression, in inverse ratio to its additional volume of efflux. This is also the case with a short pipe added to a simple aperture. This fact is important to be observed, since we do not obtain any *new force* by the addition of a cone or pipe, greater than from a hole in a thin plate; this would be impossible, but only greater volume at less velocity. The momentum of the central flowing force in this case loses the conserved elastic energy shown 32 prop. *c*, the elastic forces being directed into whirl motions that overcome the near resistances simply. Thus, if we project a jet from a thin flat plate vertically, it rises in the air to nearly the height of the reservoir by which it is supplied; but if from a cone, it simply overflows at less than half this height, as before discussed.

o. A supported conoid of persistion is equally effective in permitting a larger outflow of air to that shown for water. We should conclude this from other experiments wherein motive equivalence of air and water, or other liquids and gases, are found to hold. It is unnecessary to give the experiments I have made for my own satisfaction, to show that this is the case in this as in all other instances. The principle of making the outflow aperture of the form of a conoid of persistion is largely made use of and understood practically, although not theoretically, for the forms of horns and speaking-trumpets, wherein I find a greater angle of aperture is generally taken than I should imagine necessary for simple facility of outflow of the breath. This matter, however, is complicated in cases of air motions in cones where sounds are produced with other principles which cause the vibrations, as observed by Young in free jets, that I may at a future time discuss for the conditions of sound motions. Further, the point is not very material, for practice has possibly discovered truer angles for horns and speaking-trumpets than my theory could suggest.

85. Proposition: *If the conoids of impression and of persistion formed by the projection of a flowing fluid force be both supported by near solid matter, so as to remain static or nearly so, the flowing force will then escape in the plane of infraction only, and be deflected to make rolling contact upon the supported conoid of persistion at an angle greater than 90 degrees to the direction of original projection.*

a. This proposition indicates that a flowing force will continue in a plane of infraction when this is once opened, if this plane be supported by a certain rigidity in both the conoid of persistion and the conoid of impression; and that the flowing force will be deflected towards the rigid conoid of persistion. So far, this would be evident from the previous demonstrations, as it must be so deflected to make rolling contact upon the conoid of persistion; the projection being also accelerated by elastic compression upon the conoid of impression (76 prop.). We have now therefore only to consider the amount and nature of the deflection of the current where it is set free at exit. This is important for some cases, which, by generally accepted principles of fluid projection, have proved anomalous.

b. It was observed by Clement Désormes[1] that steam at high pressure issuing from a large hole in a plate would cause a light free body that covered the hole to adhere to the plate; that the steam would issue from the edges of the plate, but the plate would not be thrown off by the internal pressure of the steam, although this was much greater than the exterior pressure of the air. The same phenomenon was observed with a pressure of air.[2] Hachette constructed a cylindrical vessel open at one end and closed at the other. By blowing through this it was found that a disc of card would adhere to the aperture against the force of the wind. This apparatus may be very well replaced by a tobacco-pipe, or one of the glass pipes used for blowing soap-bubbles, and a small disc of stiff paper. If the paper be placed over the bowl when the pipe is blown through, the disc, instead of being blown away from the bowl, as would be anticipated by principles of direct current lines, is apparently attracted to the bowl, and supported so as to flutter at the rim, leaving only just sufficient aperture for the air to escape. In this experiment the bowl supports the conoid of persistion. The stiff paper disc at the first instant supports a conoid of impression upon which the issuing fluid forms at the vertex of the cone a pressure, equal to that upon the base of the cone, as previously discussed, 73 prop. *a*, and the air escapes at the base of the cone of impression, moving by rolling contact upon the conoid of persistion, which is in this case supported by the solid pipe bowl. Therefore the projectile fluid is directed in whirls upon the surface of the conoid of

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[1] *Quarterly Journal of Science*, Jan. to June, 1827, page 472.

[2] *Quarterly Journal of Science*, July to Dec. 1827, page 193.

persistion, which whirls, form by their deflection, currents returning upon the path of original projection, so that these, as the rolling contact continues with the adhesive force of air to the solid conoid of persistion, draw the stiff paper in the line of the inward direction of the whirls. And although the pressure upon the conoid of impression is pressing the disc forward, the adhesion of the whirls to the supported conoid of persistion is the superior force by the directions of rolling contact given by the whirl system.

c. I have found in this, as in other cases, that the same phenomena may be exactly reproduced in water, as in air; and as the motions in water are visible, I will give the mode of production of experimental evidence of the same as follows:—

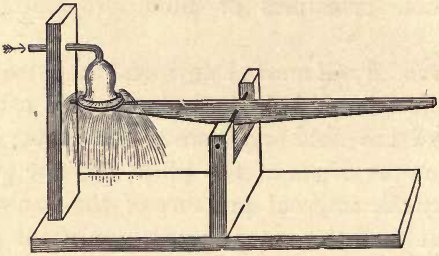

Fig. 116.—Ex.—Rigid Conoid of Persistion and of Impression.

An apparatus is made by fixing a glass pipe, such as is used for blowing soap-bubbles, firmly on a stand; a disc of talc is cut of a little larger diameter than the outer rim of the bowl. This is balanced by a wooden beam, upon one end of which the disc is cemented, so that it closes the bowl, as shown in the engraving above. The talc disc is balanced so as just to fall away from the bowl. If we now project a stream of water, containing small particles of visible solid matter, through this apparatus, we may observe the following phenomena:—Holding the disc for an instant at the aperture of the bowl a cone of impression will at first be formed upon it, as shown in the engraving below, Fig. 117, with outward whirls impinging upon and rotating towards the bowl. In another instant the whole projected fluid system will be in rotation within the bowl as in Fig. 118, and the disc will now be supported, or rather drawn inwards, by the direction of these whirls, the outflow of the water clearly indicating the complete revolution of the contact whirls. The disc will continue to adhere until the bowl is quite full, Fig. 119, when by the crowding of the water by enlargement of the whirls these will finally press the disc off and let part

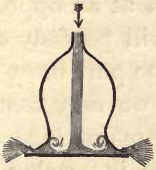

Fig. 117.—Experiment.

Fig. 118.—Experiment.

Fig. 119.—Experiment.

of the water out, so as to recommence the projection of the whirls as before.

d. The direction of the flowing fluid after it suffers the deflection by the conoid of impression, in the experiment just given, will be thrown at first at right angles to its original projection; and as this follows the plane of resistance, that is, the surface of the loose disc, it will support this disc *by adhesion of the fluid to it* as a part of the effect, irrespectively of the after inward deflection of the same jet upon the supported conoid of persistion. This adhesion we know by experiment with water is a considerable force, as shown by tractional systems discussed prop. 34 and 35. I also anticipate that it is so also in air and gases, as I only admit *repulsive* forces, as *apparent effects,* from the impressions of the elastic surface of the flexible molecules, which form the gas (4 prop.).

86. PROPOSITION: *If a flowing force meet such a form of solid resistance as that represented in contour by a conoid of persistion, the flowing fluid taking an opposite direction to that given in* 85 *prop., that is, entering from the base and moving to the vertex of the conoid; then the greater part of the mass of flowing force will be resisted by the solid conoid, but the central or axial area of the flowing force will be accelerated.*

a. By the principles of whirl force already discussed as being active in an inclosure of a solid cone of which the motion of a liquid in Venturi's tube is an illustration, I have observed that in this case, the accumulated activity of the central area of the cone is deflected into whirl motion, in which the direct force of projection of the central portion of the current is lost, as just shown, 84 prop. *n.* Therefore, if we reverse this mode of motion, that is, reverse the direction of influx to the cone, the central force should be *accelerated* by the same principle acting inversely, and the volume should be diminished. We must, nevertheless, conclude that in doing this with a solid conoid, the resistant surface is placed at an opposing angle to the direction of the current, so that it will possibly only accelerate quite the central axis of projection, or the part most free from lateral resistance. Under these conditions, we may assume that the force of the central current would be fully maintained, and probably in most cases accelerated, by the whirls increasing the central elastic force in the full area of the flowing current, being, as it were, compressed or concentrated upon a small area of the central part; thus giving a kind of directive pull to the whirls moving upon the conoid, the flowing force in this case being used for the traction or acceleration of the central current only.

b. In the case of entry of a fluid at the base of a cone, the central current in the inclosed area is isolated from the side resistance by the insufficiency of space to form large enough whirls to reach the centre, as the whirls would in this case be first engendered at the nearest points of resistance, at the mouth of the hollow cone, upon which, after the current whirls had made contact, and were returning by whirl deflection upon themselves, the whirls would be at first directed to pull the current forward, and then to set it free. During this time, the restrained elasticity of the flowing current, would accelerate the central current, which would again throw off other whirls further up the conoid by the resistance of its restricted walls, and again accelerate the central portion; therefore the central current would be constantly reinforced by the restrained elasticity of the external resisted flowing force over the larger area of the mouth of the cone, and accelerated in the limited area where the flowing force could be most active, until the inflowing current found a free vent at the open vertex of the cone. This principle would show a higher velocity in a small aperture at the vertex, than the initial velocity of the flowing force entering the cone or conoid at its base; subject to the condition of fluid accommodation to insure the development of whirl force without confusion. Upon the whole, this principle may probably give the true theory of hearing-trumpets. A conic form of entry by the base of a receptacle is also shown to be a form capable of giving acceleration to water in the form of the *vena contracta* in one of Venturi's experiments. It is most probable that this form of conic aperture to inflowing fluids would give some small acceleration of the central axis of a current in all cases, irrespectively of many interfering causes.

c. The absolute motion here proposed may be shown experimentally by causing a broad stream to contract to a narrow one by reversing the inflow into the same sectional conoid as that shown at Fig. 115, page 238. The same apparent disposition of whirls, upon the surface of the cone will be observed in the motion of coffee grounds in water, as was shown in that experiment; the result being that the central current will be accelerated somewhat above the original initial velocity of the current before entering the conoid. The normal section of which in this case should be parallel as described, 84 prop. *i.* The above diagram represents this principle of motion; A to B being the direction of the flowing force. The central *projection* in this case must not be looked upon as an elastic compression which could act by any means normal to the direction of the current; as in this case the greatest normal pressure is where, by the freedom of space, the lateral whirls are most developed, that is, towards the opening of the cone. The whirling motion is quite visible by the rapid rotation of particles quite up to the open vertex of the conoid.

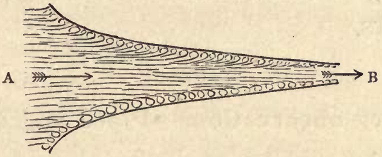

Fig. 120.—Ex.—Liquid entering a Conic Area.

d. There are some incidental circumstances which render this form of cone a vehicle for the collection of sound, which I may discuss at some future period.

CHAPTER VII.

PRINCIPLES OF CONIC RESISTANCE, TRACTIONAL FORCES, ROTATION, AND INTERMITTENT ACTION IN THE PASSAGE OF FLUIDS THROUGH THE PARALLEL SPACES OF PIPES, CHANNELS, JETS, AND CURRENTS.

87. Preliminary observations—Pipes and Channels.

a. Assuming the surfaces of resistance to cause deflections of direct force within a fluid flowing through a pipe or channel upon principles of rolling contact discussed in the fourth chapter. We must then imagine that by the conditions present, if such motions are developed, that they will be excessively cramped, particularly where the pipe or channel is of small diameter. Further, if we can by any means imagine, that there are present in the projection of fluids through pipes, active elements of conic resistance, of the same kind as in cases of projection of fluids in more free areas, of which I have given demonstration in the last two chapters; then, likewise, the whirls formed by the resistances will be also necessarily cramped and deformed by deflections from the restriction of area. Further, for the possible formation of whirls in pipes, we have not the influence of direct head resistance in the fluid as the most active cause of deflection, as it was shown to be in open areas; the resistances in a pipe being evidently principally lateral. Upon such conditions it would almost appear to be necessary that we should fall back upon the popular idea of slipping or gliding motion as the only possible means of overcoming the adhesion and cohesion of the fluid which is evidently present, and it is only in considering a fluid as an infinitely jointed system of matter, as before proposed, that we can imagine such principles of motive accommodation to be present that the conditions of conic resistance and of rolling contact can possibly be active. The tangential action of a liquid moving upon the surface of resistance by which rolling contact is assured, giving freedom to the central area of a pipe, may be made out without much difficulty; but our difficulties do not by any means end here, in that, as soon as we have a continuous flowing fluid, we have then to consider tractional forces active upon it, supported by the adhesion of the liquid to the pipe or channel. Taking the whole matter which experiment has induced me to follow, I have found the demonstration of the motion of a fluid in a pipe more difficult than any other form of fluid motion I have endeavoured to investigate. Therefore, in this discussion for the elucidation of principles, I shall be compelled to offer in many cases purely theoretical ideas; finding for them only such incidental experimental demonstrations as I am able.

b. To make my ideas of the principles of fluid motion in parallel spaces as clear to conception as possible, upon the above suggestions, the conditions of fluids flowing through such spaces may be conveniently discussed under separate heads.—1. That a particle of the flowing fluid will be resisted in its forward motion by conic areas of resistance upon principles discussed, 58 prop. 2. That a particle, as a portion of a cohesive system of matter, will be withheld by the traction of following parts, and by all adhesive surfaces, 34, 35 prop. 3. That a fluid necessarily makes rolling contact on solid surface, from its infinitely mobile or jointed system, 46 prop. 4. That all accommodations occur, that are possible, in a dense material system. For the elucidation of the foregoing conditions I make the following propositions.

Forward resistance to a fluid flowing in a parallel channel.

88. PROPOSITION: *If a fluid by the presence of lateral resistances be compelled to flow in a parallel direction to planes of resistance, every portion of such planes will, by an assumed conic series of molecules directed therefrom, offer resistance to every particle of the flowing fluid for such distances as the conic series of resisting parts may be able to act.*

a. A fluid may flow *directly forward* parallel to two planes of resistances if it be equally distant from each of them. This will be clear, as any forces of resistance on one side will be in equilibrium with those on the other. Under these conditions we may assume that a certain central volume of a fluid flowing between two equally distant parallel resistances will continue in a direct line and not be influenced to flow to the one side more than to the other.

b. The conditions of static equilibrium of a particle in a fluid, given 57 prop., page 167, assure us that equilibrium would be perfectly maintained in any position of a fluid particle by assuming equal conic areas of pressure to surround the resting particle, the principles of which have been fully discussed. Further, that upon movement of any particle within a fluid, every conic area directly and laterally extending within 90 degrees to the direction of projection, would offer some resistance to forward motion as offered in 58 prop.; but that in a free open fluid the greatest *resistance* would be where the most direct momentum of the force was directed, that is, in the conic area in front of the projection. In the present case to be considered, namely, that of a parallel space, the local conditions of resistance are materially different from those considered in the last chapter, as in the long parallel spaces which form pipes, the head resistances only may be taken to be nearly the same as for open spaces; the lateral resistances from adhesion of the fluid to the walls of the pipe or channel being relatively much greater by the solid support they receive; so that altogether the head resistance may be taken to be relatively small, and such conic resistance, as can be demonstrated, will be derived almost entirely from the circumscribing lateral resistances; where such can take directive angles of less than 90 degrees to lines of direct impulse of the fluid in the pipe, to be supported by its solid walls. In this manner the resistance will be nearly as the superficial area of restraint of the containing parallel surfaces.

c. Under the above conditions in the motion of fluids in pipes and parallel spaces, we have to investigate the action of *oblique conic areas of resistance*, which do not, as in the cases previously considered in former chapters, form direct head resistances by *condensations* directly forward of the projection, but the condensation caused by the force of projection of the fluid into the resistances, may be assumed to be immediately deflected, by the obliquity of direction of possible impulse upon the lateral surfaces. In this obliquity there will be, however, such elements of head resistance, as the continuity of the solid plane of resistance will support; which will be greater in proportion to its longitudinal extent.

d. The values of the oblique forces stated above cannot be estimated in any simple components of direct head resistance such as might be assumed to occur from partial obstruction; for instance, a thin rod crossing a current would offer equal direct *head resistance* to that of a long plane of the thickness of the diameter of the rod placed longitudinally in the flowing direction of the current; but the entire resistance to the flowing fluid would be much greater in passing this long plane than in passing the rod, as the plane may be assumed to support oblique cones of resistance at every point. There is also the further condition, that the force required for the displacement of the liquid upon the plane is largely dependent upon the extent and freedom of motions of rolling contact possible to be induced in the system, and also dependent very much upon the friction of the systems of motions already induced, which offer greater resistance at the entrance of a plane, where motive forces of rotation have to be induced than in its continuity, where accommodations to this least frictional form of motion are more fully developed.

e. The geometrical principles of construction of conic side resistances which act as modified forms of head resistances, may be conceived by supposing every particle of the flowing fluid to be supported by cones of resistance extending obliquely in front of the projection to the plane of the solid walls which the fluid passes, as in the diagram below (Fig. 121), wherein the deflection of the flowing force, caused by conic resistance, is shown by the direction of the arrows in the curved line. The resistances themselves being shown by the straight divergent lines extending to the plane represented by the horizontal line at the top of the diagram. In this manner, the resistance so far as the particular cones of resistance, that are shown by radial lines and assumed to be directed to a single point A, only are active, would cause the flowing force represented by the lower arrow to be ploughed off as it were from the general central stream flowing in a parallel line with the plane of resistance, so as to take divergence from the current to the surface of resistance as shown by the direction of the upper arrow. This motive deflection ensuring rolling contact of the flowing stream upon the plane of resistance by principles discussed 43 prop. *d*, page 133.

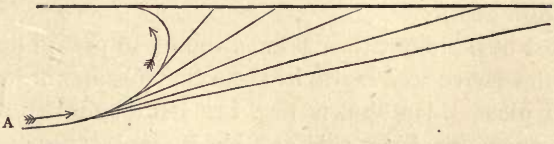

Fig. 121.—Diagram—Inflection by Forward Lateral Resistance.

f. The duplication of such a system as that proposed above, about the axis, taking one diameter of a parallel channel, would be in every way motively equivalent to the projection of a biwhirl as previously proposed in a free area. The impulse acting in this case in a line through the central axis of the channel; except that in the previous case in the bifurcation of the fluid about the axis, the whirls were shown to be derived from conic areas of direct *head resistance;* whereas in the present case, in spaces of parallel restraint, the lateral solid surfaces may be assumed theoretically to act only equivalently by extension of oblique cones, so that their oblique resistances take the place of direct head resistances. The deflection of the flowing force being in both cases in nearly the same motive form; the current having also upon the same principles a tendency to flow outwards from the axis of projection, thus inducing motions of rolling contact upon the cohesive surfaces of the channel or pipe.

g. I have not been able to obtain by experiment the evidences I desired of the lateral conic resistance of a pipe, although I shall offer further evidence in other propositions as I advance, but I presume that the collateral evidence I have previously adduced will be nearly sufficient to make it reasonable to accept the principles now offered to ensure the necessary motions of rolling contact, which are also evident upon principles of whirl force, already fully discussed. The case now proposed *varies only in degree*, according with the direction of the angles of conic resistance, which have been already shown to be derived from every angle up to 90° to the direction of projection. (57 prop., page 167.)

h. The best experiment that I have found to obtain any demonstration of the above was made in the following manner with a thin trough with glass sides, shown Fig. 122 below, similar to that described 81 prop. *a.* Into this trough I poured nearly saturated salt water until the holes at each end of the trough were nearly reached. I then covered this with a stratum of warm water, so that an invisible divisional plane was thus formed by the differences of density between the two fluids. Cold water coloured

Fig. 122.—Ex.—Biwhirl Separation of Liquid Planes.

with aniline was now projected through this divisional plane from an inlet pipe to the left of the figure, and this, by its intermediate gravity, flowed between the denser and lighter fluids. The motions of rolling contact were observed along the plane which were particularly evident at the entrance of the stream; and the angles of resistance in the planes were indicated by deflections upon the lateral resistances. There are, however, complications in this experiment that I hope to clear somewhat by future propositions.

Resistance of parallel channels by adhesion and cohesion of the flowing fluid normal to the moving parts.

89. PROPOSITION: *That the cohesive forces of a fluid moving past a parallel plane of resistance either solid or fluid, will deflect the flowing particle from its direct course in such a manner that the deflected fluid near the surface of resistance will be moved in a retrograde direction to the flowing force, but the deflected parts will be again drawn into the direct flowing stream through a cycloidal curve by traction.*

a. In this proposition I follow only the conditions of a fluid in its properties of cohesion, and of adhesion to a lateral prescribed channel throughout the distance that the fluid is assumed to move. The forward parts of any moving fluid may be assumed to resist by cohesion equally with the backward parts previously considered for traction, 34, 35 props. An equally cohesive system will move generally upon its centre of inertia by forces impressed tangentially to any part. I will now most particularly consider the functions of traction through adhesion and cohesion active upon resistances normal to the motive direction of the central particle in a current.

b. If we extend cones of resistance in the manner proposed in the last proposition shown in the diagram, Fig. 121, and take conic areas of resistance to extend forward of a moving particle by the general cohesion of the mass of which it forms a part, every portion of the fluid will then resist every other part, if it is moved against it, and every lineal or other series of particles in any direction that can be separately taken will be held together by cohesive forces extending to every point of resistance; therefore the point of resistance shown at A, Fig. 121, although not of greater resistance than any other point of the system of the fluid at equal distance from the static plane of support, may be conceived to act, as far as its force of resistance goes, as a fulcrum of resistance to every other lineal series, or conic area of molecules that extend in any direction *over* this point, that may be held by the general mass cohesion of the fluid system.

c. These theoretical views of fluid resistance may be partly shown by the following figure in which I will take lineal resistances as equal to conic ones for simplicity:—

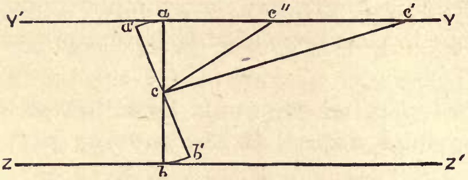

Fig. 123.—Diagram of Cohesion under Tangential Action.

Let Y Y′ represent a static surface against which the intervening fluid possessing cohesive forces adheres and reposes, reaching with equal or diminishing force of cohesion to the lineal plane Z Z′.

Let Z to Z′ represent a moving force acting upon the intervening matter between its own plane and Y Y′, and let the matter between the planes Y Y′ and Z Z′ be adhesive to these planes, and cohesive in all its parts.

Let *c c′ c″* be a cone of resistance whose oblique base is upon the static plane Y Y′ extending to the point *c*, its vertex.

d. Now if we take one transverse lineal series of molecules in the general cohesive system of matter in the interspace Y Y′ to Z Z′, as that represented by the line *a b*, and take our point of resistance as that of *c* only, then if this rigid line be deflected by flowing force moving in the plane Z to Z′, the point *c* would form a fulcrum of resistance to its progress, and would either by rolling contact or adhesive force move *b* to *b′*, and at the same time it would tend to move *a* to *a′*, for the assumed cohesive force that would extend from *b* to the fulcrum *c* would also extend towards the plane Y Y′, and if this is taken to be an imaginary direct rigid lineal series without interference it would impress its force by moving *a* in a retrograde direction to *a′*.

e. The conditions of the above would somewhat resemble the case of a straight rod in the position *a b* being floated upon a surface of water Y Y′ Z Z′. This rod having an impulse impressed upon it at *b* in the direction towards Z′, this impulse would cause a certain opposite movement at the other end of the rod *a* to *a′*. The rod as a cohesive system moving upon its axis of inertia supported by the general resistance of the water.

f. Following the conditions of the above construction, and further assuming the termination of the lineal molecular series at *a* to be adhesive to the line Y Y′, then the quantity of motion represented in the figure from *a* to *a*′ in the last diagram could not occur unless the rigidity of the molecular series could break asunder the adhesive force of the assumed rigid system at this point *a*; that is, if the adhesion were greater at *a* than that of the rigidity of the lineal series *a b* in itself. In this case the lineal series of molecules *a b* would be *bent* by the force acting from *b* to *b*′ over the fulcrum *c*, still assumed an immovable axis of inertia.

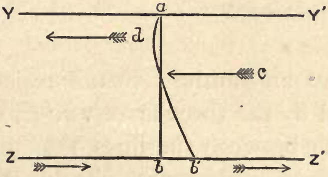

Fig. 124.—Diagram of Normal Deflection under Adhesion.

g. Now if we consider the line of static force first taken and represented by a lineal molecular series *a b* as *not* in this case perfectly rigid, but of certain elastic flexibility, as shown in the figure above by the line from *a* to *b*′, and assume the point or end of the lineal series at *a* adhesive upon Y Y′. Then we may take this lineal series to be resisted as before by the point of the arrow *c*, as an axis of inertia which takes the place of the cone in the previous diagram: and the movement of the flowing force from *b* to *b*′ on Z Z′ acting upon this elastic lineal series *a b*′ would deflect the lineal series at some point between the point of the arrow *c* and *a*, in the opposite lineal direction to that of the flowing force, moving from Z to Z′, giving a *minus velocity* to a parallel of the stream as represented by the arrow *d*, near the static surface Y Y′.

h. In the above construction it is quite clear that no fluid could, as a free system, offer a single point of resistance or axis of inertia, as in the supposition taken for a rigid molecular series. But if we suppose the whole mass of an incompressible liquid interposed between two planes, and that the liquid is static upon, or in uniform motion upon *one of these planes*, as that of Y Y′, forming a system of resistance supported by this plane, and that the moving force acting upon this system is in the plane Z to Z' only, then every point normal to the lineal series represented by the line a to b may be considered as a point of resistance to the moving force, assumed to be adhesive upon the static mobile system represented by the intervening incompressible liquid and its static adhesive plane.

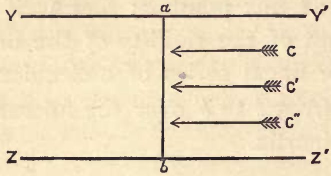

Fig. 125.—Diagram—Static Resistance.

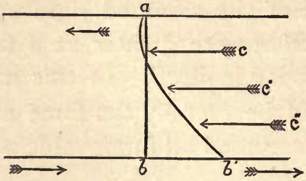

Fig. 126.—Diagram—Motive Resistance.

i. Let the points of an infinite series of resistances upon the line a to b be represented by the three arrows $c\,c'\,c''$, Fig. 125, and assume the intervening space between the lines Y Y' and Z Z' to be a fluid or system possessing a certain mass cohesive flexibility; then each parallel plane of resistance will suffer a certain deflection in any vertical series of molecules by the movement of one of the parallel planes, as that of Z to Z', for a distance parallel to itself upon the intervening space between Z Z' and Y Y'. Let Fig. 125 above represent such a system as that described before movement, and let Fig. 126 represent the lineal deflection after a movement of the line Z Z' to the distance $b\,b'$. Then will the lineal series $a\,b$, taking a line of flexure about the unstable fulcrum of resistance represented by the point of the arrow c'', deflect by its cohesive elastic force the contiguous stratum c' in the series to the point of the arrow c'. This again will deflect the lineal elastic series about the point of the arrow c' to the point of the arrow c, and this again continuing, the deflection about the point of mobile resistance c will complete the curve at a, this last being a *minus* deflection from the direction of the flowing force b to b' as it was directed in Z Z', upon the plane assumed to be static by adhesion.

j. In the above cases, if we follow the deflected lines upon the diagrams, we may observe that we have taken the conditions of flexure only; but if we consider the same cohesion to be maintained in the deflected line, so that a point of our lineal series in moving upon the resistances carries forward the lineal series of molecules it deflects, with a certain constant force derived from the flowing force, then the lateral parts will be drawn into the flowing system in the flexure lines shown in the diagram, Fig. 126, by the curved line *a* to *b′*. The above construction that I have given for matter moved in molecular lineal series perpendicular to the flowing force, with fulcra of resistance by adhesion laterally towards the head of such force, may be equally applied in principle, or extended in lineal series to any angle of direction from the central axis between the planes of resistance, or if the area be such that the solid resistance may be taken to be at infinite distance with respect to any solid parallel of the same fluid, then the inertia of the lateral fluid may be conceived theoretically to produce a static plane relatively to the impression of the flowing force, in every case, with equal results. Further, the fulcra taken to be axes of resistance supported by fluid matter forward of the projection, may with equal demonstration be taken to be fulcra of resistance, tied or supported by the general cohesion of the material system by *traction* upon matter backward of the projection. In either case the amount of the flowing force will constantly deflect any lineal projection upon lateral axes of inertia by resistances directed towards central areas of motion, and thus assure the same principles of traction lines by a general system of cohesive resistances. This connects the principles previously discussed in 35 prop., as shown *d*, page 118, with the present proposition. What experimental evidence I am able to produce for this proposition I will defer until the next.

Conditions of rolling contact of a fluid in a pipe or channel.

90. PROPOSITION: *That fluids flowing in restricted areas will be able to maintain an apparent continuous central flow under conditions of deflection and cohesion, by rolling contact upon the resistant surfaces by separating into motive units, or separate systems of motion, in which the principles shown in the two last propositions will be combined.*

a. When we compare the motive lines considered in the last two propositions, we see at once direct antagonism of motive direction of flexure in the single unit of motive fluid. That whereas at every instant the projectile forces of the central current when moving upon lateral resistance will project the force radially forward, as in other conditions of biwhirls, previously considered in 88 prop., that on the other hand the principles of cohesion of the mass system will tend to deflect the lines of force radially backwards, 89 prop. We also observe that by the principles of central projection, the whirls deflect the fluid matter from the central axis of projection towards the sides of the channels, as previously demonstrated in the one case; whereas by the principles of traction the lateral parts are drawn towards the central axis in the other. If we represent these systems again by lines about a single point of resistance on a plane, they may be reproduced as in the following diagram.

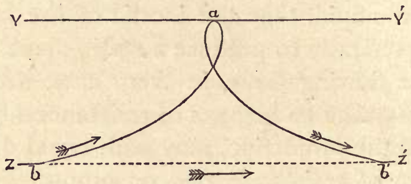

Fig. 127.—Diagram—Intermittent Action.

Let *a* (Fig. 127) on the plane Y Y' represent any point of resistance to a flowing force, moving in the direction Z to Z'.

b. Then the line of flexure from *b* to *a* would represent the line of motive force of a projectile unit of fluid projected from *b* towards *b'* by principles of conic resistance discussed in 88 prop., and the line of flexure *a* to *b'* would represent the motive line of traction by cohesion upon principles discussed in the last proposition. Therefore if we suppose these forces equally active in a system upon every particle, they must either be assumed to act in antagonism, or to direct forces in composition with their motive powers, or to be *separately active* at different periods of time. By observation as carefully as I have been able to follow this difficult matter, I have come to the conclusion that the motive fluids passing solids and resisting fluids move by *intermittent actions*, in which the deflected and tractional forces act at separate intervals of time which may be in some cases almost infinitely small. The spaces of time for the motions being those which are under the conditions the least frictional to maintain the intermittent system necessary for rolling contact upon the planes of resistance. There is, however, one thing most important in this theory, which is, that such intermittent action opens a possibility to continuity of motion through deflection of lineal parts of the flowing fluid, and assures rolling contact upon the planes of resistance as shown in the diagram at *a*, Fig. 127, by the continuity of *one line of flexure* from *b* to *b′*.

c. The system of motion offered above, which relates particularly to areas of restraint, will be a *cycloidal system* derived from whirl projections re-entering the flowing system by tractional forces. There is no doubt there may be great difficulty in conceiving so intricate a system as that proposed above, but its reality appears to be most consistent with my theoretical deductions and observations of areas of restraint.

d. For the conditions under which flowing forces moving against resistant surface may be induced to divide into separate motive systems, which are necessary to support the conditions just offered, we may conceive that deflections of central forces into whirl systems will always have directive tendencies to complete their whirls, unless these motions are resisted by superior forces (82 prop.), so that by this principle, motive forces in fluids acting upon lateral surfaces where whirls are engendered by the resistance, there will always be the disposition to divide the flowing fluid, to complete separate whirls, along the plane of resistance, as closely as the mobility of the fluid system will permit. If this were not so, supposing the adhesion of the fluid to any surface to be great, it would not flow unless urged by force sufficient to entirely overcome the adhesion present.

e. Further, by cohesion, a traction will be directed from the adhesive surface of a solid to every motive part of a fluid system near to it, so that the position of the reflex motion of rolling contact of a whirl, after completion of a semi-rotation upon a surface, will be again directed towards the axis of motive traction in the area of restraint, and be free to follow the traction by the cohesive forces in a central current. Therefore a flowing fluid will be in unstable equilibrium in moving forward upon a surface of resistance, and will divide where the direction of the angle of deflection by induced whirl motions produces a greater strain upon the direct motion of the fluid in any part, than its cohesive force can withstand. The divided parts in whirl systems will upon these principles be extensive if the pressures are small, so that the whirl action is free; but may be very close where the system is cramped, to cause rapid deflection by near resistances.

f. If we can assume the action of a flowing fluid upon resistances to be intermittent, as suggested by our proposition, the difficulties of accounting for the separation of the cohesive forces of forward parts, to permit the flow, becomes almost infinitely less than could possibly be conceived for gliding planes of molecular displacement, in any possible system of cohesive matter whatever. It may further be conceived, by the principles now suggested, that by separating the units, the whirls will be thrown out more freely from the instant effects of conic resistance, exactly as the whirl-rings in Prof. Tait's experiment, described 67 prop. *e*, have projectile force, whereas constant systems are static in space, 73 prop. *e*, so that a flowing fluid may become, as regards its projection, exactly as a close series of such whirl-rings. Upon this principle, as the projectile fluid that forms the separate whirls is assumed by necessity to be compelled to return the fluid projected by deflection to the central system, so we may imagine that, by the effort of the projection, the whirls upon contact with the resistant parallel surface will make rolling contact, and that they will be instantly, as it were, thrown off by the same whirl force, after contact, back into the general flow. The momentum of this action being intermittently constant by the impulsive motivity maintained in the central space. The interior current, by this means, will ride over the whirls, making thereby rolling contact upon them, at the same time withdrawing intermittently by cohesion the fluid deflected into the lateral whirl systems.

g. In the case of a liquid flowing with great velocity by pressure through a pipe, the liquid may occupy possibly nearly the entire internal space of the pipe, and this space may be directly projectile, but it is possible even in this case, and consistent with other cases of lateral resistance, that the liquid may be surrounded by a series of small rotary systems resembling spherical drops thrown off, as whirls rolled up as it were by the continuity of the conic deflection, and as constantly unrolled by lateral cohesion, so that the same minute globular forms are maintained for the drops to act constantly as friction rollers to the flowing force. I must admit that the whole matter of this paragraph is purely theoretical, but that it is the most consistent that I can suggest to ensure the general principles of rolling contact where the current is almost entirely projected by principles which are quite evident in slower motions. Some evidence of the above may possibly be inferred in the issue of water at great velocity, or under great pressure, from a pipe, where the borders of the issuing jet are visibly broken up into spray or drops, as also by the evidence of separation in the reflection of light, which causes rapid jets to appear of a white colour.

h. I think, nevertheless, that it is extremely probable in a perfectly mobile liquid that no system of whirl motion may reach quite to a solid plane of resistance, but it is more probable that there is a certain thin layer of liquid which is powerfully adhesive and very resistant supported by the irregularities of the solid surface, so that the liquid wets this surface and renders it smooth to the friction of the rolling parts. This thin surface may possibly have a small motive direction, by impressed forces in the *opposite* direction to the current, the conditions of which I will hereafter consider.

i. For the experimental evidence of this proposition, generally, some instances of the disposition to division upon contact of flowing liquids upon surfaces of resistance may be very fairly made out. One instance of this which is very striking to observation may be found in the projection in the section of a conoid given 84 prop. *k*, where the liquid upon contact is shown to form a series of rapidly revolving whirls made visible by the relative freedom the expanding conoid offers to frictional motion.

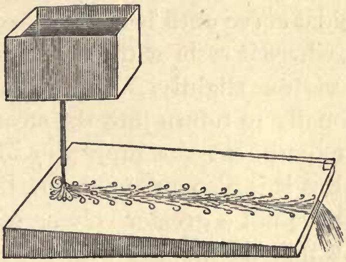

Fig. 128.—Ex.—Looped System Borders of a Current.

j. For the general experimental induction of the principle of division into separate whirl systems in parallel channels we may follow with advantage any case of the motion of liquids moving slowly through channels of small lateral resistances, for which the following experiments may be offered.

Take the apparatus described 48 prop. *h*, page 150, consisting of a can filled with water placed upon a shelf with a sloping board leading from a hole in the can to supply a fine stream of water. If we take a smooth sheet of foolscap paper and place this upon the sloping board in the manner before described, and draw the wet finger directly down the board, it will produce a moist channel of about half an inch in width. Down this channel we may project from the hole in the can a fine stream, that will continue to flow in the wet track only. Now permitting the current to flow down slowly, we may when it is steady, observe perfectly well the effect of the small amount of *side resistance* active upon it, by experiment in the manner previously proposed, of dusting the stream over with fine pepper. If we follow carefully with the eye the motion of the separate particles of pepper from the commencement of their course near the jet, we may note by these that the flowing fluid in the centre of the channel has the highest velocity, that this velocity gradually decreases towards the borders of the channel, and that the flowing fluid at its edges rests nearly quiescent, or moves in some parts with small velocity in a retrograde direction. We may further observe that every particle of the pepper is drifted by the biwhirl action of the stream more or less to the edges of the channel, from which now and then, a particle is again drawn in, but generally, at the end of the experiment, the pepper that we dusted upon the head of the stream, will be found deposited at the edges of the channel only. In fact we find that nearly every particle of pepper has followed a cycloidal curve until it has reached the more resistant edge of the channel, where it rests at the vertex of an hypercycloidal arc, or turns this vertex slightly, and reverses its direction of motion so as occasionally to return into the stream.

k. In the above experiment the projectile forces of the current are clearly evident, but it is also quite evident, that as at the end of our experiment we have not a greater volume of water at the sides of the channel, to which the pepper was directed by the moving water, than elsewhere in the stream, that there must have been throughout the entire process means for the return of the deflected water, which in this case, from the immobility of the particles of pepper, they could not in all instances follow.

l. For the separation of the system into units, if we watch the motions of the pepper for a short time, we find, that the points of greatest hypercycloidal curvature are local, and at these localities we may frequently observe the rotary motion of free whirls by the visible movement of the small particles of pepper; which shows that the rotary system of contact is divided into separate units, as we found more clearly observable at the borders of the sectional conoid shown, 84 prop. *i.*

m. We may find further experimental evidence of the same system of division, in a fine jet of water issuing into the air under pressure, by motions already induced in its system, which separate the jet into drops either directly at the orifice of issue, if it be very small, or at a short distance outward from the orifice, if it be larger. The motions of deflection, from the differences of velocity of the parts of the jet, are maintained in the air, and the liquid passes through the air upon the same principles of surface deflection as in a pipe, in which the whirls may be either incipient, as in a continuous jet, or complete, as possibly in the system of separate drops. This I shall be able further to demonstrate as I proceed.

n. In the issue of a liquid through a small hole or tube where drops only are formed, the drops most probably retain the induced rotary motions somewhat in the following manner:—

Let A′ represent the exit at the pipe or hole; a biwhirl being complete only at exit. The action of surface resistance constantly tends to produce the globular form; this being at the first instant of projection as represented at B, then at C, the complete sphere.

Fig. 129. Diagram.

o. If a small pipe has any lateral restriction at or near the orifice, the whirls have greater freedom in the direction of the greatest diameter, and they divide afterwards at the exit or *spirt.* But if the channel be smooth and cylindrical the cohesive force of the liquid will draw the parts together into globular masses to form drops, which still retain in themselves the rotary motion induced by lateral resistances acting upon them, as appears by evidence of dropping experiments, where whirl-rings are immediately projected at the surface, if the drop fall from a pipe or through the air (86 prop. *a*); whereas, these whirl-rings do not appear to be immediately formed from drops projected from positions in contact upon the surface (71 prop. *b*), so that the induced rotary system in the drop is auxiliary to the motive forces that we witness in whirl-rings engendered by small effects of conic resistance (65 prop. *a*).

p. The following experiment may be considered demonstrative of the rotary or looped hypercycloidal motion of contact of small unit systems, that continue in rolling contact, separately down the stream. If we take the slowly projected column of varnish in like varnish but of less specific gravity described in 75 prop. *e*, we may note this clear stream flowing to the bottom of the jar in the sunshine by the difference of refraction. If we carefully observe the column near its surface we may notice many small globules of air and particles of matter descending with the stream. These, as they approach the borders of the column, will be found to be set into rapid rotation. This will be particularly visible with any particles of air which, by the projection, take an ovoid form and glitter, as the differences of inclination of their surfaces reflect light to the eye. In this experiment the air globules appear to move from loop to loop in a cycloidal path, as they are carried forward by the current. If the fluid system of the globule of air were such that it permitted continuity of the induced liquid motions, as we clearly see is not the case, the air globule would then be drawn out into threads to form a part of the contact whirl, and would revolve constantly near one locality in the column; but as the air bubble is held by surface forces, 11 prop., to one form, it is urged forward from loop to loop. I was anxious to obtain the same form of evidence from observing the effect of water flowing down a glass pipe by its visible action upon solid particles of matter, but I found that if the particle were large it appeared to resist motion, but if very small its motion was imperceptible, so that in very slow motions where cycloidal action could be assumed to occur close to the tube, by easy accommodation, the small resistance to the gravity of the solid particles permitted them to descend in fairly direct paths. With greater velocities, however, there was evidently present, cycloidal motion, which could be best seen by many particles descending at the same time in water through a glass pipe, by placing a pinch of sawdust in the water, but the motion being very rapid it was not easily followed to attain knowledge of the definite paths of the particles. In this case the particles of sawdust appeared to be plaiting, and crossing in every conceivable direction, as shown Fig. 130 B. It therefore appeared evident that the motions of a particle of water could only really be observed in moderately slow motions in a pipe, by the motions of another fluid moving within the water. This was not difficult to arrange. I therefore fixed a glass tube of five-eighths of an inch bore, and

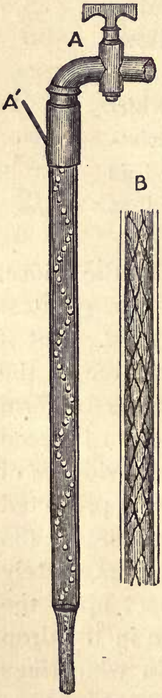

Fig. 130.
Experiments.

three feet long, to a tap from a supply cistern as shown Fig. 130 A. Now admitting air with the water into the inflow current by a small pipe A′, the particles of air were drifted in the glass pipe through cycloidal paths in constant revolution, as was clearly indicated by their glittering in the sunlight. In this case there was no doubt a certain restraint from the inequalities of density between the fluids, air and water, but the result was all I might have reasonably expected as an inference of the direction of motion in the water in so rough an experiment. In the motions of free particles and of the water itself in flowing through a pipe, there is a tendency always to produce continuity of like motion in separate parts; by these parts taking one helical direction. This is shown in Fig. 130 in the pipes A and B; but experiments in square and flat pipes show that this helical direction is a motion of composition only.

q. For another experiment to endeavour to follow more exactly the conditions of lateral resistance. I took a tall glass jar 18 inches high filled with clear water. I then drew out a piece of glass tube to a fine point and broke off the extreme end. This tube then formed a funnel with a fine stem of about $\frac{1}{200}$ of an inch bore. By supporting this funnel in a hole made through a piece of card, and placing it above the jar filled with water, so that the point entered the water, the funnel, when filled with coloured water, projected a fine stream vertically down the larger column of water in the jar, the resistance of which to the jet was easily observed by motions induced in it.

r. With the above described apparatus in projecting a dilute solution of an aniline dye, that termed *Tropoeleum* I used, but anticipate any other would answer as well. I found that as this solution issued from the point of the funnel described, it enlarged from four to five diameters. This enlargement is evidently in unstable equilibrium, for if there should happen to be any vibration in the air the column divides transversely and appears as a series of discs, one placed above the other, which resemble somewhat the annular markings of a common earth-worm, as shown in the figure by the side, which is magnified four diameters. This effect I have especially observed when a railway train was passing at about 100 yards from the position where I was trying the experiment.


Fig. 131. Ex.

s. Quite irrespectively of the separate markings, the experiment shows the directions of internal central forces to be outward movements, so that they enlarge the stream, from which we may infer the formation of whirls that would of themselves naturally cause a tendency to division in separate units in the liquid column; these divisions being only made more visibly evident under the influence of the vibrational disturbance. If in the projection of the same column the air be quite still, the equilibrium will be disturbed in a less degree; in this case the fine stream as it descends will be marked in occasional places with local swellings. These swellings descend with regular velocity until they reach a certain point, and then separate from the column, at first apparently by being drawn together into drops, but afterwards they immediately develop their true character of whirl-rings, showing that the motive system of the column moves upon whirl principles, that in a quiescent flow, are only observable, when there is sufficient accumulated local activity and freedom in the projection, to render the motive direction of the forces present visible.

t. In the above theoretical treatment of the descending column offered, if we imagine that the original motion of the jet is at all times actually intermittent as it appears to be under vibration, it would then be clear that this intermittent action might be observed in a coloured jet, or not, under two conditions:—It could be observed if the whirls or looped systems were separated *sufficiently* to give distinct rings of colour by intermittent densities, as they evidently do under the conditions of vibration; but it could not be observed if the whirls came out of an axis *one within the other* regularly, so that their general interference of projection when the jet was viewed as a transparent object would produce simply a uniform tint. We may also imagine that if the whole column were in unstable equilibrium, a small excess of interference from any cause in any part of the column would cause the following parts to assume such conditions that they would develop their whirl forces as free systems, that is, as systems free from the cohesive restraint of the constantly flowing column of a gravitating system. It is, however, most probable that from the evidence we obtain of the motions present in very small systems, we may be best able to observe principles that universally prevail; and this evidence I think may be largely found in the projection of the small jet, under the conditions just offered, by watching attentively consecutive swellings or drops as they pass down the column.

u. It has already been shown that the dimensions of free whirls vary as their powers to overcome lateral resistances, so that after the motive forces of the projectile column have by motions induced by the resistances gained greater whirl force, the whirls spread out in the liquid by the action of tangential forces. In this case, as their projectile lines become deflected more nearly normal they must possess also less force in the direction of original projection. Therefore the next drop *following*, or the next less deflected whirl, will continue its motion with higher velocity, and overtake the first more deflected whirl. Further, when a following whirl arrives at the point of motive deflection of the former whirl, the induced direction of motion in the former will tend to draw the following whirl into the front of the previous one. This I find by observation is very evident. Further, as the lines of force in the two whirls in question come together each possessing a like form of projection, a following whirl, after it enters the system of a previous one, will continue its projection, by its less resisted direct momentum, and pass quite through the first whirl, for a certain distance, or until the direction of its force lines, that are already induced by the previous whirl, absorbs the following one into the previous system. In this manner two or more whirls will be united. The general appearance of such systems is shown in the engraving by the side, Fig. 132.


Fig. 132.—Ex. Columnar Projection.

v. It will be further seen, that by the continuity of induced deflection, of the first whirl, where the following whirl enters its system, some parts of this first whirl will be left too far behind, in the general system of combined whirl deflection, to be at once absorbed. In this case, by continuity of induced rotation, the following parts will be drawn into the general system and appear to pass inwards through the second whirl, as the second whirl in its projection passed through the first, showing thereby the perfect unity of the combined whirls in the hypercycloidal or loop system.

w. By the above reasoning I conclude that however small the division of a system of a fluid in rolling contact may be, that by the conditions of area of restriction, it may be real, and capable of development at any instant, into a larger system; by the presence of greater freedom in the plane of resistance.

x. The adjoining diagram (Fig. 133) will show by observation the method of combination of whirl and looped systems in areas free from great restraint, experimentally, in projecting a fine jet with the apparatus described Fig 132.

Fig. 133.—Ex.—Section.

A. represents two whirls in a descending column in a liquid, the second advancing on the first, the flowing force of the first being deflected more nearly normal to gravitation impulse.

B. The second whirl commencing to be absorbed into the projectile system of the first.

C. The first whirl commencing to overflow and again enter the second.

D. Continuity of the same form of projection.

E. The projection of the first system through the second completing an hypercycloidal or *looped system*.

y. That the principles of division by lateral resistance into separate systems occur generally also in gases, may possibly be best inferred, by the projection of hydrogen through a small pipe, in which case, if it be lighted, an intermittent flame will be produced, particularly if the gas be carried away from the aperture by a current of air, at a greater velocity than it can diffuse itself in space. This property of intermittent action, forming thereby intermittent units of projection in hydrogen, is made evident in the well known experiment of singing tubes, the intermittent explosions of which are possibly the simplest and most direct modes of producing musical sounds. The same forms of intermittent action are also apparent in the whirl systems induced in all musical pipes, as I will endeavour at a future time to show. The same principles of division may be observed in the projection of one current of air over another, where the line of contact becomes visible, as cloud, through difference of temperature, the condensation of vapour producing series of equal small whirls, that we term a mackerel sky, as before mentioned.

z. The principle of division is very observable in air in the experiment of projection of smoke between two planes of glass, shown Fig. 104, page 222, which I left entirely to Mr. Collings to delineate, by the phenomenon he witnessed, following the means I gave. In this experiment the lateral freedom is much greater than in a pipe, so that the whirls complete a greater circuit, and do not form loops. The nearest approach to sufficiently static resistance being in this experiment near the entrance of the jet, as shown in the figure, before the whirl force is greatly diffused over extensive radial areas. The same experiment may also be used to demonstrate the vibration in jets, that Young so acutely observes in the experiment given 74 prop. *c*, page 213.

a. If we accept the general principles of this proposition with such demonstration as I have been able to give in this very difficult matter; the projection of a current in a pipe or jet will resemble very closely the continuous separate projection of whirl-rings, which are assumed to be by this theory packed as closely together as the system of accommodation of the fluid will permit, these being alternately collected by reflex action, that converts these whirls into loops, which permit the deflected fluid to re-enter the central system by traction. If we accept this as a general principle, we may still admit the influence of the special surface resistances of the pipe, which will have necessarily a force-value higher than the resistance of a free fluid to the friction of projectile whirls; and these resistances by their nearness will modify or deflect the whirl or looped systems, but not sufficiently to change the general principle of motion, as a necessary condition of the flow of a cohesive system of matter. I do not see the necessity in all cases of a set form of cycloidal motion, which is most apparent in the changes of flow by the influence of vibration (Fig. 104, page 222), as also in the intermittent local action (§ *r*) in the descending fine column. The principle of the possibility of inducing the cycloidal motions I propose, of longer or shorter periods in a pipe, is possibly the mode used unknowingly by performers on certain forms of horns, where the vibrational period represents the cycloidal space, as I will endeavour hereafter to show,[1] which produces notes of different pitch in the same horn. In any case I anticipate, under all conditions, the efflux of wind from a pipe is pulsatory, and it only depends on certain conditions whether sound is produced or not.

β. If the principles of motion of a fluid through a pipe is hypercycloidal as here proposed, the resistance and consequently the velocity of outflow will be nearly as the circumference of the hypercycloidal loop to its plane of resistance.

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[1] Now postponed for future publication.

91. PROPOSITION: *That a small flat current of fluid will develop whirls of resistance at alternate distances upon its sides. The whirls will form looped or cycloidal systems if they are cramped by nearness of resistant surface, but the current will always tend to enlarge alternate lateral whirls, for each whirl to attain the greatest circular area of rotation.*

a. For the conditions of alternation of whirls this proposition shows that whirls would have a tendency to circumscribe the greatest free areas, 82 prop.; but this is subject to the directive force of the central flowing stream, and is not possible where the friction is uniform or great enough to cause the constant necessity for rolling contact of directly projected lineal parts of the system. Nevertheless, if the current move in a free fluid system by alternation of whirls, first on one side of the stream, and then on the other, the current space would develop the largest whirls, and move thereon by the least tangential friction; and if we make the flowing force great, and the areas of resistance very free, this will be the necessary form of motion. This proposition may be partly demonstrated by the following experiment.

b. Take a white dinner dish and place it in a level position upon a table; pour water into it until the bottom surface is covered for not more than half an inch in depth. Take a pen full of ink or other colouring matter and draw this through the water with moderate velocity, the nib sliding along the surface of the dish. As the pen moves forward in the water a zigzag path formed by the ink will be observed to follow it; which will be at first of about half an inch in width. If we carefully observe the zigzag ink-path by repeating the experiment several times, it will be found to be formed of alternate whirls which are structurally consecutive oblique biwhirls, the one flowing out of the curves of the other, of which the following diagram will represent the directive principles.

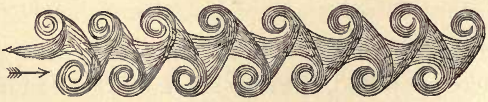

Fig. 134.—Diagram of Biwhirl Deflection in a Uniformly Restricted Current.

c. These whirls will after a short time be distorted by the general influence of motion of one upon the other, but at the commencement of the movement of the pen, a few more permanent forms will occasionally be left when the fluid is nearly at rest. Upon repeating this experiment with a different kind of writing ink to that I first tried, I found that I did not produce the same result owing to the clogginess of the ink. The first ink I used was made of logwood extract and bichromate of potash.

d. The same principles as shown in the above experiment may be witnessed as an influencing cause for the zigzag course of a river, in its flow down a nearly level plane. In this case the river at no time may be assumed to possess sufficient force to cut out a zigzag course such as is found to absolutely exist, but the tendency to attain the greatest freedom of area for the whirls, as proposed (82 prop.), such a course would be the least frictional. If such a system were once incipiently formed by the principles of this proposition so as to produce bays, by the alternate local amplitudes of the whirls, these would also alternately bring the central forces to the margin of the river, and would cause every bay to become enlarged and deepened at its circumference, until there was formed a channel in the bay in which the river would afterwards be induced to flow, so that it would ultimately change its course and flow through the circumference of the bay. In this case the friction whirls would then immediately be turned inwards and be projected forward to the opposite surface of resistance, so as to induce the formation of another bay on the opposite side of the river further down, or give the river an entirely zigzag course by the alternation of this form of whirl motion.

92. PROPOSITION: *If a small cylindrical jet of liquid be resisted by a less dense fluid, the deflection of the whirls making rolling contact upon the planes of resistances, will be less developed outwardly than in cases of greater resistance, or they may be withheld within the flowing jet by its cohesion, but they will still be motive within this jet.*

a. The superficial film of a jet may be continuous by general cohesion, so that the jet may appear bright and clear, but this is indifferent to the principles of the proposition, as the motive force may be conceived to be active within the circumscribing area of the cylinder that forms the jet, the surface of which is induced to take a reverse direction by the friction of contact.

b. The cases this proposition is intended to meet are the projections of jets of water in air. The same conditions may possibly hold for projections in very smooth fine glass tubes. It assumes that rolling contact also holds in cases of very small surface resistance which I have asserted to be necessary in all cases of contact of moving fluids upon solids or upon each other.

c. That the wind moves the surface of the ocean assures us that a frictional mode of contact occurs between air and water, when the air is moving, and it is quite certain that a like friction will form a resistance to the projection of the surface of a jet of water in air, so that the demonstration of surface friction in the one case will demonstrate it in the other.

d. I had at first some difficulty in arranging an experiment to show that whirl motions would exist in water, moving with moderate velocity through static air, until I thought of making the jet of water extremely small, so that it should present a great surface to the air relative to its very small cylindrical mass, and thus be greatly influenced by the surface resistance, but having conceived this principle of small resistances, my further difficulty was in rendering the motion itself visible, or of obtaining evidence of the same by any means, but feeling assured that motive principles were alike in all moving fluids, whether the masses were large or small, as the many experiments I have given tend to show, I thought it possible that the surface directions of motion of one jet might be observed by directing another jet upon it, as the equilibrium of projection of such fine jets would be necessarily very sensitive.

e. The experiment by which I was finally able to demonstrate the above to my own satisfaction under the condition of a dense fluid being approximately free when reduced to small projectile mass, was, by the interference of two fine jets of water in quiescent air, one of these, from which I most expected to discover directive motion, being sufficiently fine to have its bulk considerably influenced by friction at the surface of contact with the air; the motion of which I hoped to follow by the direction given by deflections after contact upon another jet. I will give my first conception, as I worked this matter out before trying any experiment upon it, which may tend to explain the principles of the experiment I afterward tried.

f. I supposed a whirl system developed in every current as that illustrated, Fig. 129. Therefore, if by resistant contact a fine current were ejected into the air, the air would form a surface of constant resistance, and a biwhirl current or looped system should be induced, within the fine stream, to overcome the surface resistance of the air. Thus if we suffered two fine streams to meet at such an angle that it was possible for one to be deflected from, or towards, the surface of the other, the influence of the whirls at the sides of the jets, as they came in contact, would carry by adhesion the whole small mass of these jets by deflection in accordance with directions of surface motions already induced in them, at least as components of directive influences. That if the whirls in the jet were revolving outwards from the centre by air contact at a point of meeting between them, they would be drawn together, by surface directions acting at the point of incidence; whereas, if no such motions were induced, they would repel each other. Before making any experiment I made a sketch diagram similar to the following illustration of my conception, in so far as I could conceive the matter, by the principles already offered of whirl motion.

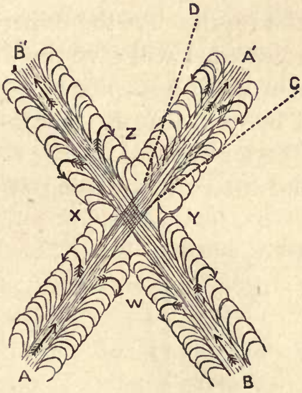

Fig. 135.—Diagram of Theoretical Crossing Jets.

Let A A′ represent a fine jet of water projected in the direction of the arrows; B B′ another jet projected in like manner across the first. Let these jets by the friction of surface contact upon the air have biwhirl or looped motions already induced within them. Then upon contact of these two jets the whirls at W and X will be drawn towards each other by rolling contact. At Y and Z the whirls would resist each other having opposite directions.

g. Now if such a jet as A A′ by its projectile force and the direction of its whirl could enter the jet B B′ so as to pass into it, the whirls at Z would tend to direct the jet A A′ to D, and the whirls at Y would tend to project the current A A′ to C, so that if the current as a lineal system were divisible with small force in its axis, the jet after division, would be divided within an arc included between the radii C to D, directed from the centre of the crossing current.

h. The only difficulty appeared to be some confusion of directions of force lines, which I should have anticipated, except from having made some other somewhat similar experiments on a larger scale, and from the certainty of biwhirl action being constant in such motions.

i. In the following experiment, although clearly founded upon my conception of the biwhirl motion of fluids, which I have before fully discussed, I scarcely expected the full realization of my theory in the projection of one flowing stream through another, and it really surprised me (although I had reasoned it out) as it appeared in practice so contrary to the ordinary experience of superficial observation, so that if I had fallen upon it by chance I should not have comprehended the result. As the experiment for demonstration is very easily performed, I will give the details.

j. Take two lengths of small lead pipe of say half an inch internal diameter, and a yard or so in length. Close one end of each pipe by burring it over, or better still, by sawing the ends of each of the pipes off obliquely, and afterwards closing it over so as to form a cone of about an inch length of axis. The meeting edges of the cone formed in this manner may be soldered together with soft solder. Now after filing off each pipe to a rounded nose, the ends may be pierced with a fine brad-awl or point so as to open them to a smooth bore of say $\frac{1}{16}$ of an inch for one pipe, and $\frac{1}{30}$ for the other. The exact proportions are immaterial. Having the pipes thus pre-

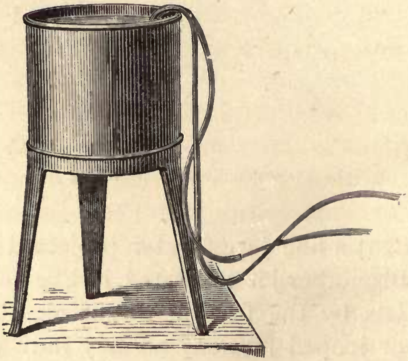

Fig. 136.—Ex.—Crossing Jets of Water.

pared, bend them as shown in the figure above into the form of syphons, and place the open ends in a vessel of clear water at an elevation of about 30 inches above the jets. The free ends of the pipes may then be turned outwards, as shown in the engraving, and be fixed by staples to a wooden stand, each at about six inches from the end; these pipes may be now adjusted by the flexibility of the metal as required. Placing the pipe with $\frac{1}{16}$ inch bore so that it will eject the water nearly horizontally, and the pipe with the smaller bore lower, so that the jet that comes from it will cut the first larger jet at about an angle of 30 degrees, the apparatus is then complete. The pipes may be set in action by sucking them full of water by the mouth. The experiment being now ready, as the jets of water flow, it will be observed that the small jet from the lower pipe will cut a clear way through the larger stream and suffer no deflection at the point of contact, even if the smaller pipe be reduced in size, so that only a small spray of fine drops issue and strike the larger jet, these drops also pass clearly through it without deflection. Further, the larger jet will not be materially disturbed or have any swelling or parting at the point of contact of the smaller one; but the smaller stream will appear as a clear rod of crystal moving through it; however, after passing through the larger stream, the smaller jet will often divide by the whirl forces at points of exit, in which case only the upper part will continue in the direct line of the original projection.

k. It will further be observed in this experiment that the velocity at the point of contact of the small jet upon the larger one is no greater than that of the larger one, as the fall of water is alike in both. The angle of contact of the jets being small we might superficially expect that the two streams would unite and form a single jet, or be deflected from each other, but in this case the small jet striking the larger one suffers only a slight recoil at this point of impact, where the surface particles are revolving by rolling contact upon the air, to be impelled into the larger jet, by the revolution. To accommodate the intrusion of the small jet, the larger one must therefore have its bulk in some way increased, and swell out against the force of cohesion of the water of which it is formed; but the surface of the larger jet being also in revolution by rolling contact upon the air, and this revolution being in an opposite direction to the surface of contact of the smaller jet, it therefore tends to eject it again as soon as intruded, so that the whirls become most unstable opposite the point of contact of the small jet, by which means this is ejected at exactly the opposite point on the larger jet, to that at which it entered; it therefore appears to pass directly through the larger jet without material loss of initial velocity, or in some cases of volume.

93. PROPOSITION: *To every current flowing in a parallel channel or in a pipe, there will be a lateral counter current flowing in the opposite direction near the plane or planes of resistance.*

a. The general cohesion of liquids will always tend to induce continuity of surface, prop. 38 *a*, and in this continuity, the motive effects of rolling contact are still presumed to be active, so that any induced movement from conic deflection, capable of producing a complete cycloidal arc or loop, will tend to give reverse direction to the fluid resting upon the surface of resistance.

b. That by the flexure of traction lines deflected by lateral resistance, a retrograde motion will be induced in a fluid near the plane of resistance, by the principles discussed in 89 prop. and shown in several diagrams.

c. That by principles of conic resistance, simply as head resistance, biwhirl forces induced in parallel channels by the direct resistance of the forward or most free portion of the flowing force, will complete their whirls in direct proportion to the force of projection by which they were formed, in continuing beyond the plane of resistance, so as to produce a retrograde motion upon lateral surfaces as shown, 90 prop.

d. From the roughness and immobility of every solid surface relative to the smoothness and equality of the system of a fluid; a moving fluid in contact with a solid will be more resisted upon the solid than upon any imaginary plane in the fluid, at any parallel distance from it, however small.

e. The adhesion of liquids to solids, where such liquids *wet* the solid, appears in all cases to be greater than the general cohesion of the fluid, and to be generally of the nature of an attraction, 22 prop. It will therefore follow that the conditions discussed, *a* and *c*, of this proposition will not exactly hold if the conditions of adhesion at the surface here proposed be true; as the reflex action by this adhesion would be less frictional at any plane of cohesion throughout the fluid more distant from the solid surface, and the retrograde action would be also less resisted in any less adhesive plane. The conditions of this adhesion were taken in functions of general cohesion in 89 prop., quoted *b* in this proposition. The general equation under these conditions, from surface resistance and reflex action, would remove the retrograde direction of the fluid proposed to a small distance from the plane of resistance, so that if a counter current were formed this would flow the opposite way, *near*, but not directly upon the planes of resistance as given in this proposition. The principles here offered may be shown by a diagram, which may be taken conveniently as a refinement of the functions of biwhirl action of a fluid acting directly upon a resistant adhesive plane solid. I take in this case the action of the projection only, not the traction which completes the curve.

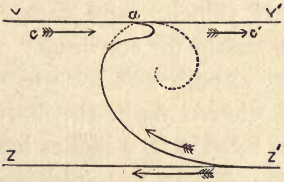

Fig. 137.—Diagram.—Deflection of a Cramped Whirl near a Solid Resistance.

Let Y Y′ represent a plane of resistance upon which a flowing fluid is adhesive, in a greater ratio, than the general cohesion of the fluid system.

Let Z′ to Z be the direction of a flowing force in a fluid that extends to the plane Y Y′.

Then a whirl of projected fluid deflected from its plane of projection Z′ to Z, as shown by the direction of the lower arrows, would in equally cohesive and resisting matter make contact at *a* upon the surface of resistance. But if the flowing force in the whirl be more resisted near the plane of which *a* forms a part, than in other parts of its trajectory, it will be retarded and deflected at this place, and the direct momentum of the following parts of the whirl will carry it forward by flexure to smaller radius, the place *a* now being assumed to be static, so that the reflex action to the direct line of projection would be in the plane represented by the arrows *c*, *c′*, not at Y, Y′, directly upon the plane of resistance, but near to it only. If we now compare this matter with the directions given by *tractional forces* to a current near a plane of resistance, we have the same directions given both by whirl forces and by traction as shown

Fig. 126; so that by both principles, a reverse direction of a current should occur *near* a plane of longitudinal resistance.

f. I think the above may be safely accepted as a principle of projectile fluid motion. It may be shown experimentally by the counter currents upon any extensive flowing stream or river, though these counter currents may in this case not be equal on both sides, from an unequal disposition of resistances; but in all cases this may be taken to be a principle of *relative motion* in a flowing current.

g. Perhaps the least demonstrable conditions of a counter current would be in the cramped whirls of a liquid flowing through a pipe, as in this case the reflex action must be brought very near the resistant surfaces. I have therefore sought experimental evidence for this particular case, by giving to the flowing current great velocity, and such an amount of freedom as was possible for continuity of projection, as in the following experiment for which I constructed apparatus. A cistern A, in the bottom of which a brass tube was fixed 2 ft. 9 inches long, and ¾ of an inch in diameter. A boss was soldered on the brass tube at about 9 inches from the lower end, and a small hole drilled through this into the tube, the hole being made smooth inside. A small glass tube was inserted, and bent as shown in the sketch. This was about 6 inches long, the lower end was inserted into a vessel of water B. On filling the cistern and allowing the water to run away freely through the brass tube; the water was drawn up from the small vessel so long as the flow continued. This experiment is similar to one of Venturi, wherein the pipe is placed at the *vena contracta*, but this position, shown by Venturi, is entirely unnecessary for the effect. This same direction of motion in the small tube takes place in both cases, which I take to be evidence of the presence in the pipe of a counter current throughout its length. This experiment may be also used as a demonstration, that the flowing liquid does not *fill* the pipe entirely, but leaves space for intermittent action, as proposed, 90 prop., sufficient to include the small current flowing upwards from the vessel B. The pipe B may extend to the depth of the outlet or deeper with like effect; or the system may be placed horizontally, the like evidence of suction being in all cases present.

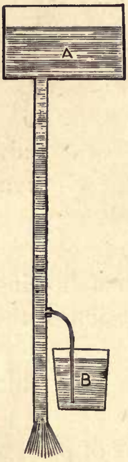

Fig. 138.—Ex.

94. PROPOSITION: *A flowing fluid in a channel will be deflected from every resistance by motive whirls until the greatest volume of the flowing fluid moves in the tract most free from resistances by equilibrium of rolling contact.*

a. Demonstrations that a liquid will be deflected from a resistance in cases where there is not continuity of planes of adhesion in the direction of the force were shown 42 prop. It is evident also, where there is continuity of solid surface, and the projectile force of the fluid is sufficient to overcome the adhesion to the solid. I will take both cases.

Let a stream of water, say of half an inch in diameter, fall vertically as from a pipe, and place in this at an angle to the course of the stream the edge of a razor, so that half the current may pass free, and half be deflected by the surface presented by the side of the blade of the razor. By the resistance offered to this, the stream will swell out slightly half an inch before it reaches the edge of the razor, and by the general elastic forces in the flowing system, which will be conserved after passing the resistance (39 prop.), will fly off from it.

Fig. 139.—Ex.

b. For the demonstration of projection under the influence of surface adhesion we will permit an undercurrent to enter an open parallel channel; in this case it is clear that the central area of the upper surface will be the most free part of the channel from near resistances. Experimental evidence may possibly be made more conclusive by a celebrated experiment of Bossut, which is as follows:[1]—

A reservoir of water of about 11 feet in height is kept constantly full or at one level surface. An orifice is made in the vertical surface near the bottom of the reservoir; the opening being five inches wide horizontally; the depth being adjustable by a brass sluice valve, which can be opened to any required distance from quite close to two inches by the movement of an adjusting lever. A trough or canal 105 feet long, 5 inches wide inside, and 8 to 9 inches deep, is placed at the orifice quite horizontally, and the bottom of the orifice from the reservoir is in a line with the bottom of the inside of the

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[1] Abbé Bossut, *Traité d'Hydrodynamique*, vol. xi., pages 196 to 206.

trough. The sides are also in a line with the edges of the orifice. The trough is made of strong fir planks polished inside, with quite smooth joints.

c. In the experiment I wish to notice, the sluice is opened exactly half an inch. I translate Bossut's words: "It is necessary to observe that in every experiment the velocity of the current is not uniform, that is, that each separate division of the canal will not be traversed in equal time, and that the velocity diminishes as the water recedes from the reservoir. This movement has some particulars which merit observation. When we raise the sluice the water is ejected along the bottom of the canal, and at first keeps only in this direction. But as it proceeds it meets with resistance, swells on itself, and its surface takes the form shown by dotted lines in the above engraving E M G. Then it falls by its own weight from the highest point M, and *a part of the water returns towards the reservoir*, following the direction M N. There is, therefore, in the part C M of the canal two currents which are going in different directions—the one formed by the deeper water which goes in the direction C F, and the other by the surface water which returns in the direction M N, and which is very apparent at the commencement. This effect terminates at a point N at about 12 feet from the orifice E C. Little by little the swelling of the water diminishes, although it always exists, until finally it takes the form of the water represented in the engraving E R G, where the point R is the highest from the bottom of the trough. The water which arrives at each instant from the reservoir strikes continuously at N O, the mass before it, N O F G, mixes with it, and forms the mass which renewed incessantly returns to the same figure. The currents of which we have spoken are forcible examples of those which form in rivers or the ocean, in all instances where the water is retarded by obstacles. We see in this case that the water swells at first and then its weight

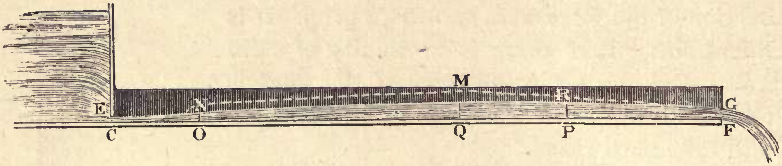

Fig. 140.—Ex.—By Bossut showing Reverse Surface Direction.

causes it to spread itself out, from which it results that currents may take any possible direction."

The motion of the water in Bossut's experiment may be explained upon principles of resistance, in the following manner. For the general flow of the current the central surface horizontal plane would offer the least relative local resistance, whereas the surfaces of the trough would offer local resistances in proportion to the pressures upon them, and the air surface would offer a certain small resistance at its line of contact. The current would, therefore, be deflected to form contact whirls in every direction, upon which the flowing force would ride by rolling contact, but as the local resistances would be less upon the upper surface than upon any other, the general whirl force would deflect the current mostly upwards and cause a general central elevation.

d. For the entire resistance to the flowing mass, not local and transverse only, against gravitation, we may take it that the greatest amount of resistance would be from the surface of the bottom of the channel, by which every particle of the liquid would be retarded by that in front, 42 prop. It would, therefore, occur, that the stream in its flowing force would have a constant tendency to split upon the head resistance, and upon principles of conic resistance, to have its force deflected by the local side resistances, so that whirls formed by the general system of resistances would curl over and roll backwards almost frictionless upon the eentral part of the stream opposite to its general flow.

e. Now taking the directions of the surfaces of resistance that surround a flowing liquid stream, the aerial surface must in all cases be nearly the area of least resistance. In the case of the trough it would be particularly so; therefore we have a much greater force of adhesion of the water to the bottom. Now from any cause whatever a moving fluid actually moves away from points of resistance to the point of least resistance offered to it, as our proposition states, therefore in this experiment the air presenting little resistance, the fluid would move into this with all the energy its internal accommodation permitted. This energy may be computed by the amount of matter it would be able to carry against gravitation. Having this plane of least resistance, at the aerial surface, presuming that there is an increase of resistance with depth of water from nearness of the bottom, the liquid would take a rounded form upwards in the air. The mass carrying this force having moved to the surface of the water, it could not again *sink below it*, as in order to do so, it would have to move to points of *greater resistance*, which is impossible for any free body, and especially so for a liquid or fluid of any kind that contains within its own matter great solidity of resistance. Therefore the part of the body which has now received the impressed force flows forward with the stream, but the part which impressed the force suffers reflection and returns with its reflected force, accompanied by the impulse of natural gravitation, from its now elevated position as a surface motion back in the reverse direction to that in which it was originally projected.

f. Bossut's experiment shown upon the principles of the proposition might be represented by the diagram below:—

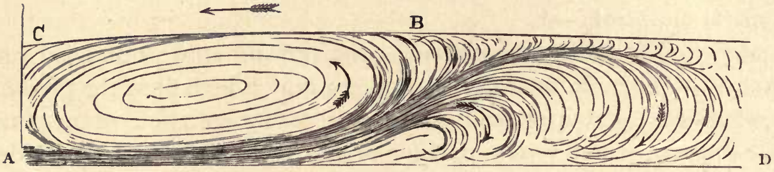

Fig. 141.—Diagram—Whirl Theory of Bossut's Experiment.

A, a stream projected along the bottom of the channel toward D and deflected by the resistance of the channel to B, where an oblique biwhirl is formed; from the centre of the oblique biwhirl system B, the reflex current falls by whirl force and the potential of gravitation to C.

g. It would at first sight appear from the above that an undercurrent could not exist, as the resistance would carry the greatest motive flow to the surface, and this, I presume, would be true for a fluid of equal gravity. But we know that undercurrents exist in the ocean and in the air, so that we must assume that the undercurrent of water exists from greater density, as the aerial currents also have greater densities derived from the pressures they receive from the air above. To this matter I will return in the discussion of systems of motion in terrestrial areas in the next section.

h. Where the water is deep the aerial resistance would form a larger factor, so that absolutely the projectile freedom would be greatest at a distance below the aerial surface; where the axis of the hypercycloidal projectile system would be placed.

CHAPTER VIII.

PRINCIPLES OF MOTIVE RESISTANCE TO THE PROJECTION OF FREE SOLIDS IN EXTENSIVE FLUIDS.

95. Preliminary remarks.

a. Having fully discussed in the previous chapters the principles upon which motive directions are given to fluid forces moving under the influence of solid resistances, which are shown to produce similar or equivalent deflections in whirl-forms, complete or incipient, in all cases; it will now only be necessary, as regards the projection of solids in open fluids, to consider the conditions under which the fluid has greater freedom to complete its motive force lines by the uniformity of mass resistances in contiguous parts. This matter, although I know it to be an important part of my subject, as it concerns the impulsion of ships, I have been unable to investigate as I wished, and I can only direct attention to certain particular conditions, relative to previous propositions, which may, I think, be usefully followed as principles.

b. I have endeavoured to demonstrate by theoretical deduction, as also by experiment, that when any particle or unit mass is projected in a fluid which may be conceived, as regards the unit projection, to be of infinite extent, that the projected unit will engender within the extensive fluid a conic area of resistance, and cause lateral deflections of the forward parts of the fluid (58 prop. page 170), which lateral deflections produce annular whirls, if central to a mass, or biwhirls, if the fluid impressed is a plane (77 prop. page 219).

c. It is also affirmed by other propositions that the same principles of motion, or their equivalents, will engender whirl motion in the projectile unit itself, if this be a fluid, as shown in the single drop (66 prop.); and the same motive forms are also observed in the projection of liquids in pipes and in jets, of which instances are given in the last chapter, but more particularly in 90 prop. In the consideration of the projection of fluids within fluids, taken in chapters 5 and 6, all parts of the projectile and the resistant are evidently alike mobile. In these cases, by principles of conic resistance, whirl systems are induced which diffuse the unit projection entirely and extensively within the resistant fluid. In the case of the projection of a solid, we have a simple unit, held by its cohesive forces constantly under motive restraint, and this being of fixed form, all resistances and motive directions for accommodation of fluid forces must therefore occur wholly outside this, that is *within* the surrounding resisting fluid, so that by this principle of motion, we must conclude that comparatively to the motion of a projectile fluid previously discussed in these pages: *That a solid projectile body moves in a fluid under the impression of forces that would have separated its mass, as a projectile fluid is separated, if it were not held under the restraint of its cohesive force.* In this manner we may, by principles already discussed, fairly estimate the natural strains from the direction of forces active upon the solid, and contra to these, the strains and directive forces that are taken in the fluid to resist them.

d. We have further a most important condition evident by experiment in the projection of a solid in water which renders the projection of the solid in a certain degree equivalent to the projection of a fluid, namely, that the water is so powerfully *adherent to the solid and cohesive in itself*, that the solid carries with it in its projection a considerable portion of the contiguous liquid also. Therefore we must conclude that the solid and contiguous liquid form in a certain sense *one unit projection.* This contiguous liquid part of the projectile system, when the velocity of the projectile has been maintained for a certain time, forms no mean factor of the entire projection. This is made quite evident by the important experience of Mr. J. Scott Russell given 34 prop. *d* page 116, wherein the projection of a boat in a canal engenders a motion in a mass of contiguous water that possibly quite equals the boat in volume-force; which is made evident by suddenly stopping the boat, which represents one factor only of the projectile system. The liquid in this case by its internal mobility continues a certain part of its projection after its frictional detachment from the boat. By this theory of projection of a united solid and surrounding liquid system, being assumed to form one unit of projection, we may conceive also that motive forms of whirl, or other systems, may be induced by the liquid part of the projectile within the resistant liquid. This principle will offer much less frictional modes of motion than those that would be produced by any form of slipping or gliding motion, if such were possible; and will bring the condition of solid projection much nearer in principle to fluid projections than would be possible if the evident adhesive and cohesive action between the solid and fluid were to be entirely overcome at the surface of the solid.

Equilibrium of forces about a solid projected in a liquid.

96. PROPOSITION: *As a body placed in a liquid will be surrounded by equal pressures at equal depths of the liquid; this body, if moved horizontally, will after the first impulse encounter very little greater head resistance than it will receive pressure from behind; but as the body to continue its motion must displace a certain bulk of the liquid from the front to the back, the force required for the continuity of its movement will be as the friction of this displacement.*

a. From the general instability of fluid matter by which near particles under the impression of small forces may move quickly to equilibrium; moving masses slowly impressed by constant forces might attain motion with very small resistance where the general surrounding equilibrium was not much disturbed. But as higher local velocities were attained, the accommodation could not be so immediate, and the compressions engendered by the forward motions would extend in the fluid to much greater area and move in this area with less freedom until the resistance might approach the general conditions of the elastic reactive forces of a compressed fluid. Where such a system would not offer the entire solution for the effect of a given forward fluid resistance, is that the fluid being an elastic system of freely jointed matter, the resistant parts would by deflection under the compression attain set directive motive forms of projection, which would be those in the path of least resistance for the displaced fluid to move to the position of the displaced solid projected therefrom. These displaced parts would have also directive energy, and produce, as I will hereafter show, rotary systems of motion in the fluid, consistent with the velocity of the moving body, which, by deflection of the fluid to backward parts, would either *increase* or *decrease* the general resistance to forward progress of the solid. In this manner the resistance to the motion of a body in a fluid would be proportional to the powers of the set motive forms, either to assist or interfere with, the system of accommodation that it would be possible to set up for its impulsion at a certain velocity, and not as a function of the *transverse section* presented to the adhesive fluid resistance. The motive forces engendered in the fluid about the solid would form a part of the system of general projection as before stated, and these forces would be active upon the projection of the solid, into the resistance according to the directions of motions induced.

b. The researches of Messrs. J. Scott Russell, Rankine, Froud, Stokes, and others, have demonstrated that certain forms of vessels will offer less resistance than other forms, these not being necessarily proportional to the volumes of displacement. These conditions will be better developed by future propositions. This proposition need only be taken in a very general sense.

Projection of whirl forces by free solids moving in fluids.

97. PROPOSITION: *That a free solid moving in a fluid of infinite extent will carry forward with its motion the nearest portion of fluid in front that resists it, and with this portion, will open out a cone of resistance in the fluid and engender whirls, that will by their rotation ensure rolling contact upon the general mass of the resisting fluid.*

a. If a solid move forward in water it must of necessity occupy the space formerly occupied by water, and as the water remains during the movement at nearly the same level at its surface, except for such small differences as I will hereafter consider, it is clear that a certain quantity must be constantly moved from the forward to the backward part of the projectile solid, as discussed in the last proposition.

b. The pressure upon the water by the movement of a solid below its surface, upon the above conditions, will in every way resemble the pressure of water upon water, because I infer that a certain volume must constantly be moved forward by this pressure, that is, it must move until it is displaced by the solid as given, *a* above. Now if such a volume be pressed forward this must act in its pressure exactly as though it were a projectile current of water, and the same forms of motive resistance will be engendered in the resisting water as were shown in principle in 73 prop., page 209, except, in this case, that the projectile fluid will be held under a certain restraint, by the presence of the near solid to which it will adhere with a certain force.

c. It may also be conceived from previous experiments, that the water at the head of a moving solid, as, for instance, a boat, will impress the resistance of the extensive fluid in front, which offers by surrounding hydrostatic pressures conic resistance, upon principles given in 58 prop. This form of resistance is also evident under conditions given in 40 prop. *b* of a peg in a stream, the water in this case being motive, which is equivalent to the solid being motive, the liquid being in both cases elevated at a distance in front of the solid, or in the place where the motive resistance is deflected. It is also clear, as before shown, that such a solid as a boat cannot separate its own mass upon the conic resistance, except in so far as it is a system of motion including the fluid carried with it of which the liquid projected in front forms a part. Therefore after a certain amount of deflection of the water carried forward by the boat, which is displaced, the boat then enters the cone of resistance by a motion that is practically a fracture of the resistant cone in its axis.

d. In the above case of fracture of the cone of resistance by a solid projectile, the projectile will encounter the least resistance in its lateral parts and near the aerial surface of the water, particularly for impressions of forces from the boat at positions below the surface of the water. Therefore into the smallest resistances or weakest parts of the aerial surface the projectile forces in this case will be deflected, and towards the weakest points of resistance the projectile water carried by the boat, will be best able to complete such circulatory forms of motion as are necessary to ensure rolling contact to its projection. It will also follow that as all parts of the aqueous system have equal density, that in proportion as the head of the boat by its forward protrusion penetrates the water below the surface, the cone of impression formed by resistance of the moving water in front of the boat will be more resisted; since the surface will relatively offer but little resistance to the conic areas directed from the stem of the vessel. The deflection of the water therefore by the minus resistance will be towards the surface both forward and laterally, so that the whirl system formed at the surface will be in the plane of the surface nearly, or *biwhirl.*

e. The visible evidence of the generation of a projectile biwhirl system about the head of a boat in full motion will be seen by the conic pressure at the surface of the water at the bows of the boat, throwing off the forward water from the boat laterally immediately before it comes in contact with the full resistance due to the pressure upon the level surface of the water. This is exactly equivalent to the motion of the water before the peg in the stream (40 prop.) just quoted. If the whirl force did *not* deflect the water from the bows of the boat, but that the boat came upon the resistant water as a direct compression, then the mass of the water in front would be raised above the general surface directly at the head of the boat, exactly as a wave is elevated when it impinges upon a solid vertical coast or other surface of resistance; the elevation being in direct ratio to the immediate compression upon the nearest part of the mass of surface water about the area of contact.

f. By the above principles the constant forward motion of a solid in a liquid will as constantly overcome the forward liquid resistance by the motions of deflections it induces. This continuity of projection will resemble the constancy of projection of small forces given in 75 prop. *b*, wherein the projection deflects the whirl backwards, or more correctly speaking leaves it behind in continuing its direct forward projection, the conic resistance separating the projectile force in a much less area, as is shown in the engraving Fig. 100, page 216, where the whirl-ring is left behind.

g. In the same manner the forward projectile whirls from a solid will be left behind if the solid be constantly pressed forward with a constant force. Therefore this fact will open out to us a method of observing the projectile whirls engendered by the forward motion of a solid as we may make the solid so short lineally to the direction of its projection, that the whirls will be separately developed, and visibly follow it. Having this conception, experimental evidence becomes more simple.

h. If we take a piece of flat board and move it along vertically in the water, it will answer the purpose of the experiment I wish to demonstrate of the projection of a very short projectile as proposed above; we may if we please give the board any other form in its forward part, but as it is not the direct resistance we wish now to observe, but the path in which the water is carried forward and deflected by the projectile. The shape of the solid projected will be of no consequence, provided it is short enough in the direction of projection, that the whirls developed may be left behind, to be visibly shown exterior to the influence of the lateral surface movements of the solid when the rotational systems are once induced.

i. The experiment I tried was as follows:—a trough 20 feet long, 11 inches wide, and of the same depth, was filled 9 inches deep with water. A piece of board of convenient size to hold in the hand, 15 inches long, 5 inches wide, and 1 inch thick, was held upright in the water in the trough near one end; the end of the board being sunk about 6 inches below the surface. This board was moved along at walking pace, keeping it vertical and at equal depth. The whirls induced by the forward pressure could be clearly observed deflected behind the board and moving backward to fill up the space of displacement. The general forms and directions of these whirls are shown in the engraving, Fig. 142, which is taken from an angular solid made in section to represent the head of the boat. In this case the whirls that are thrown behind the projection are not spread out as in the case of projection of small forces previously shown Fig. 102, page 221, as the surrounding forces are not quiescent but laterally motive relative to the projection, from traction of the water behind the projectile solid. The conditions of the whirls as following forces I will hereafter consider, the present proposition is used for the demonstration of the existence of whirls *simply* being induced by the displacement of solids in fluids.

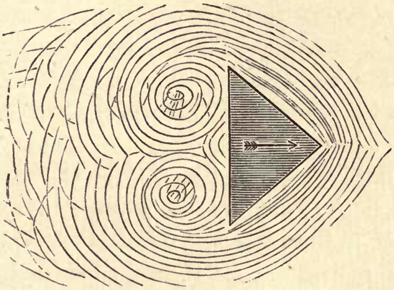

Fig. 142.—Ex.—Evidence of Whirl Motion by following Whirls.

98. PROPOSITION: *That a free solid plane moving longitudinally in an extensive fluid will engender projectile whirls in the fluid by adhesion of the fluid to its surface, such whirls will form means of rolling contact to all lateral resistances in the fluid.*

a. The principles of lateral resistances discussed in props. 88, 89, and 90, will answer for the present one, except where conditions of restraint are pointed out in these propositions, in which projectile solids in an extensive fluid will, from the fluid being more free from rigid resistances, cause the motive lateral forces to be developed upon a larger scale as more free whirls.

b. If we consider the solid plane of resistance shown Fig. 121, page 249, as a *projectile* plane relatively to the motion of the water. This plane in moving forward will produce exterior pressures at a distance from its mass by conic areas of equivalent projection to the cases of resistance previously discussed (88 prop.).

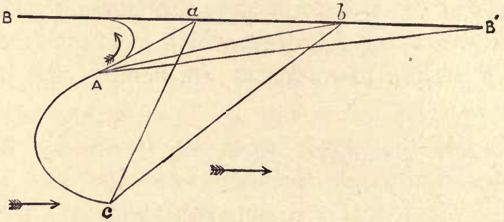

Fig. 143.—Diagram—Conic Deflections near Motive Solids.

c. Taking the diagram given Fig. 121 to represent the part of the diagram above B B′ A; A being the point previously taken for the extent of resistance in fluid moving parallel to the plane of resistance upon B B′, which resistance, as regards forward parts, was shown to be derived from conic areas of adhesion A *a b*, A *b* B′, the flowing force being by the resistance thereby deflected in the direction of the incurved arrow as shown near A, turning the current inwards towards the plane B B′. In the case discussed in 88 prop. the point of resistance of the central area or most free part of the current was assumed to be only at A. If we now take the case of an infinitely free fluid, the lateral resistances may be assumed to extend to any distance, but the values of the forces, derived from the angles of effective resistance, will decrease in like ratio. Thus if we assume a distant point *c*, and take the resistance of the same surface as before taken *a b*, this oblique cone in an infinitely jointed cohesive system of matter will offer much less resistance to the motion of the plane B to B′ in ratio of its constantly increasing angle of divergence from the direction of projection. It will also be seen that the diagram, Fig. 121, shows in this case the same form of directive lines as that given in Fig. 136, page 272, and the same forms of motion as given in 93 prop. will be necessarily followed in this form of projection as in that in a restricted area, except for the effects of deflection due to the restraint, in the closeness of area previously considered, in the one case.

d. Upon the above principles of conic projection it will be seen that a moving solid in a liquid, particularly after it is surrounded by established whirl forms, will not have the deflected liquid which it opposes thrown up against it as a wave against a sea wall, but the moving solid will rather by conic projection press the liquid back from its surfaces of contact into lateral space.

e. If the adhesive whirls upon a moving side surface are *projectile,* the fluid in contact will be projectile, therefore for a moving body to continue its projection in a fluid there must be means of supply for such projection. In this case we must imagine some of the conditions given 90 prop. as active, and consider that the surface receives supply at such positions as are most free from projectile whirls, or from fluid forces otherwise directed, to enter the system of motion; the condition of which will vary under different circumstances.

f. The best means that I can suggest, to ascertain the presence of projectile whirls being thrown off the surface of a solid moving in a fluid, would be to endeavour to measure the loss such fluid sustained near the surface of the solid by the whirl projection, in some particular case, where the supply of fluid could not be brought, or could be brought only by a very frictional course, so that the deficiency would indicate the consumption of the fluid in whirl projections. This would no doubt be difficult to do directly, but by restricting the supply of the fluid to form the necessary whirls, by any means, this restriction would possibly produce a strong impulse to draw the fluid towards the static plane of rest upon which it must be motive by rolling contact. This may be perhaps best shown by the principles discussed 51 prop., page 156; where it is shown that it is mechanically possible by whirl systems, which are there described as volutes, to continue a force of projection in a fluid without great loss of velocity by constant disintegration of a part of the flowing force to form the involuted system of the whirls.

g. It will also be clear that if in the projectile whirl system of a solid and a fluid, whirls are thrown off in the fluid and carried forward by the solid moving longitudinally, as a necessary friction-saving motion upon resistances at parallel distances, that the friction of a moving plane will be greater than that of a moving fluid in a current of like extent that could supply freely such a whirl system as that shown 51 prop. *c*, from its own mass.

h. Accepting the above, if we move a solid body having a plane under surface over and parallel to another solid plane of more extensive area, between which planes there is intervening an equal depth of liquid; the moving body with its adhesive liquid would then form one of the planes capable, as previously discussed, of engendering a projectile whirl system in the intervening liquid, tending to throw its whirls as shown in the last proposition, Fig. 142, out of the motive system at the back of the solid projection. So that if, as is the case, the projectile solid is unable to supply from its mass the necessary fluid whirls to secure rolling contact, the intervening fluid will be projected backwards or some other very frictional mode of supply must be induced, as that of a gliding motion.

i. By previous theory I have assumed in all fluid motions that sliding would be excessively frictional and that rolling contact would be almost the only possible form of motion for the movements of fluids upon themselves or upon solids. Therefore if the system of rolling contact is persistent in the case proposed, for the contact of the liquid upon the resistant surface, the *direction* of such motion will be persistent also, excepting as it is withheld by restraint or by some other force.

j. In this manner we might theoretically separate the effects of the action of the two solid surfaces between which a resisting fluid is placed, and assume that so far as one of the surfaces of the moving solids is concerned, that the intervening fluid would supply all necessary matter for integration, thus forming the whirl system of rolling contact, as assumed to be the least frictional on *one* of the planes. In this case, supposing the intervening liquid disintegrated by rolling contact upon the lower surface of resistance, that is, upon the bottom plane, and that a motive solid is placed above this, which will evidently be unable to supply liquid for this disintegration, it is then quite clear that if there is no direct means of supply of the liquid for whirl production, that the upper plane, that is the solid, would be forcibly drawn down to supply the deficiency of fluid matter to form the friction whirls. We ought therefore to be able experimentally to obtain evidence of the truth of this proposition by the motion of any solid parallel surface moving at a limited distance over another like surface with any intervening fluid body whatever filling the interspace.

k. To obtain experimental evidence of this proposition we may assume the following conditions:—that a boat carrying an adhesive liquid with it, and moving along a stream would engender a system of projectile whirls by constant friction upon the bottom of the stream, upon principles already discussed; and that the whirls thrown off from the liquid which was adhesive to the bottom surfaces of the boat upon conditions similar to the volutes of 51 prop. would be retained with certain force upon the bottom surface of the stream, and if there were no frictionless exterior means of supply of water, that the boat would follow the projectile direction of the whirls, as a part of the flowing force of the whirl system, and be drawn downwards.

l. For experiment the following was arranged:—A boat-shaped log with vertical sides and a perfectly flat bottom, of 18 inches in length, 5 inches in width, and 3 inches in depth was made out of a solid piece of wood. The top and bottom surfaces were therefore flat and of equal dimensions. A piece of sheet-lead was nailed upon the entire bottom surface so that the log-boat was in stable equilibrium within the surface of the water. The upper surface of the boat floated at about half an inch above the water line, horizontally. A piece of string was now attached to the upper surface of the log-boat by means of a nail, so that the boat could be drawn along the trough. Placing the boat upon my trough, 21 feet in length, previously described, which contained 8 inches in depth of water everywhere throughout its length; I now held the string at an angle of about 30 degrees to the surface of the water and drew the log-boat along with it, after me, at walking pace. In observing the boat I found that the upper surface immediately sunk to the level of the water, notwithstanding the force that I used to draw it along, was tending to pull it *upwards*. When I quickened my pace a little the entire boat sunk beneath the surface of the water. The point by which the boat was drawn in these experiments was directed to the centre of inertia of the boat for the angle taken, so that the log-boat remained approximately horizontal throughout its movement.

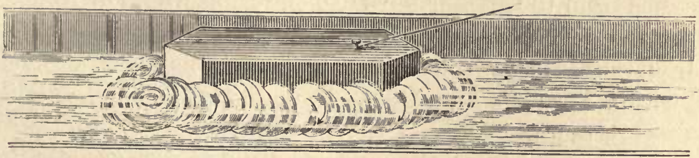

Fig. 144.—Ex.—Evidence of Projectile Whirls about a Solid.

m. The above diagram was drawn to represent this system of motion according to my theory. The boat is shown in the engraving much too high above the water, and the whirls do not appear to extend to the lower surface of the trough which they should do. I suppose my sketch for the diagram was not very clear, but anticipate the engraving will sufficiently indicate the principle. This motion of the boat downwards would be entirely anticipated upon the principles discussed for the directive influence and energy of projectile whirls by lines of rolling force engendered by its motion under the conditions given, but I do not see how it could be accounted for by any other system elsewhere proposed. If the water could slip or glide upon itself or upon the boat, gravitation being constantly equal, the boat would move *upwards* towards the direction that it was pulled by the string, and not sink *downwards* as in the experiment given.

n. It may be observed that the restrained curves of fluid force discussed above, would have their functions materially changed by any other form of boat than one with a flat bottom and vertical front. Thus suppose there was a directive angle in front as we observe actually in ordinary boats, the surface water would then be deflected downwards by this angle, and the deflected water would supply the necessary material for friction volutes (51 prop.). In this case quite the reverse effects would be observed, for instead of friction of contact consuming and throwing backwards the fluid between the frictional planes, it would be introducing fluid between them by continuity of contact upon the lower surface of the boat, and thus cause the directive angle to draw in water to maintain the whirls upon which the boat would ride, and be therefore *emerged* instead of *submerged* by any increase of velocity. In this case also, the resistance from the greater power of inducing whirls of rolling contact by the deflected plenitude of material at the head of the boat, the friction would be considerably reduced, that is if the velocity were within the ratio of possible accommodation in the liquid for this form of motion.

o. By the above principles, to attain the least friction for the impulsion of a boat of a given volume, it should be built in no part parallel to the direction of motion but rather of ellipsoidal form. The whirls formed at the forward parts should complete their projections on the backward parts with re-entering curvature. This will also meet conditions I will point out further on.

99. PROPOSITION: *That the lateral whirls upon a moving plane in a fluid will be developed to the greatest amplitude at a certain moderate velocity. That at lower velocities the fluid will find accommodation for whirls of smaller projection. That at higher velocities the projectile forces will separate the cohesion of the fluid system and cramp the whirls. The resistance will be to the velocity inversely proportional to the amplitude of the whirls.*

a. That an infinitely jointed system of matter would move by very small forces the near parts, and by moderate forces the distant parts, was discussed by the conditions of a hanging chain (53 prop. *b*, page 160), wherein it was shown that a small force when first impressed on the lowest link of the chain would move the lowest link only, but a larger force would set the whole chain in motion. It was further shown that a force at high velocity would also move the lowest link, that is, it would move upon its nearest free part or axis, as the inertia of the near part would be entirely overcome without materially disturbing that of the whole jointed system. The like conditions were also shown for a more rigid jointed system as a pile of single bricks in 53 prop. *d.*

b. Principles equivalent to these may be assumed to hold good in the generation of lateral whirls about a solid plane moving in a liquid longitudinally; so that in the impression of intense forces the system of possible accommodation in the fluid may necessitate separation of the molecular parts from their systems of continuous jointing, to produce a motive displacement that may contain certain elements which are nearly equivalent to fracture.

c. The whirls from a projectile surface moving laterally in a fluid will be generally developed proportionally to the force of projection, and the freedom from lateral resistances conjointly. The friction of resistance to the fluid will be for the most part distributed in area, that is, where the whirl system can take the greatest amplitude; but if the system of rolling contact can be equally developed without great friction in a smaller area, there will then be less inertia of the surrounding fluid to overcome, in order to induce whirl motions, and the entire motion will be also less frictional in such slow projections.

d. For the conditions of the slow projection of a plane moving longitudinally in a liquid, we may be assured, that as the solid will possess only a certain length of plane, the *flow*, as we may term it, of the liquid that is adhesive upon this plane will only engender a motion for a certain distance laterally in an extensive fluid; for this contiguous fluid, considered as an infinitely jointed system of matter, will move under slow motions by accommodation within its near parts, within such space of diffusion as that in which the distributed projectile force will be best able to overcome the inertia of the resistance of the mass of the static fluid. Therefore we can imagine that in the projectile whirl system of a solid moving in an extensive fluid, if the movement be slow the whirl projectile force will be *weak*, and not widely distributed against the resistance or inertia of the liquid; and that under such circumstances, the resistance will be *less*, from the smallness of the whirls necessary to be produced to ensure rolling contact, that is, than it would be if the projectile amplitude of the whirl were such as to carry the projectile force into more extensive space, to move a greater mass of the contiguous fluid.

e. Upon this principle of lateral whirl projection, the projection of a solid may be resisted more or less than in proportion to any direct function of velocity-projection of the solid simply, or as the amount or weight of fluid to be displaced; and thus the velocity of a boat in relation to the propelling force used to project it in the water, may be more or less than any given function of the force employed. This condition of development of small friction whirls here proposed, will be the condition of any slow constant projection when small disturbance only is evident, as in the projection of a boat at low velocity on smooth water, when the surface of the water is undisturbed at a very short distance from the boat.

f. We may now consider another case, given in the proposition, of projection entirely different from that discussed above, where the velocity at which a solid moves in water may be such that it will overcome the *cohesive force* of the water entirely, so that the water will be *fractured*, or separated into such forms of motion as philosophers have considered for the conditions of gliding motions. I do not believe such forms of motion as *gliding*, can by mechanical means be produced about a ship by known forces possible to be applied. But certain high speeds may approach the principle of this form of motion by restraint of the whirl-forming space, as was shown in principle by instant impressions on our hanging chain just referred to, where a system moves by intense force on its nearest joint, so that this near motion may be taken practically to be the same as a motion of projection for very high velocities, such being also a motion in degree equivalent to fracture, or release of instant near cohesion, that the general *principles* of *fracture* may in a certain small degree hold; and that as far as the fracture is concerned the body will be free from all further resistance, and its velocity relative to this fracture be as of a free body. But if we imagine partial fracture to occur at very high speeds by lineal displacement, the projectile body will be only *more* free for extreme velocities, as it will encounter less relative resistance by the amount of partial fracture of the cohesion of the liquid, than will occur in moderate velocities in which very extensive whirls are developed by the continuity of contact of cohesion in the system.

g. If we admit the partial conditions of fracture, or the possibility of this as equivalent to a restricted case of gliding, then whirls engendered by a motive surface acting upon the resistance of a fluid may, from insufficiency of time, be imperfectly developed in the distance or in such a manner only that their curves would be cramped, and return by a small radius to the surface of the moving body. Then in the near distance the fractured parts of the fluid system may be assumed to be separated from the fluid mass, and the velocity of the moving body suffer less restraint proportionally to the amount of fracture produced, which will be really in this case less inversely as the radius of development of friction whirls.

h. Assuming as a separate condition the movement of a solid in a fluid to engender forms of motion in the fluid that are the least frictional, at the velocity at which the solid moves, and that induced motions always have a tendency to continue, so that long surfaces will offer less proportional resistance than short ones per area; then upon purely theoretical grounds, to recapitulate the whole matter considered, the value of side resistances might be estimated as follows:—

Firstly, if the motions be slow, whirls of small amplitude would be engendered, causing small displacement of the parts of the fluid upon themselves, or if very slow, such ample time for accommodation would be given, that the solid might be conceived to roll forward upon very small masses, or even upon molecules only.

Secondly, in motions strictly within the velocity of the continuity of the power of adhesion and cohesion of the fluid, whirls would be engendered of extensive proportions, the evidence of their existence being visible at the liquid surface by the projection of ridges, or waves formed laterally upon the surface; which ridges by reaction partially, form the planes of resistance upon which the whirl system afterwards makes rolling contact. This velocity would be proportionally very frictional from the volume of matter moved beyond that discussed above.

Thirdly, the possibility of a high velocity at which the whirls would decrease in amplitude by fracture, or cramping from want of time to complete their forms, so that they spread out less relative resistance in extension of the motive system. The following diagrams will express types of these ideas, of which, on account of some disturbing causes, I am not entirely confident.

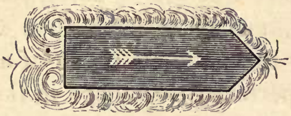

Fig. 145.—Diagram—Slow Projection.

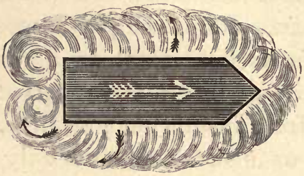

Fig. 146.—Diagram—Moderate Projection.

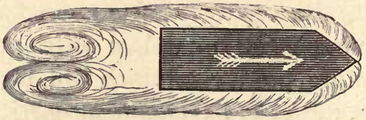

Fig. 147.—Diagram—Rapid Projection.

i. Fig. 145. *Velocity of slight resistance.* Motions of rolling contact having ample time for fluid accommodation in small space, to engender thereby a *close system of projectile force.*

j. Fig. 146. *Velocity of highest relative resistance*, for the mass of fluid displaced; the cohesive force upon the vessel being sufficient to engender contact whirls to the greatest possible distance. *Broadest system of projectile whirls.*

k. Fig. 147. *Velocity of diminishing resistance or of fracture of cohesive force*, the whirl system being incipient for want of time, the cohesive force partially *breaks* the liquid, and the vessel is set relatively free in proportional to the amount of fracture; the mode of motion approaching, in this case, to slipping or gliding. *Cramped system of projectile whirl force. Friction constantly and relatively proportionally less as the velocity increases.* Some evidence of relatively diminished resistance with velocity has been found experimentally in the resistance of air by Smeaton.

100. PROPOSITION: *That the system of whirls established about a solid moving in a fluid are projectile forces which can only be brought to the equilibrium of rest by equal resistances.*

a. It was shown in 34 prop. *d*, that the contiguous liquid forms a part of a projectile force with any solid moving within it by adhesion; as demonstrated by the stoppage of a boat in Mr. J. Scott Russell's experiment before alluded to. If we assume in this case of a projectile boat, that whirls are engendered in the surrounding liquid by that portion of the liquid which is carried forward by adhesion to the boat, these whirls will then, by the uniformity of the surrounding resistances, finally assume *set forms* which will be those that are the least frictional in movement, and these forms will therefore remain persistent. Now by the continuity of such motions in which the uniformity of resistances assume a sort of motive quiescence, the whirl forms could not be *observed.* But if they are really projectile forms as here assumed they could be made evident by offering greater resistance at some part of the induced whirl system. This principle I have endeavoured to investigate experimentally as follows:—

b. Using the trough before described which measured 20 feet long, 11 inches wide, and of the same depth; and a log boat made of a piece of solid wood 18 inches long, 4 inches wide, and of about the same depth. The trough was placed in a level position with about 8 inches of water in it. I drew the boat along the trough by a piece of string, at walking pace, for ten feet or so, and then caused it suddenly to stop, by means of a bar placed across the trough above the water line, with which the head of the boat came in contact. In this experiment the side whirls continuing their induced momentum,

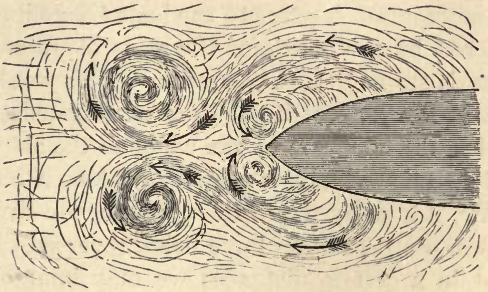

Fig. 148.—Ex.—Reversal of Lateral Whirls.

derived from the projection of the boat, were thrown off by the sudden concussion, and projected forward with some velocity. This projection of whirls appeared very evident by careful observation, as the whirls formed when thrown off formed two distinct whirlpools, at a distance of about fifteen inches in front of the stoppage; where previously to the stopping of the boat the water was quite smooth and undisturbed.

c. The adhesive system of water upon the boat appeared to unroll and reverse the whirl direction as it existed upon the side of the boat when in projection. This reversal may be made quite apparent in some cases by observation of the surface motions which in this case somewhat resemble the following whirls, before described, in their natural forms for the near part; but the forward projection encountering more resistance the projected whirls are afterwards thrown together, bifurcate and form outward and inward revolving whirlpools; the general directions of which are shown in the diagram, Fig. 148, on the last page.

d. The volume of the whirls about a vessel will be proportional, other conditions being equal, to the means of supply of water to them if this supply be effected by some less frictional mode of motion in the water than would be occasioned by a loss of amplitude by cramping the area of the whirls. In the case given 98 prop. *l*, the whirls are cramped by want of facility of supply, and the motion of the boat is proportionally resisted.

e. The mass of water in an entire whirl system moving about a boat at a given velocity might possibly be approximately measured by the momentum it gives to the boat above its mass into the resistance offered by the inertia of the mass of water it encounters when the boat is stopped by any means. This would possibly be as the volume of Mr. J. Scott Russell's wave of translation, described 34 prop. *d*, plus, the adhesion of the water to the sides of the boat. The same might be ascertained possibly by a boat moving forward at a given rate, urged by a propelling force, the motive value of which is known, the propelling force being reversed, until the vessel comes to rest in a certain distance; we may imagine that in this distance, the directive influence of whirl force about it will be neutralized. Let the vessel move from rest this same distance, and ascertain the propelling force used in volumes of elastic force of steam, or otherwise, then the difference observed will be from the momentum of the whirls; the resistances of the inertia of the water overcome being approximately the same in both cases, as an equivalent motive system is induced in reversing, to that of starting.

101. PROPOSITION: *The whirls engendered in a fluid by conic projection of the forward and side parts of a moving solid are forces set in motion by the forward projection of the adherent fluid which are restored with equal force in the opposite direction in other surrounding parts of the fluid or upon the solid to bring the contiguous fluid system to equilibrium after the solid has passed.*

a. The principles of action and reaction suggested in this proposition are important in suggesting the best form of a vessel to diminish resistance of the water in which it moves; and for the consideration of forces exerted upon rudders by which steering direction is given to the vessel.

b. Applying this proposition for the consideration of the best forms of vessels, the circuit of projection of the whirls will be representable by force lines, which originally projected by the vessel, will by their circuitous motion, return again and impress like forces, or a part of them, in an opposite direction upon a backward part of the vessel, or these forces may be thrown in collision with opposing whirls behind the vessel, and exert no compensating forward directing force upon it to that used in their projection. It would appear for instance, that if the circuit of the side whirls projected by the head of a vessel had such directive force given to them that they could complete a circle nearly, so as to impress reflex whirl force directly upon the stern surface; these whirls might then upon return contact have motive direction capable of urging the vessel forward. It would also appear from this mode of motion that there is a possible angle or curve for the stern of a vessel in which such whirls would meet the surface most directly to the forward motion of the vessel. On the other hand, if the whirl is cramped or overflows the vessel, and does not meet the surface in a direction capable of propelling the vessel, it may then otherwise cause a dragging action by a general traction upon the water behind, which forms a part of the unit projection.

c. We may have inference when the right form of a vessel is attained when the whirls formed at the sides of a vessel have freedom to exert their forces on the lateral parts by reflex action, as the liquid surface will then be little disturbed from its level plane. The force of propulsion of the vessel being wholly used in the projection of the whirls that are necessary for the displacement of the bulk of the vessel to ensure rolling contact.

d. There is another point that I think very important, namely, the principle upon which the back whirls by reflex action act upon the rudder of a vessel. I definitely assert in this particular case that the back whirls *act*, because their action may be very clearly observed in the surface motions at the stern of any moving vessel; especially in cases where, in the construction of a vessel, the lines have been well considered with regard to velocity. These stern whirls, when the vessel is going at full speed, appear as large discs, before described, which follow close behind the vessel near the position where they are thrown off after a complete revolution in a constant whirling motion, induced by the lateral surface; the evidence of which is clearly observable in the track of the vessel long after it has passed, not only by the absolute circular surface motion engendered, but by the relatively less vertical surface motion of the track, the surface waves being levelled as it were by the induced horizontal motion of the whirls which retain their whirling motion for a long period.

e. In a very important paper by Professor O. Reynolds read before the British Association in 1875, in reports, page 141, "On the Steering of Screw Steamers," Professor Reynolds has discovered the effects of the rudder, in cases of stopping or reversing the engines, by direct experiments with models, which experiments are of the greatest possible practical value. In the cases offered by Professor Reynolds I am not aware of any cause being attributed to the effects observed, or whether these cases rest in the *terra incognita* of fluid motions generally. But it appears to me that they can very well be fully explained by the principles of whirl motions which in this particular case may be made visibly evident by careful observation of outward forms only, without reference to any theoretical consideration. I may first state the important practical conclusion drawn by Prof. Reynolds, which he states in the three following laws:—

"1. That when the steamer is going ahead she will turn as if she were going ahead whether she has steerage way or not.

2. That when the screw is reversed the rudder will act as if the vessel were going astern although she may be moving ahead.

3. That the more rapidly the boat is moving in the opposite direction to that in which the screw is acting to drive it, the more

nearly will the two effects on the rudder neutralize each other, and the less powerful will be its action. It would be reasonable to suppose that a boat may move fast enough to overcome the effect of the reversal of the screw; but this was not the case with the models."

f. Taking these facts to be explained by principles shown in the action of stern whirls upon a vessel, we find that in the first place, when a vessel is moving straight ahead the stern whirls are compact complete systems of motion, entirely motive in one direction about their centres, one large whirl being located near each side of the stern of the vessel. Therefore to move such a whirl it would require a force sufficient for its complete displacement that would displace an equal volume of the water. Further, by the induced rotary motion of the whirl if it were in any way pressed as by the flat side of the rudder, it would be constantly rotating upon the plane of pressure, and would act upon the surface of the rudder as an elastic body when compressed; the rotational force by tangential action producing the effect of elasticity. Therefore in the direct motion of the vessel the rudder when moved towards one side would be pressed by one stern whirl, and would by this pressure move the stern of the vessel by its local force and elasticity, to the opposite side where the whirl system by the direction of the rudder would be less active as a pressure.

g. Now if the vessel be stopped the whirls will be thrown forward and reversed. This stopping could not occur directly upon a vessel except in case of collision, but if the screw were reversed the flow of water in the tractional system of the whirl would project it towards the head of the vessel, and by this action the whirl would be thrown so far forward that it would not possibly be any longer active upon

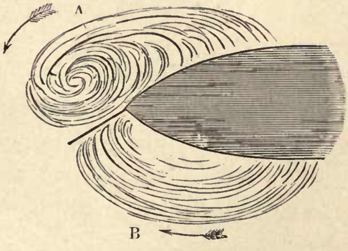

Fig. 149.—Ex.—Action of Lateral Whirls on Rudders.

the rudder. The general principles of these actions upon a rudder may be shown in the diagrams.

h. Fig. 149. Represents the vessel going at an equable rate, the rudder being turned perpendicular to the direct impulse of the whirl as shown on the lower side of the diagram at B, the directive force of this whirl will therefore press the vessel over. In this case the whirl represented at the lower part of the figure may be conceived to be completely displaced by its pressure upon the rudder. The freedom given to the whirl shown at A, in the upper part of the diagram, permits it to pass further astern. This whirl A may be thrown at such a distance astern that it may by its reflex curvature even assist the whirl B by its direct momentum in pressing the stern of the vessel over.

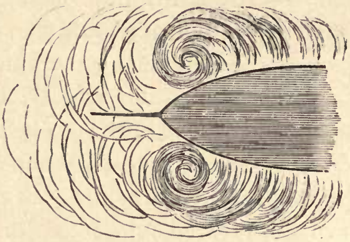

Fig. 150.—Ex.– Non-Action of Lateral Whirls on the Rudder when a Vessel is stopped.

i. Fig. 150. Represents the vessel when stopped, in which case the whirls are thrown forward. The same effect would be produced by the screw of a steamer being reversed; the whirls being now entirely inactive as a pressure upon the rudder.

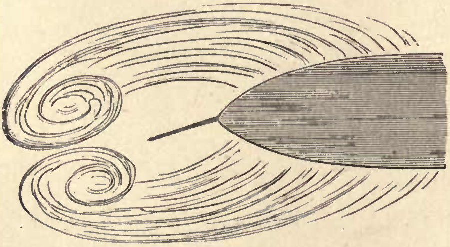

Fig. 151.—Diagram—High Speed the Rudder misses the Direct Impulse of the Lateral Whirls.

j. Fig. 151. Represents the case in which a sudden impulse is given to the vessel, the whirls are thrown backwards and miss direct impulse upon the rudder and the stern of the vessel, the line of forces still being directed towards the rudder, which would not receive much impulse from them. This is possibly an imaginary case, as such a high velocity as required cannot be attained; it is given in order to complete the system.

k. It is generally conceived, upon perfectly logical grounds, that upon gliding principles of fluid motion, the rudder simply directs the lateral stream lines from which it receives its impulse. If the rudder of a vessel were active in this manner, its proper place would be at the head of the vessel and not at the stern; but if the force on the rudder be derived from the pressure of the stern whirls, there will then be one general law, namely,—that the momentum of the moving liquid upon the rudder will depend entirely upon the direction and momentum of these whirls upon contact, such whirl direction being engendered from any cause.

Helical whirl deflections.

102. PROPOSITION: *That a floating solid moving horizontally in a liquid will deflect its friction whirls upwards in surface motions and produce waves. Such whirl motions by after deflections will become helical.*

a. In the several propositions in this chapter I have taken into consideration special directions of whirl motions about a projectile solid floating in a liquid, as being projected in a plane. I have followed the matter in this manner because demonstration was much more easy for a special case than for general complex principles; nevertheless, we must assume that from the nearly equal densities of water, for instance in every direction upwards, downwards, and laterally, that the resistances in all directions from the inertia of the fluid will not be materially different, and the same will hold for every divergence from the point of projection; therefore the whirl-generating force will be approximately equal throughout a *circular* area opposed to the projection in the liquid. This principle was fully discussed under conditions of fluid projection 68 prop., wherein it was shown that a whirl cut by a liquid surface would continue its perfect form beneath the surface until it was deflected upwards by the minus resistance of the aerial surface.

b. Although by the above conditions, the resistance to a floating solid, by the *inertia* of the resisting liquid, will be equal in all directions within the liquid, the mobility of the system will be very unequal, as the aerial surface will receive very little relative support by contiguous adhesion of heavy mass, compared to the deeper parts, and any form of induced motion once deflected to the surface or areas of less resistance will continue in this path of deflection; so that any open space that is necessary in the liquid for motions of accommodation for the friction whirls to be produced, must necessarily continue the projections into such space. In the projection of a floating solid body in a liquid that is adhesive to the solid we may assume as before that every point of adhesion will be resisted by a conic mass of the liquid at a certain angle forward of its direction of motion up to 90 degrees 58 prop. Now if we take one angle of resistance only, say that of 45 degrees, and assume the rigidity of the liquid such that it will resist the impulses of a flowing liquid at say one yard distance, then at every distance of one yard from the submerged part of the moving solid, at an angle included within 45 degrees of the forward direction of motion, there would exist an area of compression in the liquid.

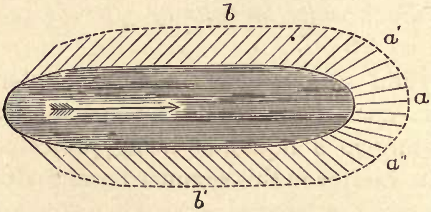

Fig. 152.—Diagram of Adhesive Projection about a Moving Body in a Liquid.

Let the equal lines surrounding the solid represented as in the model boat in the diagram, Fig. 152, be of one yard in length, then the conic resistance of the boat may be assumed to press back the liquid to this line, so that there will be an area of compression represented by the dotted line surrounding the boat in its horizontal plane at one yard distance, the resistances being assumed all equal, the surface water would be pressed back and elevated by the compression at this distance.

c. If we now take the vertical conditions of resistance by the pressures engendered in the motion of the boat, on the same principles as just discussed, then by the same arguments it will follow that at a certain line below the water as represented by BB′, Fig. 153, it may be assumed that all resistances will equally take direction of 45 degrees as *one* angle of resistance.

Let A A′ represent the surface of the water, B B′ a line below the surface, and C C′ a line at an equal distance deeper in the water. Now taking the same angle of 45 degrees as before, about this line

Fig. 153.—Diagram of Resistance about a Moving Vessel.

and the same lengths of radii; the pressing force from adhesion to the plane B B′ may be assumed to be upwards and downwards. In this case as all the points of pressure to the surface line A A′ would have no other resistance than from the air above, and all the points of resistance at the line C C′ would have the resistance of the water supported by continuity of lines to the land surface upon which the water rests, the axis of the lineal series of pressures represented by the plane B B′, by the minus resistance at the aerial surface, would then cause this plane to be deflected upwards to find equilibrium at some distance outwards from the adhesive surface of contact upon the boat; so that the deflection of the water caused by the projectile force of the boat, would thrust this plane of direction outwards, and throw it upwards towards the point A′ in the diagram.

d. If we now return to the diagram, Fig. 152, upon the plane, we find that the pressures at *a a′ a″* become pressures outwards and upwards by deflection of consecutive pressures that the boat meets. The pressures at *b b′* are pressed upwards but more outwards by the same forms of motion. Now in the above diagram the flowing force of the water meeting the projection about A will encounter the lineal resistances of the forward parts of the water, and there will be in this meeting part a hydrostatic pressure; the constancy of which will further press the elevated water to the most free part of the surface, which will be again outwards from the boat. By this cause a wave will be formed near the head of the boat directed outwards from it.

e. If we now consider the constancy of resistance by the opposing force of the liquid to the projection of the solid, on every point of the direct line we have taken to represent axes of cones of resistance, we shall find this line will be deflected to whirl forms; for instance, the lines of conic resistance directed to A′ in the previous diagram, Fig. 153, will decrease in force outwards and will by this means be curved over at this weak point by the action of the flowing force upon it; there will therefore be a deflection, to the form shown below, in front of the boat, and this deflection may even curl over at the highest point. Now, as the upward direction of motion will be by the hydrostatic pressures also outwards, by the constancy of resistance of radial forces, these forces will project the water by continuity of the resistance from the front, directly backwards and outwards, producing surface waves which will be by composition of these motions *helical* at the exposed edge, as shown complete in the sketch diagram, Fig. 155, below.

Fig. 154.—Ex.—Forward Projectile Wave.

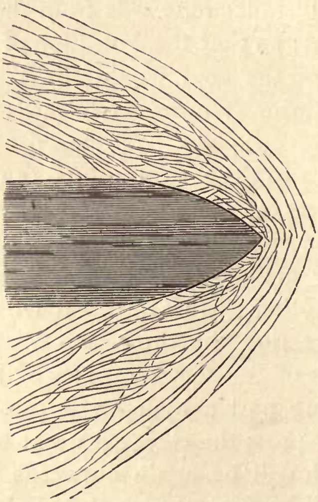

Fig. 155.—Ex.—Helical Deflection.

f. In this manner a complete *helical cylinder* will flow outwards from the bows of the vessel, and this will induce other following separate rotary systems, so that one outward flowing wave will generate another upon principles to be discussed further on. These helical motions will generally compound with the system of whirls discussed in previous propositions, so that the whirls themselves will be in most cases in a certain degree compounded of helical forms of projections.

103. Remarks.

I regret that my opportunities have not permitted me to follow this matter of free solids in infinite fluids to a further extent, especially as the subject appears to me to be full of useful results. I intended to take a voyage for observation of free liquid motions, but my bodily constitution is such that my mental energies are quite suspended on the liquid element, so that these investigations of practical results I must leave to others. I hope, however, that I have adduced sufficient principles by following previous propositions, to open a way which will show that the methods and formulæ generally devised for the resistance of solids by fluids, by some form of slipping motion of the surface water in contact, is probably erroneous. If the measurable adhesion of water were really to be entirely overcome, the value of which adhesion is shown in Mr. J. Scott Russell's experiment 95 article *d*, it appears to me, that available mechanical forces would be insufficient to move a floating vessel, at any rate, by such forces as could possibly be contained within the bulk of the vessel. But in a liquid system it is quite clear that principles of accommodation may occur, that may permit rolling contact of a not very frictional nature which will avoid the necessity of assuming slipping motions in any form.

CHAPTER IX.

DIFFUSION OF FLOWING FORCES IN FLUIDS. TERMINAL PROJECTIONS. RESISTANCE TO PROJECTION IN FLUIDS DURING DIFFUSIONAL MOTIONS.

Diffusion by interference.

104. *Diffusion of fluids in like fluids—general principles.*

a. The principles discussed in the fourth and seventh chapters, with the experimental evidence given, assure us that a flowing fluid will dissipate its energy upon lateral resistances, maintaining the central axis of projection at the highest velocity. We further have evidence that by the necessity of rolling contact, if we accept the principles offered in previous propositions, there will be division or intermittent action, deflecting the direct energy of the flowing current at its planes of contact, either with solids or with relatively quiescent parts of the fluid, into whirls or looped systems. These deflected parts have been shown to produce a retrograde motion in the lateral parts lineal of the current, and such retrograde motion we may imagine, reacting in like manner to ensure rolling contact, would produce again a retrograde action in lateral parts to its new direction which would tend to restore the current to its former direction. So that there is a possibility of deflection and redeflection by which currents may be flowing in contact in opposite directions in lineal sections, their motive forces being nevertheless derived from one direct projection. We have further evidence that by lateral resistances, whirl systems will be developed to the greatest extent in the most free areas. Where such areas are large the circumferential parts of the whirls may be practically considered as direct flowing forces, and thus meet all the active conditions of direct currents, and as such, divide by tangential contact into separate whirl systems, thereby ensuring continuity of rolling contact. So that lateral resistances in open areas become sources of separation or of diffusion of direct flowing forces.

b. Upon the above conditions also, direct head resistances will cause bifurcation of central forces in every radial plane from the axis of the current, and the deflected parts will again bifurcate in proportion as they may again act as direct forces.

c. If direct projection of a fluid be made within a like perfectly quiescent homogeneous fluid, the whirl system induced remains very complete, but if there be interference or other direction of motion in parts of the fluid, the result is generally a diffusion of forces in which the projectile fluid is *split* up into many whirl systems. These general conditions I will now separately consider.

Lateral Diffusion of Whirl Systems.

105. PROPOSITION: *Every part of a fluid in rotation will have a tendency to throw off its circumferential parts into the open surrounding fluid; these rotary parts being withheld by the cohesion of the motive system only. Thus a whirl in an open fluid will separate at its circumference, and engender like, although smaller, whirls by parts thrown off tangentially. These deflected parts will afterwards move in the reverse direction to the original whirl by rolling contact upon the surrounding fluid, and the reversed parts may again divide by like tangential action, constantly, until the flowing force is entirely diffused.*

a. The most perfect self-contained whirl system will be that projected by a single impulse, in which case, a projectile whirl-ring will be formed, the involution of which will maintain the circumferential parts very compactly (67 prop. *d*, page 197). It is nevertheless evident that the exterior circumference of such a system, except in so far as the cohesion of the fluid withholds its matter, must encounter conic resistance in every free space of the same kind as that shown in many cases, as in instances given in 82 prop. *d*, page 231. So that if we do not imagine the cohesion of the fluid projectile system to be very great, the circumference of a rotary whirl of such a system will be constantly disintegrated into smaller whirls, that will be split off, as it were, from the circumferences of the larger ones in all more free or less resistant spaces; forming thereby surrounding circulatory systems; and these whirls will again be split into smaller still, if the resisting fluid space be open. This will also be the necessary form of motion to ensure universal rolling contact in the surrounding fluid by principles of whirl or biwhirl deflection. This principle was discussed as being evident in the first series of contact whirls 82 prop. *c* for irregular spaces, and is shown effectively at the edges of the section of a conoid 84 prop. *j*, page 238, as also for the condition of jets and currents in the last chapter.

b. Some approximate conditions to the above I have often observed in the rising tide of the Thames, in the inflowing water passing through the arches of London Bridge, where the current is restrained by the piers of the bridge, which cause it to issue with considerable projectile force tangentially to the quiescent water behind the piers, upon the side of the bridge that is protected by its position from the force of the direct stream. Here the current moving past the quiescent lateral waters, engenders a complete system of hundreds of little whirlpools and still smaller whirl dimples, which are each the centre of a complete whirl whose system often extends in depth to the bottom of the stream as a cut whirl-ring (79 prop., page 224). This complex whirl system spreads out a hundred yards or so beyond the piers, each whirl forming a comma-like depression upon the water surface, the general superficial appearance of which I endeavoured to delineate on the spot from observation of which the figure below is a reproduction.

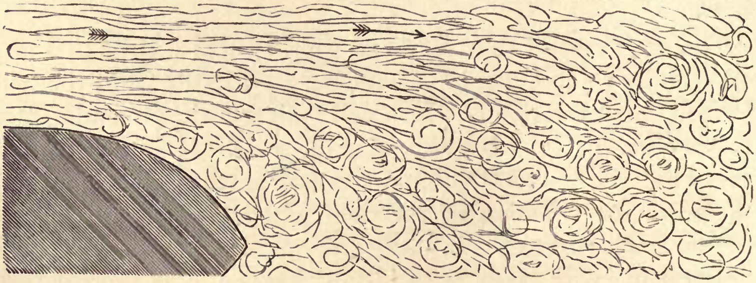

Fig. 156.—Ex.—Complex Lateral Whirl System by Projectile Current through one Arch of London Bridge upon rising Tide.

c. The whirl dimples in the above described phenomenon, if carefully observed, will be found to be generally in pairs (biwhirls), the separate dimples of which constantly diverge from each other further and further as they are carried down the stream by the current until they finally die out in plate-like discs (whirl discs), or separate into smaller systems. There is further evidence of the continuity of tangential rotary action upon the quiescent waters where the tangential force is insufficient to form *whirl dimples* or *whirl discs,* shown in the evidence of a general horizontal rotary motion, whose visible effect is to make the rippled surface of the water more calm for several hundred yards down the stream.

d. In flat wide coast lines of shallow water, at the entrances of rivers into the sea, the vertical surface motions in whirls cause waves or ripples present gradually to disappear, and there is left generally a system of horizontal bifurcation and general horizontal rotation, visible by the presence of whirl discs, caused by the tangential action of the current.

e. The results of this horizontal diffusion of flowing force being in this case, as in all other observable like cases, that the surface of the water is calmed by the horizontal action of the whirls, so that the wind has much less influence to cause ripples or waves upon the surface than it would have even upon still water. This exactly resembles the conditions before pointed out, that occur after the passing of a steam-vessel over a wavy sea, wherein horizontal whirl motions are necessarily induced which subsequently move by tangential division, so as to leave a broad less wavy path in the ocean visible often for many miles.

Capital Diffusion.

106. PROPOSITION: *That a projectile, or a flowing force forming the head of a current before the continuity of projection can rotate a large volume of the resistant fluid laterally, will divide its direct current by constant bifurcation, until the projectile force becomes in equilibrium with the forward resistance.*

a. The conditions proposed, under which direct flowing fluids will bifurcate or become deflected by head or conic resistances, so that the deflected force can react after separation tangentially upon the surrounding fluid as a direct projection, and in like manner again divide upon forward resistance, is a process by which there appears to be brought about a constant diffusion of projectile forces in separate units of fluids acting upon themselves; this form of motion being special to fluid matter. The same motive principles will also offer the smallest conceivable friction to the first impulse of direct projection in extensive areas of resistant fluid matter. It is therefore possibly the manner in which diffusion of forces in fluids is at first mostly brought about, whether for such large masses as I have mentioned 41 prop. *d*, page 129 or for the smallest visible parts or possibly for the ultimate division in molecular systems before direct currents are induced by projections through the resisting fluids.

b. I have observed by evidence of experiment, that the biwhirl division of flowing force would permit continuity of motion in a fluid where the constant force was so small as to be imperceptible. I have taken a weak ammoniacal solution of carmine, the specific gravity of which did not exceed that of water at the same temperature by $\frac{1}{1000000}$ part of its volume. This, when placed gently upon quiescent water, entirely floated at first, but gradually small protuberant nipples were projected by solution. These nipples possessing only a very small plus momentum, due to excess of gravitation in their projection into the liquid system, formed immediately whirl-ring systems, and these again others, by constant action of gravitation until they reached the bottom of the vessel or floated by attenuation midway.

c. If the surrounding resistances in a fluid are constantly equal and there is no separate maintaining force as there is in the case of gravitation projections, just given, to exert its action constantly upon a vertical projection from the central current, a simple whirl-ring or biwhirl system will be the only result; except for such lateral whirls as are necessarily thrown off by principles here discussed as friction savers to ensure rolling contact. But it will only rarely happen in any large mass of fluid that its parts will be in such a quiescent state, and its resistances so equable, that such motions can be preserved in simple uniformity, and if there are any exterior resistances greater than those in the average open fluid, upon these there will be division, even if the general system of whirl action gives an equation of constant direction. In this manner any inequalities of liquid surface as of ripples or waves will engender biwhirl systems in an inflow of water which will separate the whirl system induced into smaller motive units. And in like manner, by inequality of resistance, any broad current entering slowly on an extensive liquid area will have its forces constantly divided and subdivided by the consecutive head resistances it necessarily encounters.

d. Upon this principle a typical form of consecutive bifurcation may be represented as in the next engraving, Fig. 157, in which the lines of flowing force directed from A towards B may be conceived to separate in motive units of direct projection consecutively as resistances are encountered until exhaustion of the primitive impulse.

e. This tendency to constant bifurcation may be very well shown by an experiment of the projection of a very weak current, which we may follow by the same experimental means as before proposed, of rendering the resistance about a projection relatively greater by near adhesion of a liquid to a solid, through restriction of area, in reducing the projection to a thin section as follows:—

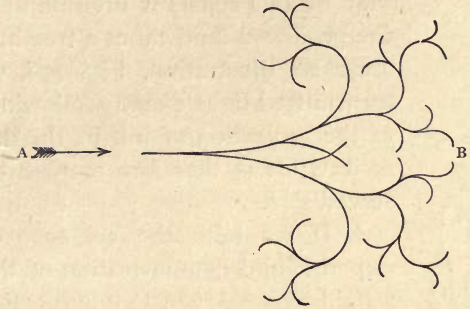

Fig. 157.—Diagram of Diffusion (theoretical).

f. Take two plates of perfectly clean glass, say of about six inches square, and firmly cement a border of card of about $\frac{1}{18}$ of an inch in thickness round three sides of one of the plates with marine glue. Then cement the second plate to the first. By this means a very thin waterproof trough will be formed, open at the top. If we now fill this with clean water and place it upright in a groove in a piece of stout wood, the experimental apparatus will be complete. To observe the desired effects, the trough should be placed before a window to transmit light through it, when the following phenomena of original fluid projection may be observed by the projected liquid diffusing itself by the constant force of gravitation under the resistance of the near surfaces of the glass. We shall need for the experiment in this case a fluid of much greater specific gravity than the carmine just mentioned, as the projection has to entirely overcome the friction of the surfaces of the glass. I have found ordinary black writing ink answer very well.

g. Take a pen full of ink and place this gently upon the surface of the water in the trough. The ink as it descends slowly by gravitation will be found to divide constantly upon the conic resistance which opposes its direct projection, the divided parts having insufficient momentum of projection to move the lateral fluid tangentially, as in cases discussed in the sixth chapter, to produce extensive biwhirls. After the first division, the divided parts again divide, and these again, so that by this constant division and subdivision, the projected fluid takes a tree-like form. The annexed illustration, Fig. 158, was taken by transmitted light exactly following the outlines of the projection of ink in the thin trough described; it is therefore represented of the observed size.

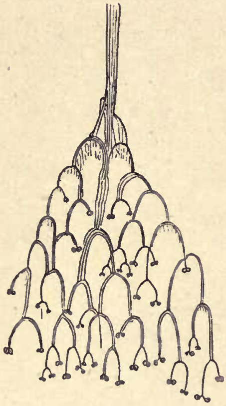

Fig. 158.—Experimental Diffusional Forms.

h. If we take the evidence of the above experimental demonstration of the diffusion of a fluid by weak constant projection, and under equal lateral resistances upon the conditions given, to represent a natural *law of diffusion,* we should then reasonably expect to find that it would be present as an influencing principle in the diffusion of fluid forces in nature. Such, for instance, as we find present in diffusion of liquids in the animal body, and the vegetable also, under certain conditions, possibly more especially in the early development, or *setting out* as it were of the circulatory systems. This we may possibly infer, although, at the same time, we are compelled to admit that the mysteries of vitality are as yet unsolved. But so far as we have advanced, we find the evidence ot mechanical law being followed, as in the perfect mechanical structure of the bones and the application of the muscular system thereto; the structure of the eye, which is as perfect as an optical instrument as it is a vital one; and numerous other instances. Therefore, by like mechanical law, the principle of diffusion here demonstrated may prevail in the formation of the circulatory system within animal tissues, that is, assuming the animal tissue to be at first in a fluid, or semi-fluid state, formed of matter possible of after coagulation to the set form we witness. The intrusion of blood or other liquid with small force would, of itself, by the natural fluid forces here defined, open out through any inequality of resistance, or by a constant pressure in one direction, a *reticulated system* which would not only be composed of canals for future transmission of the fluids, but the forms of such canals would be those that would be the least frictional for a constant flow through them afterwards.

i. Further, this diffusional experiment shows that a path of bifurcation is not necessarily a very frictional one for fluid matter, as it is shown evidently in this experiment to be the mode of projection least affected by the resistance of a uniform surrounding fluid to the direct action of a small gravitative impulse.

j. It is most probable that if the fluid systems of animal life were set out in homogeneous matter by principles here discussed, under certain conditions, this evidence would not always remain under the motive changes of animal life; but it might be anticipated to be evident in such parts of the animal tissue where there would be little strain by muscular action. In following observations as carefully as I was able, I found very generally some evidence of such a principle often or generally apparently differentiated by other causes. Thus as an instance, Fig. 159 is copied by the camera lucida upon the microscope from the injected arteries of the bladder of the shrew. The bladder, although muscular, is possibly in this case not under great or unequal strain after its first development. I omit in this illustration the finer divisions (capillaries) which continue in the like forms.

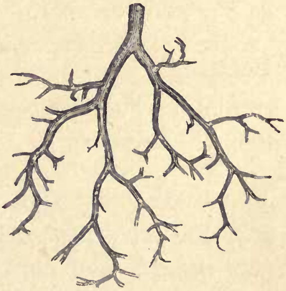

Fig. 159.—Arteries on the Bladder of a Shrew.

k. The same form is well developed in hepatic veins, which, possibly from the quiescent general state of the liver, from its absence of muscles is very little strained or differentiated by further development after generative formation. One complex system of which is shown in the engraving, Fig. 160. The same principle is also beautifully shown in the convoluted layer of the brain of which an illustration is given by Dr. W. B. Carpenter in his "*The Microscope and its Revelations,*" Plate 25, Fig. 3.

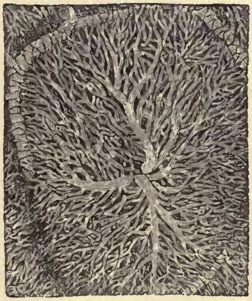

Fig. 160.—Hepatic Veins in the Human Liver.

l. It is on this principle we may possibly trace the true theory of the formation of tumours of the *sarcoma-*

tous class which often attain large dimensions, and contain a perfect circulatory system entirely abnormal to the natural animal form. The same principles may also influence the development of mammillary glands. It is also possible upon this principle that animals increase in bulk by the plenitude of food supplies, by continuity of vascular development, particularly through the influence of inheritance, after several generations of similar growths. The vascular system of a small mammal as that of a mouse being but little finer, *per massa*, than that of a large mammal as a deer.

m. In the above theoretical deductions, it must be strictly observed that for the production of forms entailing constant bifurcation, there must be a constant force in *one direction* in the system such as a *pressure*, or as the *influence of gravitation*, or there must be by some separate cause, an inequality of density, or of resistance in the resisting fluid or surface; for if the projection is final and the surrounding fluid perfectly equal, a unit projection will produce a single whirl-ring only.

Terminal projection of weak currents.

107. PROPOSITION: *That the lineal central area of a current by its higher velocity will maintain its projection longer than any lateral more resisted part. Therefore a terminal projection will produce an annular whirl within the volume of its projectile mass. Such a terminal projection will generally swell out as an elastic system until it comes to rest.*

a. The small whirl system above defined we may term a *terminal whirl of projection.*

b. If a liquid with excess of specific gravity over the surrounding liquid be projected by its weight, then conic resistance will divide the axis of projection as before shown, and move the projectile liquid through whirls of a certain radius by the constant impulse of gravitation; but if the projectile force be too weak for this continuity of projection, it will then, by encountering relatively greater head resistance, come to rest. Therefore the whirl, instead of forming a complete free spiral as in cases given for projections of greater force, as taken for instance in 65 prop., page 192, may complete its projection by coiling spirally within its own terminal mass by continuity of the greatest motivity in the central axis of projection.

c. This terminal motive form may be observed incipiently in the projection of ink in water between two plates of glass described in the last proposition *g*, at the instant of time before the bifurcation of the extreme points of the divided currents. I have produced the same effects in solutions of gelatine, albumen, and other homogeneous resistant fluids. The illustration in Fig 161, A, is taken from the terminal projections in the same ink experiment as illustrated in Fig. 158.

d. That this terminal form shown at A in the annexed figure is an incipient whirl-ring system may be observed by further continuity of projection by the action of gravitation as in the experiment illustrated by Fig. 158. In this case the terminal whirl by further projection opens out to flat section into a biwhirl as shown in Fig. 161 at B, and by further continuity of the projection in the current, it again divides by the conic resistance; the separate whirls being connected by a stirrup-like neck as shown at C, which in continuity again forms incipient biwhirls as in the terminations shown at D, which are only a little more advanced than A. If the projection continue by gravitation, or a constant pressure in one direction, this subdivision is constantly developed as before Fig. 158.

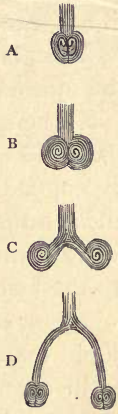

Fig. 161.—Terminal Biwhirls in Fluids.

e. If as suggested in the last proposition, nature avails herself also of this form of terminal fluid projection, such a form possibly will be permanently evident in the animal system. This appears to me to be in a certain degree the fact, although by the complex principles of vitality there may be considerable differentiation in the finally fixed forms, in such cases as this of sensitive equilibrium. It would possibly be therefore less evident in the blood-vessels, where in terminal projection in capillaries, the influence of the granular condition, as it were, produced in the blood by the presence of corpuscles, would destroy the perfect fluidity of the current, quite independently of the evident vital motivity of these corpuscles. But in nervous matter in fine nerves, there appears to be much greater homogeneity, and I find in the termination of some of the nerves the form here proposed of final fluid projection is fairly indicated, as in the Pacinian bodies forming the nerve endings of some of the lymphatics and some other cases. The same principles possibly hold in the formation of the Malpighian bodies in the kidneys; afterwards much differentiated. However this matter is so purely hypothetical that I will not follow it further.

Resistance to projection of solids during diffusional motions in fluids.

108. PROPOSITION: *That a fluid broken up into minute rotary systems by diffusional forces, will offer less resistance to the projection of a unit mass within it than when in a quiescent state of equilibrium.*

a. This proposition being purely theoretical, should be taken only in a limited sense. The resistance in a fluid beyond the inertia of its mass is as the momentum of the motive direction of its parts to the motion of any other body or part in contact with it.

b. The special conditions I offer for this proposition are that where the head of a body or a current has broken into a quiescent liquid, by engendering whirl motions, the liquid will then offer less resistance to the projection of the body or current for it to continue its forward motion. It is assumed that in a current after it has broken up the resistant matter, in front of its projection, by inducing small rotary systems, that it will afterwards pass through the broken liquid mass with less resistance. This may be roughly conceived to be in a certain degree relative to the resistance to intrusion within more solid bodies which admit of easy penetration after they are in a state of division; for instance a plough impelled by a certain force would pass readily through a soil of broken rocks that it could not enter if the rocks were in their natural undisturbed condition.

c. In the summer of 1876 I was much struck with the effects of difference of resistance in fluid masses broken by induced and quiescent motions, in comparing two cases of broken and unbroken liquid, as visible in surface motions, when I was going by steamboat through the Caledonian Canal in Loch Ness; which I compared mentally with the river Thames near London, which I see nearly every day. In these observations upon Loch Ness, where the water was as smooth as a sheet of glass, the steamer threw the whole surface into a commotion of resistant waves, which extended behind the vessel as far as the eye could follow them, attaining at the banks, lateral to the vessel, an altitude of about a yard. Now comparing this with the busy part of the Thames where the river is broken up, and the water in constant horizontal rotation, by the effects of diffusion of projections caused by the traffic, a wave can scarcely be detected upon the bank from the passing of a steam vessel of equal bulk to the one in which I was aboard in Loch Ness, both vessels moving at about equal velocity and at equal distance from the shore; the water surface not being apparently disturbed on the Thames except in a small degree close to the vessel. I have no doubt that the motion would have been less, had the water been shallower in Loch Ness, from the distance of resistance of the lower surface of the earth, so that the effect I have pointed out is only partly due to the causes shown.

d. The breaking up of the ocean surface into separate motions, which the system of each wave may possibly be, as I will hereafter discuss, will, upon the principles proposed, offer less resistance to the projection of a vessel than that which would exist in a perfect calm surface so far as the above principles are active; although it is most probable that the interference from directive wave motions to the production of a uniform connected whirl system which is assumed to be necessarily engendered about the vessel (97 prop.) to save friction of adhesion to the water, would more than compensate for any gain that might be derived from the disunion of connective fluid matter, particularly with so penetrating a solid as the sharp prow of a vessel. The difference of velocities of solids projected with equal power on smooth and on wavy seas might possibly decide this matter.

109. Remarks. Interference with Diffusional Systems.

a. In the diffusion of flowing forces, when these are active upon or against solid resistances, whirls will be established in all areas of greater repose, or of protection from the action of the general currents. The circulation of the fluid near such spaces of repose will establish systems of rolling contact for the less frictional continuity of motion of the projectile current. This principle was discussed, 50 prop. page 153, but in application to diffusional systems, many cases in nature become evident where there are present no bays or deep hollows as before considered, wherein we find that every surface irregularity appears to act with a certain directive and deflective force, to produce the minute diffusional forms now under consideration, near the surface.

b. Further, every solid projection of any kind from a surface opposing a part of a direct flowing fluid, however minute, will engender biwhirl action by the solid projection supporting a cone of impression, the principles of which have been discussed for larger systems in 39 and 75 props.

c. The activity of projections of solid matters in any form as interfering forces with the uniform resistances which cause equal diffusion, may be observed in many cases by deflections about the solid projections, by rotation of the fluid in the parts protected from the direct impulse of a current. Thus if an even stream of water flow down a vertical surface, every very small irregularity will produce a relatively large rounded protuberance or ripple, and the surface of the water will be covered with these protuberances which indicate the foci of rotary systems that produce visible units of local rolling contact of the water over the plane.

d. The space or shade necessary for the establishment of a rotary system will be any area of surface that presents a hollow, smooth, or less frictional space, the unstable equilibrium of the flowing fluid being such as to suffer deflection from very small resistance as I have shown with pipes in the seventh chapter. The like forms of motion engendered over every opening, pore, or even discontinuity of surface, appears not only to induce rotary systems over the less resistant space, but to influence the general motion, so as to create division or vibration of the flowing system inducing continuity of like rotary systems, for a considerable area in the flowing fluid contiguous to these. I have traced this in musical pipes by the effects of openings. For the demonstration of a whirl system being established in hollows, openings, pores, and open spaces, I may mention a simple experiment I observed at an open railway-carriage window when the train was running, the air at the time being very still. As I was sitting I constructed a small flag with a leaf out of my pocket-book by first cutting the leaf nearly into two parts, leaving the part near the outward edges only, so that half the leaf moved freely as upon a joint; securing the one half by thrusting my penknife blade through one free part, it formed a flag which I could hold in any part near the open window, the opposite window of the carriage being closed. As I moved the flag horizontally past the opening it indicated the fact that the air entered at the most backward part at about one-third of the extent of open space. That it flowed equally outward at the forward part of the opening, and an intermediate part near the centre of the window could be found where the flag was either constantly fluttering or in rotation upon the joint. Now moving the flag to different parts of the carriage near the opening, I soon discovered that the flowing air had an established path of about the same circumference within the carriage as could be described within the opening of the window. Through this space the current was in constant rotation, entering at the backward part of the opening to the direction of motion of the carriage, and passing out at the forward part, which was much less defined in its direction, and of more expansive area, there being also evidence of intermittent action. Placing my flag in various parts of the carriage I found indications of perfect diffusion throughout the whole carriage; this was also rendered apparent by the particles of dust visible in the sunshine that entered the window.

e. The circulation of fluids in areas of repose is no doubt most important to vegetable life in bringing its food supply through the atmosphere, particularly in nearly still air, where whirl systems may be formed in every pore, more concave or more quiescent space, which the shaded parts of the leaf will secure for the faintest zephyr. The same principle will be active in hollow or curved spaces upon the leaf, where the apparent contractions caused by the ribs, form interspace hollows or relatively quiescent spots, wherein the air will be constantly circulating in whirl motions, bringing continual change of food supply from the passing current.

f. There is no doubt in my mind also, that numbers of interferences produce like shadings and divisions of direct flowing forces, or biwhirls, in the most shaded parts of the vegetable surface. Thus hairs, the serrated edges of leaves, and their pointed forms are such as will engender cones of resistance inducing whirl motions about the leaves, or the entire plant, and although the general principles of separate resistances, that cause diffusional motions, in the air may not be traceable to the functions of a single leaf, still the forms of plants may be such as to enable them to induce the circulation of the air necessary to bring food continually to their organs of reception. A field of grass may not in each blade receive the full value of its whirl-generating force to split up and divide the currents of air that pass over it; but upon the whole, it will be eminently adapted to establish such diffusional motions to attain this object by the number of points the blades of grass present.

g. There is nevertheless, I anticipate, some powers of engendering motive forces in the air in every plant for its own sustenance, as we find that a single tree grows much more vigorously if separated from others although there may be apparently the same conditions of soil and light, or even greater protection from injurious influences of cold dry winds. Further, we observe that every leaf and branch separates from others as far as possible; and trees will suffer almost any deformity to reach clear air spaces, as we may commonly notice in trees near the borders of woods.

h. I imagine also, that there is always present in the atmosphere diffusional forms of motion caused by unequal expansion of the air by heat-forces. The sun acting more directly on parts of the land surface most exposed to his direct rays, as also more upon open than clouded spaces; whereas radiation is greater or less according to the nature of the local surface. By these means local currents are established by the expanded air directing its impulses upon the radiational spaces, causing thereby diffusional and intermittent motions, possibly at all times ensuring a constant admixture of the air which is most important for the health of both animal and vegetable.

i. Although the above conditions will be those naturally induced in the atmosphere, there will nevertheless be a certain resistance to diffusional motions, in the weak polar forces of the molecules of the atmosphere, if we admit the presence of such, upon conditions discussed in the first chapter. The influence of polar forces will be to establish a state of equilibrium of rest, so that such polar forces will limit diffusional projections, that these projections will not attain the powers of constant forces but be produced and maintained by impulses only. The condition of greatest polar tranquillity will be that in which the molecules approach to nearest central contact *en masse*, which will consequently be also the state of greatest *density relatively to pressure*, therefore a still atmosphere will have the greatest density. In this possibly, we find part of the value of a barometer as a weather-glass; diffusion of vapour-bearing air at varying temperatures being a cause of deposition of rain, as well as of producing a lighter atmosphere.

SECTION II.

DISCUSSION OF NATURAL PHENOMENA IN CONNECTION WITH PREVIOUS PROPOSITIONS.

CHAPTER X.

GENERAL CONDITIONS OF MOTIVE FORCES OBSERVABLE IN THE DIRECTIONS TAKEN BY NATURAL CURRENTS, PRODUCED BY HEAT, THE ROTATION OF THE GLOBE, AND GRAVITATION. REACTION OF NATURAL MOTIVE SYSTEMS.

110. General Conditions of Natural Systems of Fluid Motions on the Globe.

a. In offering propositions for the motion of projectile fluids, in the previous section, I have been compelled very generally to demonstrate principles as well as I was able by small table experiments, and in so doing, have constantly felt the necessity of reference to natural phenomena to give my ideas wider and more tangible elements of reality. This I have felt most important, to ensure the general applicability of the principles of fluid motion offered; as otherwise the motive effects, that I have discussed as principles, would possibly impress the mind as motions active in small close systems only. To obviate this defect in writing the previous section, I made at first, an attempt to give some illustrations by natural phenomena, but found that in proportion as I introduced such experimental evidence, to support my propositions, the natural phenomena themselves required for illustration so much general description, that the continuity of my ideas were obscured or lost, and the special principles of fluid motions I wished most particularly to impress, at the time, became complicated with other effects which represented only disturbing elements. I therefore determined, as I was unable to overcome the difficulty, that I would discuss such portable experiments only, as I was able to make, to illustrate my ideas; and to set aside the consideration of more extensive natural phenomena for discussion in a separate section to support the same, after I had considered the active principles that I concluded were evident in the displacement of fluids generally; and which would therefore, I conceived, not depend in any way upon the *dimensions* of the motive systems considered.

b. A further inducement to follow this course of demonstration was found, in that, I could obtain considerable support from the observed directions of general surface motions of fluids upon the globe by the observations of others; as also in some of the theoretical deductions taken therefrom. This I found particularly the case in the discussion of the general principles of air motion in the trade-winds by Sir John Herschel in his *Physical Geography and Meteorology*, in the eighth edition of the *Encyclopedia Britannica.* I found also general support for my propositions in the theory of oceanic circulation, of Lenz of St. Petersburg; which has met with such practical demonstrations in the valuable researches of Dr. Carpenter. These matters appeared to me to lead to demonstrations of the establishment of a system of aerial and oceanic circulation, that was evidently active upon the same principles as those I have discussed for other cases of projection and resistance in fluids shown by my small experiments.

c. Following the same principles in natural phenomena as are evident in experiment:—We may assume that tangential forces which have been shown to produce rotation in small masses of fluids placed laterally to motive parts (50 prop.), will, in superior volume forces, act in like manner upon the larger masses of fluid upon the surface of the globe, moving these also, in like manner, by rotation about their centres of inertia. We ought also to find that direct projections impinging upon resisting parts of a fluid system would produce whirls and biwhirls (77 prop.). Further, that in biwhirl action there would be a general continuity of diffusion of motive fluid forces throughout all areas free from great resistance (105 and 106 props.). I shall particularly devote my attention in this section to show how far these rotary or whirl systems of motion are consistent with observations, which, upon the large scale of nature-work, go so much beyond manual experimental possibilities, but which, nevertheless, I feel are so relative, that the experimental evidence induced from such small effects, may be shown to be *principles* which are active in all cases, quite irrespectively of the mere dimensions.

These facts may also tend to establish a principle, I think exists, namely, that fluids *move* as perfectly homogeneous systems of matter in which impressed forces in unit masses act as the amplitudes and velocities of impression into the masses impressed, proportionally, whether the masses be the aqueous or aerial systems of a planet, or systems as small as those given in my experiments; or by *inference* in almost infinitely small systems, such as are possible in the circulation of the smallest animalculæ. In this matter I consider the smaller motions sufficiently demonstrated in the last section, so that I have now only the conditions of the larger natural systems to consider.

d. For the *generative sources* of motion in fluids upon the globe, which may act afterwards in the manner given by the demonstrations of my propositions, and which I wish to further assure, we have principally, the presence of ever-active forces of the sun's heat and the earth's rotation. The air and water volumes upon which these forces act remaining constant, and thermal forces cause local expansions, where the sun's rays fall most directly, thereby producing fluid displacements. We may anticipate in these displacements upon the surface of the globe, whose velocity in parts varies as the difference of circumference of rotation, that some general motive principles will be active in the direction of the fluids upon it, by which whirl systems will be formed upon every resistance to the direct impulse impressed. Such whirls or biwhirls being *free* or projectile if produced by *intermittent* impulses (67 prop.), or *located* in space, if, by the conditions present, the forces act with *constancy* (73, 77 props.); always assuming the fluids upon which such forces act to be in a state of equilibrium. In the operations of nature such freedom of quiescent equilibrium in the resistant fluid to meet the above conditions will not frequently be found, and we have then to consider such composition, or diffusion of forces, as may occur (109 remarks). Further, the conic resistance in one plane may meet with other forms of resistance which will deflect and *deform*, as it were, the equilibrium of forces in the entire system, as, for instance, the projectile central force of a flowing river will meet with very unequal resistances *vertically*, as above the central projection we have the air, and below the solid earth of the bed of the river, so that the equilibrium of resistance, by which equilateral biwhirls can be formed, will be possible in horizontal surface planes only, where equilibrium is produced by gravitation. And as regards *vertical motion*, whirls will be deflected downwards only upon the resistance, as whirls of rolling contact. Further, as the bounding aerial surface of water approaches the condition of a perfect plane, and that the land surface meets this plane everywhere with nearly equal resistance *per linea;* horizontal resistances will be always in approximate equilibrium, as regards liquid surface motions, and, approximately, equilateral biwhirl systems will be found to be generally induced in this surface plane. On the other hand, if a liquid or aerial system have considerable depth, and there are from any causes extensive under, over, or intermediate currents occupying broad bands of moving fluid, some cases of which I will hereafter discuss, such currents will form biwhirl systems, in *vertical section only*, in central parts, and generally to our powers of local observance of such systems, will appear as continuous flowing forces taking one uniform direction; so that in this case we shall generally have surface motion quite distinct and often opposed to the direction of currents above or below certain gravitation planes.

111. Forces which move the fluids of the globe.

It will be convenient before entering into discussion of natural whirl systems, to point out briefly the motive forces that act directly upon cosmical fluids; the principles of which in a general sense are very well understood by those who have studied the subject; this will afterwards prevent the necessity of special description when reference is made thereto. The only motive forces necessary to be considered, and which act directly and perceptibly upon the fluids upon the earth—for we may very well omit the minor influences of chemical, electrical, and magnetical forces—are: 1, those of heat; 2, the momentum of the earth's revolution; and 3, gravitation. The composition of these forces being quite evident in all movements of air and water upon the globe. Some special effects of these forces which concern us may be now conveniently and separately discussed before considering the necessary directions that such forces induce in natural fluids, upon principles given in the last section. In addition to this, it will also be convenient to consider, as preliminary matter, the action and reaction of air and water upon each other.

112. Heat-forces active upon cosmical fluids.

a. The heat-forces that act upon fluids upon the globe are derived from one source only—the radiation from the sun, if we omit the consideration of terrestrial heat derived from volcanoes and springs, which forms but a small element of heat-force in the great economy of nature, evident in superficial systems.

b. The radiation of heat from the sun has three distinct functions:— 1. It acts directly to *expand* the air vapour and water, heating these bodies at the same time to a certain temperature, this expansion being greatest in the formation of vapour from water. 2. It acts indirectly by giving the heated bodies forces of radiation, from the temperature that they have received, which gives them again, *at another time*, an equal force of *contraction* or of condensation, which acts motively in the inverse direction to the original expansion, and with force equal to it, all bodies being assumed to radiate and receive heat according to Prevost's law of exchanges. 3. Heat acts also in producing currents, by rendering the expanded fluids *lighter*, and the radiated condensed fluids *heavier;* the parts of the fluid affected, seeking gravitation equilibrium, by the one part underflowing or overflowing the other.

c. By the important experiments of Joule we obtain knowledge of the values of heat expansions, as material forces, in the laws he has established for the equivalence of heat and mechanical force, in which we learn that every degree Fahrenheit of elevation of temperature of a pound of water equals the mechanical elevation of one pound weight to a height of 772 feet; and if we consider the known action of the sun in raising the temperature of fluids upon the earth, we at once have data for conceiving the immensity of the forces active in causing expansions; which will be at all times sufficient to account for the movement of masses of air and water, with the forces we witness, against the powerful resistances present on the earth's surface, and in the fluids themselves forward of the position to which they are moved.

d. The exact values given by Joule, if taken superficially, in relation to measurable inferences from cosmical phenomena, represent possibly only about one-third the absolute forces derived from heat communicated from the sun as ascertained in the manner that we are able to measure it by the thermometer; as the temperature of water, for instance, elevated one degree, has, during this elevation, exerted an expansive force, and performed the outward work of pressing fluid masses aside and upwards, as well as the intermolecular work of increasing palpable temperature. And the like work is again performed, after a certain period, by radiation and contraction of the same heat, on another part of the earth, or at night, to bring the fluid again to the initial temperature from which we at first imagine it taken; the heat-forces from surface waters being also largely transferred to the air and not lost. In the same manner the equatorial temperature in the ocean is observed to be only about 82 degrees Fahrenheit; but the quantities of heat-force utilized in the elevation of vapour over these regions is perhaps fourfold that left measurable by the thermometer. This vapour-force being made evident only by forces witnessed principally in reaction, active elsewhere, in winds, currents, and rainfall.

e. The direct local influence of the sun upon separate regions of the earth is shown by Sir John Herschel,[1]—in that, "the same sunbeam which, at a vertical incidence, acts on a surface equal to its own sectional area, when incident obliquely on the earth (including its atmosphere), is spread over a surface larger in the inverse proportion of radius to the sine of the obliquity. It needs little consideration, then, to perceive that at the poles, where the sun is below the horizon for half the year, and where during the other half it never attains a greater altitude than 23½°, and *that* only for a short time, its effective warming power on a given horizontal surface must be very far inferior to that which it exercises in the equatorial regions, where its meridional altitude never falls short of 66½°, and where the days and nights are always nearly twelve hours in duration; nor that in the intermediate latitudes the increase of its altitude, and the length of the day as it advances along the ecliptic from the winter to the summer solstice, should bring with it that accession of general temperature which we observe."

f. By the above it will be seen that heat expansions of the fluids resting upon the globe develope most powerfully within intertropical areas, and under this condition, motive forces engendered by these expansions over the globe will have in these regions their powers, at least as superficial forces, most constant. In principle, we may assume that these regions will act as the boiler of a steam-engine acts—as a projectile force to move surrounding resistances; the force taking motive directions, according to the general mechanical principles of all force movements, through the space of least resistance, as in the movement of the piston in the case of the steam-engine, while other parts of the engine remain stationary.

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[1] *Meteorology*, § 9.

g. As the earth maintains upon the whole a mean constant temperature at all times, or as approximately so as we are able to measure it, it will be clear that the *earth's radiation* of heat-forces into space, from displaced fluids, is as great as its powers of receiving such forces by radiation from the sun. So that as the sun's heat-force diminishes towards the poles, whence fluids are directed by thermal expansions, the excess of the earth's radiational forces in these regions must be equal to the entire amount of heat thus received. And as the contraction of a natural body by loss of heat-force, in any manner, is inversely equal to the expansions per unit of heat-force; such contractions react locally upon surrounding parts with equal energy to the original heat expansions, as before stated.

h. By the obliquity of the sun's rays to the surface of the globe near the poles, we have here regions at all times covered with ice, therefore of constant condensation, whereas in the tropics we have regions of alternating expansion and contraction, or of *absorption* and *radiation* of heat, day and night, caused by the earth's rotation. We have, therefore, the heat expansions of fluids as directive forces, from the tropical area, *pulsatory*, and the polar forces of contraction, *constant*. The pulsatory action gives the fluids projected by heat-forces greater penetrating powers, upon principles discussed in the fifth and sixth chapters (73 prop. *c*).

i. The diurnal expansive effect of the sun's rays, upon the above principles must drive forward a wave of condensation, in both air and water, which traverses the fluid parts of the globe in its rotation, acting auxiliary to the tidal wave caused by the sun's attraction conjointly with that of the moon, these waves, being intermittent, are pulsatory in diurnal periods, and therefore locomotively projectile.

j. From the above it will be clearly conceived that we have established upon the globe, a thermal diurnal pulsatory force system, which expands fluids at the equator, causing them to flow or overflow outwards in the paths of least resistance, and near the poles, an area of constant terrestrial radiation of heat which acts as a condenser to bring over the expanded fluids; condensing and causing them to return towards the thermal equator, by their gravitation forces in their superior densities after condensation; by any path that offers at the time the least resistance to the establishment of equilibrium of the fluid systems as they rest upon the globe. Such thermal force under any conditions gives fluids upon the earth a general reciprocal action in northern and southern directions. The principles of action, evident in such motion in these fluids, resembles the circulation of water which we find in common practice in the heating of buildings by pipes, where the heat at one end of a system causes the constant projection of an upper surface current, whereas the cold or radiation of heat at the opposite end of the system constantly effects the supply of a denser underflowing one.

k. The warm air present at the formation of vapour, by direct action of the sun's rays and by reflection from the earth and ocean, rises with it, by the elastic forces of the expanded compound aerial fluid thus formed, to greater elevation under the pressure of lateral denser surrounding air, which is by this cause directed to underflow inwards beneath the lighter fluid from the nearest direction in which it can find a free or less resisted path to supply the loss. In this manner we have direct undercurrents projected towards the areas of thermal expansion and evaporation. To complete such a system, where the heat-force is practically constant, if taken for the whole globe, we have also necessarily a locality of condensation of vapour forces; otherwise such evaporation, if constant, would engender cumulative resistance equal to its elastic force by heat expansion. From this cause the condensed water in rainfall in temperate and polar regions produces inflows to partly equilibrate the excess of vapour-force active upon oceanic areas where the evaporation is in excess of precipitation.

l. Between the two areas of greatest expansion and contraction formed by the causes given above, we need the constant action of a system of reciprocally motive forces, embodied in the fluids acted upon, which in their movements seek the least frictional courses, that the accommodation permits to maintain a circulatory system, establishing means of supply in proportion to demand, to the aerial and aqueous thermal systems. These special local conditions I will consider further on.

113. Thermal Forces in Vapour Systems.

a. The formation of vapour as representing an expansile force, and of rainfall as a contractile one, are of such importance as elements of motive forces upon the globe, that this subject may with advantage be further considered for certain particulars.

b. First as regards the formation of vapour. We may possibly assume that the sun's rays, acting directly upon the aqueous surface, may be divided into two distinct functions. 1. A part of the sun's rays is evidently active upon the surface film producing *reflection.* This reflection may be conceived to be partly active in the form of heat and light, and partly in the molecular action of *evaporation;* possibly in the manner suggested in the 16 prop. 2. A part of the sun's heat enters a liquid by direct radiation, as other heat forces do in passing into, or through, transparent media. This radiation force of the sun's rays has been detected to be measurable by thermometers in clear water within a depth of 600 feet, as some experiments on board the *Challenger* show.

c. By the first condition offered above we have evaporation direct upon impact of the sun's rays, which, as regards the water evaporated upon the surface of the globe, may be considered to form an *intermittent* or diurnal vapour system. For the second, in which we have a general heating of this aqueous surface to a considerable depth, and in so far as the surface reacts afterwards by radiation, we have a *constant* evaporative force active at all times, day and night, which is particularly evident within intertropical regions, where it is found that the surface temperature of the ocean varies very little. This secondary constant force of evaporation is supported by the high specific heat of water, by which the heat-force is conserved with small outward elevation of temperature to react afterwards motively upon its reduction to lower temperature. There is also active, to support the constancy of surface heat, the presence of convection currents; which by interchanges, tend at all times to maintain the surface temperature as high as any part of the liquid mass.

d. Reserving a fuller consideration of the part that vapour plays upon the globe in special and local vertical circulation, and taking only general particulars, we may take it for granted that evaporation represents a constant expansile force over tropical areas by the formation of a *constantly renewed elastic fluid system*, which is intruded, and takes partially the place of the air. It will also follow that either at some part distant from or near to the tropics there will be an area of condensation where the vapour returns to water, its place in the aerial elastic system being only made up by ·001 of its original volume, which the water approximately represents in relation to its vapour.

e. Dalton found that evaporation forces rapidly decreased by loss of temperature in water. Thus a certain surface of water at boiling temperature, evaporated 40 grains per minute, at 180° Fahr., 20 grains, at 152°, 10 grains, and so on, until at about 39 degrees the evaporation was very slow, although it continued at all temperatures. By this and further experiments we infer that the tropical heat-forces consumed in raising vapour are immensely greater than the same forces under greater obliquity of the sun's rays in the temperate regions. Practically the air at the freezing temperature of water in a free state contains very little vapour; so that we may imagine that in all areas, condensation will take place to a certain extent by the lowering of the temperature of vapour, but that at freezing-point this operation will almost practically cease, so that in areas below 32° Fahr. we have generally to consider the effects of thermal forces in producing contractions upon the air only, as before stated.

f. By the continuity of the flow of air over water it will finally become charged with moisture from surface evaporation. This moisture, as invisible vapour, will be a conservation of heat-force which will be partially restored upon after condensation. It will therefore occur in the projection of vapour currents that these may advance far into the polar regions, where the average cold would in no way support the vapour-force, were it not for the heat restored by this partial condensation. Further, as we find that all polar directed thermal fluid forces possess eastern momentum relative to the latitude velocity of the earth, by conditions to be discussed, the vapour currents as those of aqueous and aerial ones will thus be directed to easterly courses. It would also follow that at the northern or colder sides of these currents condensation would constantly occur, and as this condensation produces either rain or fog, it would subtract the elastic force of the vapour. Therefore, circumpolar areas would be areas of contraction to the vapour, exactly as the tropical regions would be areas of expansion, and this contraction would, by minus elastic force, permit the continuity of projection of the vapour-force indefinitely for some distance into the cold area.

g. We must nevertheless under the above conditions rest assured that after a certain amount of refrigeration, the aerial vapour would become very attenuated, so that its effective projection through condensation would not proceed very far after condensation below 32° Fahr., the freezing-point of water, which is also the point of deposition of snow; whereas a purely aerial system would constantly condense proportionally by decrease of temperature. It becomes therefore probable that vaporous projections into polar areas fall generally somewhat short of purely aerial ones, so that assuming such vaporous forces to produce whirls, as I shall endeavour hereafter to show they do, by original thermal projection and condensation in separate regions, they would form such whirls of less amplitude than those formed by a purely aerial system. Some evidence of the above I will discuss further on.

114. Obliquity of the earth's axis in relation to heat-forces.

a. The obliquity of the earth's axis of revolution which causes our local summer and winter temperatures permits the sun to act in the expansions of the fluids presented, alternately, in the northern and southern hemispheres; the direct expansion over the summer half of the year being possibly about double those of the winter half. The intensity of the heat-forces increasing up to the summer solstice, and decreasing in like manner after this until the winter solstice, upon principles defined 112 art. *e.* The direct effects of the sun's heat-forces largely disappear in work exerted upon the mobile fluids of the earth. Thus, in the advancing summer the expansive force of the sun's heat only appears to increase the amount of cold in the temperate regions by acting upon the air and expanding it, a portion of the cold air being driven from the regions where the sun's rays now penetrate, against the resistance of the friction of its gravitation and adhesion to the earth. At the same time, the contractions of the air and water in the opposite polar area, by the radiation of the terrestrial heat attained in the previous summer, is drawing over and condensing the aerial and aqueous currents to equilibrate the missing elastic force derived from circum-polar surface radiation, which leaves the winter temperatures much higher than they would be if derived directly from the local effects of the sun's influence into the earth's radiation, particularly in early winter when the vapour condensations are greatest. From this cause, the sun's rays may even be absent for long periods, and yet the climate be quite bearable to humanity; thus in certain positions we find regions quite habitable within the arctic circle, where the cold would be intolerable were it not for the effects of this general principle.

b. The local motive values of heat-forces, modified by the obliquity of the earth's axis, may be partly shown by the winter and summer pressures over large continents where surface radiation is most active. Thus we find in the centre of Asia, as the researches of Buchan show, that the atmospheric pressure, evidently from the inflow and condensation of the air, is on an average 30·4 in the cold season of January. Whereas, by summer expansions it is only on an average 29·34 inches in July, or about equal to one inch difference of mercury pressure.[1]

c. During the time of expansion by heat-forces over continental areas, when the air is driven constantly to lower latitudes, and therefore to regions of higher temperatures in the advance from winter to summer, these expansions represent immense forces; the increase of volume of air at the same pressure being $\frac{1}{273}$ part for every degree Centigrade of additional heat. But this altogether forms only one item, as the vapour forces are relatively greater in expansions and condensations, wherein the mass-reduction from vapour to water is over one-thousand-fold; such expansions and condensations aid directly in forming local winds.

d. In air over oceanic areas, by the difference in the obliquity of the sun's rays, we have no similar directly measurable thermal expansion or loss of atmospheric weight shown by the barometer, comparable to that derived from the direct radiation of heat upon land, which causes expansion over continental areas; but in the aqueous areas the active vapour-forces as they are increased by summer heat must constantly increase also the amount of vapour, proportionally to the evaporation, and add this weight to the aerial fluids; but as vapour is lighter than air, the vapour-weight is not all measurable by the barometer, so that the intruded vapour practically increases the expansive forces, making these greater by the entire amount of this addition over the aqueous areas in summer; and we need not from this cause assume absolute atmospheric displacement from continental areas to produce the relatively *plus* pressure we observe. The general direction that heat-forces take in aqueous and aerial systems on the earth, beyond the many conditions of displacement here considered, I will endeavour hereafter to show. The tendency in all cases of seasonal obliquity being to direct fluids towards the winter hemisphere, by thermal expansions in the opposite hemisphere in the summer, and to draw over and condense such fluids in circum-polar areas during the winter. The *active* forces being much greater than the measurable thermal forces, that we can perceive by our thermometers and barometers

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[1] *Trans. Roy. Soc. Edin.*, vol. xxv. p. 575.

as before stated. Such immeasurable forces being also largely derived from the latent heat absorbed in the one case, and released in the other, in the formation of water from vapour, and ice from water, and *vice versa.*

115. General effects of latitude velocities upon cosmic fluids.

a. There is no direct momentum of fluids at the poles relative to the earth's revolution, the earth turning upon an apparently immovable axis once in every twenty-four hours nearly; whereas at the equator the direct momentum is about one thousand miles per hour, the intermediate spaces being as the latitude velocities of the respective parts of the globe. By this we comprehend that by any transposition of a fluid north or south, the fluid will carry with it the velocity momentum of its mass from the latitude of the part of the globe from which it was displaced; which velocity momentum would be plus or minus that of the earth's velocity, according to whether it was directed north or south. The theory of this principle of motion was first pointed out by Hadley, *Phil. Tr.*, 1735, and was shown satisfactorily to give an eastern or western impulse to fluids relative to the earth's revolution, in all cases where these were expanded or otherwise directed northward or southward from any cause. The effects of revolution-velocity giving in most cases, particularly in nearing the tropics, greater direct momentum to the air, vapour, and water, than the direct effects of expansion and contraction, by the radiation of the sun and earth, in moving these fluids north and south by diurnal and seasonal expansions and contractions. Upon this principle the trade-winds are accounted for. There is one point in the effects of revolution velocity, in relation to thermal displacements, that I am not aware has been previously observed, which is important, namely—that the revolution of the globe being a circular motion around the poles of the earth; *that the influence of all thermal forces, which are direct or longitudinal motions to or from the poles, will engender elliptical orbits in displaced fluid matter.* These orbits may be complete or deflected by composition with other forces. This principle we shall find will materially affect the direction taken by aerial elastic fluids, but with water, the cohesion of its mass will resist motive ellipticity with a certain force; although the same influences will nevertheless be manifest in this also in degree.

b. As fluids upon the globe cannot move constantly in one direction by the action of heat and revolution forces without materially disturbing gravitative equilibrium, a fluid pressed to one part of the globe will form a resistance to following parts. In such resistances the momentum of projection will be largely conserved, for deflection of the fluid into the path of least resistance, for the continuity of projection; whirl motion being the general resultant; the whirl taking any direction in space, horizontally, vertically, or obliquely, according to the composition of forces present.

c. To follow particularly the effects of the earth's revolution upon fluids at its surface, we may with advantage take the conditions of an imaginary case—and suppose for an instant the earth perfectly stationary in space, that is, with no rotary motion, with its fluids resting quiescently upon it, as formerly believed. If we now imagine that it suddenly commences to rotate upon its axis, by movement of its solid parts, in a west to eastern direction as actually. Then under such circumstances the inertia of the fluids resting near the equator would be able only *slowly*, from the mobility or infinite jointing of their systems, to take up the revolution velocity of the moving earth; in fact they would resist movement as the entire inertia of their mass into the mobility of their systems. Under such conditions it would be very clear that the equilibrium of the polar system would not be greatly disturbed; but the inertia of the mobile fluids resting near the equator would throw them apparently, that is, in relation to the earth's revolution, with great violence in a westerly direction. In this case the liquids would be powerfully projected upon their western boundaries, or coasts, which they would overflow until the action of gravitation on the elevated masses caused them to be deflected to parts of the globe carrying less velocity momentum, and until gravitation forces into rotational momentum were equilibrated.

d. Now supposing the oceanic coasts perfectly resistant at their western boundaries, then as the equatorial projections would have greatest velocity, the waters would move away from the greatest resistance, as in all other cases of fluid projection, and as this movement would cause deflection towards the more inert polar systems, to which they would communicate their impulses, it would therefore occur that by the revolution velocity alone of a globe set in motion by movements communicated through its solid parts, that liquids resting upon it in equilibrium of gravitation would have a relative minus velocity, so as to appear from the solid parts to be in motion in an opposite direction to that of the revolution, and that the waters, if resisted in such motion by solid parts of the globe would be deflected upon the resistance, and move under this deflection towards the polar parts where their velocities would be first brought to equilibrium with the rotation, and then by continuity of pressures causing further deflection, in excess of it; producing an apparent movement in an opposite direction. It would further be clear that such a system of drifting of the oceans upon their western boundaries, and of deflections to polar parts would materially disturb the equilibrium of gravitation by abstraction of water from the eastern equatorial parts; therefore gravitation being active proportionally to the elevation, the disturbance would cause the elevated waters to flow to equilibrium from the parts where the directive impulses were least active; and in this instance, the water deflected first to a western boundary, and then towards a polar area, would produce a return supply current to establish gravitative equilibrium along the eastern coasts, thus producing a complete surface rotation of the ocean.

e. If we now again return to the effects of a thermal force as discussed 112 art., assumed active on an aqueous system in projecting liquids on the globe to and from the equator, we may assume that every particle of liquid matter that may be moved as a supply current towards the equator will carry with it a minus revolution velocity, therefore it will be, in relation to the velocity of the latitude to which it is drifted, as *new matter*, which will be subject to the conditions considered above, of fluid *that commences to move* upon a rotatory globe, in the ratio of the minus rotational velocity that it will have in flowing towards the higher velocity near the equator.

f. We may now therefore confine our attention, for one condition of the motive principles discussed above, to a liquid *commencing to move* within a prescribed area, surrounded, or partly so, by resistances, which act to the liquid as a containing vessel, in preventing a general propagation of direct circumferential motions. We may take such containing vessel to be one of our great oceans, or any area of smaller dimensions, provided that the differences of velocity of parts of the system are sufficient to give a difference of momentum to the polar and equatorial sides of the fluid taken.

g. Under the conditions proposed above, experiment becomes much simplified, for we have in this case, instead of assuming a number of conflicting motions to represent the directions of surface forces, the condition of a fluid *at rest which commences to move.* We have at the same time demonstration in natural phenomena in the certainty of the inertia of fluids reacting near the equator, as we find in the direction of the trade-winds and surface oceanic drift, which transports the fluids themselves, and that must act, in regard to rotational velocities, as *new matter* which has not acquired the latitude velocity of the place whence it is transported.

h. If we now take the case of a liquid at rest in a containing area of resistance, for which conditions we may assume that of the Southern Atlantic Ocean; this area being convenient as it is included in moderately parallel meridional planes of resistance on two sides by the coasts of S. America and Africa. Then assuming that thermal forces direct the oceanic waters south from the equator, as just proposed, and that these waters in their new position now act as new matter which has not attained the minus latitude momentum of its displaced position. We may then represent the conditions I propose experimentally as follows:—

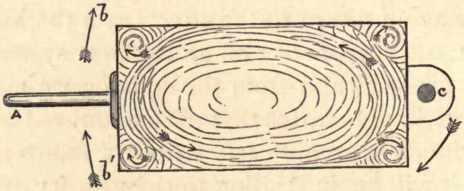


Fig. 162.—Ex.—Circulation by Rotation.

The trough shown in the engraving, Fig. 162, was made for trying this experiment; its dimensions were 30 inches in length, 20 inches in width, and 6 inches in depth. It was constructed so that it could be moved round upon a centre *c.* This centre may be assumed to represent the position of the terrestrial axis at the South Pole, in the illustration taken of the Southern Atlantic, when the apparatus is moved round by a handle fixed at A in the direction of the arrows *bb′*. For convenience of experiment the centre of the apparatus was supported upon a post of about a yard in height; motion being given to the vessel by taking the handle in the hand and walking around. The water flowed in the direction shown by the arrows inside the trough, which was made visible by particles of sawdust floating upon the surface.

i. On continuing to walk around, after a certain time, the water ceased to rotate, the friction of the vessel finally retarding it, and all parts acquiring an initial radial velocity. But now a current projected in the direction from A towards the centre *c*, assumed to represent the thermal effects proposed (112 art.), by means of a supply pipe (not shown in the illustration), the rotation recommenced *de novo;* and the same took place from a like projection of water in the opposite direction, from near the centre. In all cases the difference of revolution velocity in the projected parts constituting the motive force according to the minus or plus velocity derived from the revolution.

j. In a system of fluid of large extent the momentum of revolution would support its continuity, and smaller efforts of the forces derived from equatorial or polar directed flowing matter, would act cumulatively upon the rotary system.

116. Influences of gravitation in density systems of fluids.

a. The action of heat upon water is exceptional, in increasing and decreasing its volume; its greatest density being at 4° C.; the entire range of oceanic temperatures from 4° to 43° C., increasing the volume of pure water by about $\frac{1}{108}$ part (·0092) at atmospheric pressure. From this there is some variation of which I have no data for sea water, the greatest density of which has been found to be about 3°·2 C. at atmospheric pressure. This temperature was reasonably assumed by Sir J. Herschel and others to be the lowest at the bottom of the ocean. The experiments in the voyage of the *Challenger* have shown lower temperatures to 0° C., so that if there are in these observations no instrumental errors, we must assume that the density of sea water increases with loss of heat under great pressure to the freezing-point of pure water; under any conditions the thermal forces in open water, by the density and mobility of this liquid, will produce thermal currents, which are also made evident by experiments to which I have already referred.

Not
Sea

b. The density of quiescent air and vapour resting upon the globe is found to vary inversely as the compression it receives, that is, as the mass resting directly above any area according to Boyle's law; the force of gravitation decreasing also with the altitude. It has been computed that from these causes, by the average pressure at sea-level, half the entire mass of aerial matter resting upon the earth would be contained in an average stratum of about 3·4 miles in altitude if there were a continuous ocean covering the entire area. By the intrusion of the land surface above the oceanic level this quantity is possibly contained in about 3·6 miles above the average oceanic level as it exists; within this lower denser strata the expansile and contractile forces of radiation and conduction of heat, and of evaporation and condensation, are most active.

c. The action of the sun in the diurnal revolution of the earth will, as stated, cause the air to expand and form vapour upon the side of the earth upon which its rays fall, and this expanded, or lighter fluid, will seek gravitation equilibrium with any contiguous more condensed or heavier fluid. Now as the globe is covered partly by water and partly by land of various inclinations, in some parts in valleys and flats, and in other parts in mountainous districts, the expansile and contractile forces which are actively represented in currents or winds, will suffer considerably more resistance to gravitation impulse in passing over irregularities of hill and dale and saliently inclined surface generally, than upon more level plains. The most level and least frictional planes being uniformly those of water, therefore the larger systems of aerial forces moved by thermal and gravitation impulses will seek the most uniformly level or least frictional planes, which are the large open oceanic areas. Upon these lower areas, and proportionally to the general limits of freedom from the resistance of land surface, there will be established the strongest and densest currents of aerial matter, which carry their impulses also inland from the liquid surface upon which they move.

d. Under the above conditions, if we conceive the establishment of such lower dense aerial currents in the areas of least friction, that is over oceanic areas, these will also have a powerful momentum and carrying power against the inertia of lighter currents above, which will be supported by no exterior static resistance. It will therefore be almost entirely to the influence of the movements of these lower currents that we must look for the establishment of the motive systems of the entire atmosphere, and partly so also for the oceanic surface.

e. By decrease of density in the superimposed air above any given level plane of equal altitude, it may be conceived that the lower stratum of the air will somewhat resemble a separate stratum of a *denser fluid* resting upon the globe. Under these conditions any land of great altitude above the sea-level will act as a static point of resistance of greater effective force than its mere height in relation to the general air space. This particularly applies to the special movements of the lower active carrying stratum. In this manner every point of land upon which a wind is directed from the sea is a powerful supporter of a cone of impression.

f. We may further conceive upon the above principles, that the lower stratum of air will also have a constant tendency, from its superior density to that above, to *continue horizontal impressions* of force engendered at the *oceanic level*, at this *level only*, unless moved upwards or deflected by some other exterior force which changes its gravitation plane of motion, such as the inclination of land surface, or the elastic reaction of compressed air in front of a directly resisted current.

117. Effects of evaporation and rainfall as gravitation systems.

a. The great heat of the sun's rays falling vertically over equatorial regions causing continual evaporation of the water surface, would if the water were static, constantly lower its surface over these regions; but as the fluidity of the aqueous system is such, that equal gravitation surface is possibly very nearly maintained, the abstraction by evaporation must engender a certain constant force of locomotion of the liquid mass towards the areas of evaporation, to supply the loss it occasions. The evaporation over tropical areas being constant, the air above such areas can be capable of maintaining only a certain amount of vapour at the temperature it possesses, at any time, until perfect saturation. It would therefore follow that the vapour must be projected indefinitely upwards to occupy space *per se*, or that it must fall as rain; but as there are always cooler areas contiguous to the regions of most intense evaporation, and that there are alternations of the most intense evaporation periods from day to day, and of relative cessation from night to night, we find that by the constant action of gravitation, contra to any possibility of unlimited upward projection, that a large portion of the water evaporated falls as rain upon the same or contiguous parts where there is only very slightly less heat force to maintain this vapour tension.

b. Under these conditions rainfall as superimposed liquid on the ocean represents at the instant of its fall a small directive force which aids the general direction of forces of expansion by heat, to overflow from the tropical area. This particularly occurs from the rain being warmer, and from its lacking salts, being thereby a lighter fluid than that of the oceanic surface upon which it falls. The rainfall over land areas, reaching the ocean by rivers and streams flowing into equatorial oceans, represents also, by difference of density, a directive projectile force. This is markedly the effect produced by the outflow of the Amazon and Mississippi rivers into the Northern Atlantic system near the equator.

118. Salinity of oceanic water as influencing gravitation systems.

a. The conditions I have assumed for heat circulation are taken for a uniform fluid expanded in equal proportions for equal increments of heat, approximately. In sea-water at atmospheric pressure the greatest density as before stated is said to be at about 3°·2 C., a temperature to which open sea-water never appears to descend, the temperature below arctic ice, which is possibly the lowest near the surface, being nearly uniform at about 28°.

b. In regions where evaporation is in excess of precipitation, as, for instance, in the regions of the dry trade-winds, the salinity of the sea appears to be greater at the surface; the evaporization being of pure water only. Such regions generally have a clear atmosphere, and being near the equator the sun's direct radiation has greater power of penetration than in regions of moister air; water in all forms being eminently athermous. In some experiments of Melloni, distilled water was found to transmit 11 per cent. of heat rays from a luminous source (an argand lamp), through 9·31 millimetres of water inclosed in a glass cell; water saturated with rock-salt, under the same conditions, 12 per cent. The experiments of Sir Robert Christison show that water was sensibly heated by direct rays of the sun for 600 feet in the clear water of Loch Lomond; sea-water equally clear would be heated by inference of Melloni's experiment to greater depth.

c. The immediate effect of evaporation upon sea-water is to increase its salinity and render the surface water more dense, thereby creating descending convection currents; but as the temperature of the mass which the currents pass through in descending is less than its own, the descending saline water, supposing it to retain its heat in descent, would come to equilibrium where the density of a certain colder underlying stratum of less saline water equals in density the salter warmer descending convection current. The velocity of changes of position of density strata, being possibly as the gravitation forces into the fluid resistances.

d. By experiments I find that heated salt water of *less density* than cold fresh, or less salt water will not remain upon the surface of the colder water, but the surface water will immediately lose part of its temperature upon contact with the colder water beneath and descend in convection currents, the surface heat being diffused with great rapidity. This experiment I have only tried roughly in a tall beaker of cold water coloured with ink, and hot salt water placed above it coloured with milk, wherein a diffusional system was at once established. By this experiment we may assume that where the oceanic surface becomes more salt from evaporation the heat forces present will be more quickly distributed, and produce an equable temperature to a greater depth. In this case the diffusion will only be limited in diffusional force by the difference of salinity falling off, when, by solution in the descent, the density of the saline part becomes nearly equal with the lower gravitation plane it reaches. We may assume from this reason, partly, (I will offer elsewhere more important conditions) that the range of surface temperature for a depth of about 1000 fathoms, nearly over the evaporating regions of the trade-winds, is higher than near the equator, where there is no excess of average salinity. The salinity in equatorial regions, where there is greater evaporation, being kept down possibly by the nearly constant diurnal rainfall. A greater depth of warm water at about 20° north and south latitude is observable in the section given by Dr. Carpenter of the Atlantic between the parallels of 38° N. and 38° S., page 11 of a paper read at the Royal Institution, March 20, 1874.

119. Reciprocal Action of Air and Water upon each other in Horizontal Movements.

a. Air and water as all experiments show are adhesive to each other. The entire average atmospheric pressure on any part of the globe represents an adhesive mass of about 2000 lbs. per square foot. Taking this as active upon a large surface of the ocean in its continuous movements, it may be considered to communicate its momentum to this surface by adhesion, so that we find actually in many cases, where the directive forces in the water are not very great, that the influence of the wind directs the surface flow; and this may be in opposition to the thermal effects of heat expansions or radiation contractions, upon the aqueous surface system, *per se*, as just considered. On the other hand, aqueous projections have a carrying force upon the superimposed aerial fluid, so that aqueous and aerial fluids act reciprocally upon each other. We may very well consider these conditions separately, although fluid forces generally conserve their energies, so that the one force in contact, at all times, enters into composition with the other.

b. If we take the entire momentum of the air in an extensive moving mass to be equal to about 32 feet of depth of surface water, that is, nearly as its superimposed weight at the surface of the ocean, and consider both systems of air and water as adhesive mobile fluids, with no other resistance than the inertia of their masses:—Then if the moving air gives by a constant direction an impulse to the water as its momentum value, that is, as its entire weight, and the water resists equally the momentum of the air, the average movement of the liquid surface would be equal to half this; the air being taken to move the surface water to this depth at half its own velocity for constant forces, or equal to half the depth in the entire water moved; this is entirely neglecting the mass cohesion of the water. In this case it will be readily seen that the wind can only be represented as a *surface force* in relation to the thermal force in the entire oceanic system; the thermal force being active in clear water for a much greater depth, or as some experiments made on board the *Challenger* before mentioned show, even to about 600 feet.

c. Under certain conditions, where the air is not the superior motive force, but may be assumed to rest in equilibrium upon the water, the water will carry this by its motion as it would any other floating body. Now, as the weight of the air is about 2000 lbs. per square foot, we may readily conceive that its direct momentum, where it is carried constantly forward in superimposed masses of hundreds of miles in extent upon the great oceanic currents, represents an immense force, the general impulse of which will be felt for a considerable distance inland, where such force impinges upon a coast which entirely resists and deflects the direct momentum of the water. The general conditions of the above I will hereafter more particularly consider.

d. We gain very material assistance, in our general investigations of the principles discussed above, in that our best powers of observation may be directly applied to exactly the opposite relative positions in natural aerial and aqueous systems of fluids:—In the aerial we can observe with exactness the lower surface, living, as we do, in this stratum; and in the liquid the upper, in living immediately above it. This must be in every way an advantage to us in arriving at a knowledge of the motive principles active in natural systems. It also greatly increases our powers of observing the separate conditions of fluids, both under restraint, as in the lower aerial surface, and of relative freedom, proportional to density, as at the surface of water; that is, assuming all fluids are comparable, and that they move upon like principles, irrespectively of density, of which I think my experiments leave no doubt.

e. In some cases, which are quite common in established cosmical currents, the contact of air and water in motion of terrestrial displacement is quite frictionless, from the equal velocities finally induced by reciprocal action of these fluids upon each other. This arises from the constant tendency of separate fluid forces to induce equality of motive velocity and direction at the plane of meeting of two fluids moving against each other. It is almost entirely from this cause that we observe in certain localities oceanic and aerial currents to be persistent in one direction by the constant action of forces in *one of the fluids*, some causes of which I have already discussed. This reciprocal action may occur in certain localities for longer or shorter periods. The aerial momentum, as that of the trade-winds, where directive forces are powerful and constant, may be assumed to act by cumulative efforts upon the aqueous surface, until the direction of the wind is induced in the aqueous surface. In all extensive oceanic currents, in like manner, where these are from any cause the superior or most constant forces, they carry onward the gravitating air resting upon them into their own directions, so that the great currents of air and water upon the earth are mostly nearly coincident in one direction. By the constancy of active impulses, derived from the same set of causes, certain systems of aerial currents become approximately fixed locally, over certain liquid areas. Such constant systems are not often found over land areas unless the projection of the aerial fluids can be clearly traced to the continuity of momentum of motions induced on oceanic areas; or that they are necessary as supply currents to replace local projections, as, for instance, the prevalence of south-westerly winds on certain western coasts of this country; these being even much less frequent at a short distance inland, than on the western coasts actually. Generally, as a principle, the land surface being immobile, the separate impulses of aerial momentum are here lost as cumulative forces, and represent the effects of immediate causes only; whereas the separate impulses of directive forces acting on a mobile system, as over water, act cumulatively to its directive energy. We may see further that for the higher strata of aerial motions, the lower denser aerial fluid may act as a mobile system to this, and in some way resemble, as regards the friction of resistance, the action of the lower aerial system upon the oceanic area.

120. Division of the Subject to follow Natural Phenomena.

It will now be convenient to divide the subject to investigate the actual conditions whereby we may be enabled to follow the principles previously offered into natural phenomena. For this division I will take

1. Horizontal Motions in Aqueous Systems.
2. Horizontal Motions in Aerial Systems.
3. Vertical and Oblique Motions in Aqueous Systems.
4. Vertical and Oblique Motions in Aerial Systems.

As I consider in this chapter that I have sufficiently discussed the motive forces which produce oceanic and aerial currents I shall henceforth only refer to such currents as forces existing, except for certain special cases depending upon peculiar conditions or local circumstances.

CHAPTER XI.

HORIZONTAL WHIRL AND BIWHIRL SYSTEMS OF MOTION OBSERVABLE IN NATURAL PHENOMENA IN WATER:—OCEANIC AND FLUVIAL WHIRL SYSTEMS.

121. Projection of Rivers into the Ocean.

a. The outflow of a river upon entering an ocean or a lake may be taken as a very definite form of horizontal projection for a large mass of water subject to less interference than in open currents generally. It will therefore be convenient to consider this first. A river has commonly a transverse section of less depth than that which would form a complete half cylinder of projectile fluid. Hence any biwhirl produced by this projection would be formed upon principles given in 79 prop., where the section of a whirl is found constantly to be actively extending its radius. We can also imagine that by rolling contact of a stream upon its bed, the higher radii of motive forces in the central or in any deeper or more free parts of the current, will produce higher velocity in these parts, causing them thereby to act as effective projectile forces to the more quiescent or shallower spaces lateral to the current. In this manner the central part will possess the highest velocity where it enters an ocean or lake, and must therefore at this point encounter greater head or conic resistance than in the slower flowing lateral parts; so that the conditions will be such that an entire biwhirl motion will be a general resultant; the whirl system in this case flattening itself out, as it were, to meet the extent of free area opened out to it at the mouth of the river. It will also follow, that in the outflowing of a wide stream, the active tangential force upon the still lateral waters, within the ocean or lake, will have a constant tendency to set the quiescent water in horizontal revolution. But if the central outflow have considerable velocity, it will after a certain time, carry its direct force far into the ocean, making rolling contact upon the lateral whirls first formed; or if the area be free from local resistance, it will engender a more extensive biwhirl about the cone of impression upon the resistance of the static oceanic water only far forward. The lateral whirls so produced may be of any extent, according to the volume and velocity of the projected liquid and the quiescence of the lateral waters. They will, if extensive, include all minor biwhirl systems engendered by local resistances, upon principles previously discussed for diffusion, 105 prop., from whatever cause. Upon these principles a stream projected into the ocean will at all times represent a *biwhirl generating force*, which will be highly developed if the stream possess great volume or great central velocity; the whirls being also extensive if the resistances are slight or distant, as in the projection of a river from a straight sloping coast in calm water.

b. I may further observe that a biwhirl being a planic system of motion, it will project its force into the ocean with very little surface disturbance in composition of forces; its motive direction being horizontal, it will also tend to calm the vertical motion of surface waves, this being a distinct means of indicating the extent of whirl action upon the liquid surface. But the most direct and certain means of observation will be that the biwhirl stream will carry with it light floating objects, which will be projected for a certain distance into the ocean, and be afterwards deflected from their direct courses to the right or to the left of the projection. These floating objects move slowly towards the central areas of the lateral whirls by spiral paths, often taking, with the current, a circuit of many miles in extent and returning to the mouth of the stream by skirting the coast inwardly; if there are not present cross currents or other forces that cause a general displacement of the whirl system.

c. From various causes in natural currents the above principles cannot generally receive full development, as in many cases one arm only of the biwhirl after projection may be a free system, and be fully developed if there be present little resistance; whereas the other arm may be absorbed or lost to observation, or in reality, by reflections and local resistances; or if the outflow of the river be very extensive and take an easterly or westerly direction, this will occur from the difference of latitude velocity alone, active upon the deflected parts. Oceanic currents, we shall find, are active upon the same principles as rivers entering the ocean, but are subject to more interference from compositions with cross currents, undercurrents, and winds.

122. Formation of Deltas at the Mouths of Rivers.

a. Upon the principles above discussed, as being active in biwhirl systems, at the entrance of a shallow river into the ocean, there will under the same conditions of conic resistance be formed forward an area of *quiescent compression*, upon the resistance of the oceanic water, which will be of such an extent as to equilibrate the flowing force of the river; this being the condition shown in 73 prop. for a cone of resistance. Now, this being the case, every particle of solid matter brought down to the ocean by the river will have its velocity retarded at some forward point of resistance, and as a current is found to carry alluvial matter nearly proportional to its velocity, it will occur that at this point of retardation, portions of solid matter, that are only held in suspension by the motion of the water, will be deposited. It will be seen further, that deposition taking place by solid matter projected into the cone of resistance in an oceanic area that there becomes developed a conoid of impression of the rigid kind shown 65 prop., and as flowing forces are easily separated by resistance (75 prop.), a biwhirl opens over the resistance formed by the deposited tertiary matter in this conoid.

b. We may further conceive, that after a local biwhirl system is formed, every particle of dense solid matter brought down by the river and deflected into lateral whirl systems, will, by its superior specific gravity to the water, have also superior tangential force in the whirl into which it is projected; that is, its direct momentum will be greater than that of a less dense body, *per volume*, as a part of the flowing system. It will therefore occur with tertiary matter carried by the current that the heavier particles will be ejected towards the circumference of the whirl on all sides. Upon the same principles also, any lighter floating particles will be drawn inwards towards the centre of the whirl system.

c. It will be clearly seen from the above that the relative retardation of the flowing forces and consequently the greatest resistance is upon the vertex of the cone of resistance, which is placed where the stream possesses the highest velocity, carrying therefore by this velocity the largest amount of alluvial matter. As this vertex is also in the axis of deflection and that the flowing force conserves its momentum although resisted; this vertex becomes the starting point of impulse that acts motive to the entire biwhirl system so formed. Therefore by this cause the greatest amount of alluvial matter is thrown into the circumference of the whirl, or more particularly upon the cone of resistance, where the retardation of direct projection and conservation of elastic force takes place. In this cone, which is relatively a quiescent space (70 prop. *e*), a delta will be most readily formed, which afterwards takes the place of a conoid of impression to the projected liquid.

d. Experimentally I have found it very easy to form deltas, with small flat currents of muddy water flowing upon shallow still water, upon principles the same as those which hold in natural phenomena.

e. It will also be apparent that as soon as a cone of resistance is formed by alluvial matter, the matter as it is deposited will react and strengthen the resistance of the cone. Further, as the action of the whirl is to eject more solid matter by tangential force from its centre, it may, by the continuity of this action, form pools or hollow spaces laterally upon the underlying land surface, in which whirls will afterwards rotate with less friction.

f. Upon the above principles, deltas will be formed by every shallow outflowing stream carrying mud, unless there are present rocks lateral to the outflow, or transverse currents. The delta being a solid conoid of impression, which permits by deflection the continuity of the flowing force by a system of rolling contact upon lateral resistances; which really exists in principle, throughout the entire stream, at all parts, as fully discussed 90 prop. If there were not present this mode of accommodation, a river would be much resisted at its outflow upon a shallow coast, and thus form a lake by the tertiary matter brought down, which does not occur frequently; but generally the tangential force of the projected current deflects its own waters into lateral whirls, which again throw off lateral whirls, and the system by the continuity of the current soon attains a velocity in a permanent whirl system, which henceforth offers perfect lateral rolling contact to the projectile current, so that it finally flows outwards into the ocean without great loss of its initial momentum.

g. It may be observed that as soon as a whirl system is established, although the deflection of flowing force conserves its elasticity, and consequently its greatest velocity at the most exterior or eccentric free portion of the whirl system, that this exterior is also the plane of greatest resistance to the whirl. The induced velocity in the central part of the system constantly eliminates force into the external resistance; therefore a whirl once formed is also at its complete formation in a state of dissolution at its exterior surface, where the resistance is gradually reacting upon the projectile force. It thus occurs that a certain part of the tangential force in the whirl may, at any open point in its course, become a current in mobile lateral equilibrium, and in this state be split off from the whirl system having tangential force left direct, for this deflection into the open fluid. In this case it would in continuity of projection immediately meet again head resistance, liquid or solid, so that it would again bifurcate and form a new biwhirl system, upon principles similar to those discussed for diffusion in the ninth chapter, so that in any case of the entrance of a river into the ocean on an extensive plane of equal resistance, but not of great depth, there would be formed a *series of deltas*, or at least a series of biwhirl systems, which, if they contained tertiary matter, would develop into deltas.

h. If we apply the same principles of motion to larger systems of oceanic currents we may imagine these to be active in forming land areas, which by their pointed forms meeting the oceanic currents directly, will resemble deltas. In this manner the establishment of a powerful biwhirl system from any cause will leave a delta of rotative repose where whirl forces are inactive, although they will be most active about the outline of the delta (cone of resistance). I think it possible to this cause, and the general influences of whirl action that we owe the pointed forms of continents and islands which are directed very generally towards such currents proceeding from the more open oceanic areas, wherein directive impulses are most evident. The whirl action in these cases, tending by tangential force in the masses of oceanic waters projected, to wash away prominent coasts that are frictional to the whirl forces, and to deposit the matter detached in exterior areas to the whirl system, where the least interference occurs. Thus for instance, currents directed from the Southern Ocean would establish whirls upon any headland, and these whirls when established, as for instance in the S. Atlantic, Indian, and Pacific oceans with free oceanic areas for projection from the south, would have a certain tendency to conic pointing of the continents of South America and Africa. In like manner, whirl-forming currents directed northward by thermal causes in the Arabian Sea, and Bay of Bengal, open to the impulse of the Indian Ocean, would tend to the conic pointing of India, and the same would occur also in Greenland to the Atlantic currents. And generally, the greatest oceanic area lying actually southward, the pointing would be generally in this direction from the greater magnitude of the fluid forces; the motive heat forces, summer and winter, giving, as before proposed, direct impulse. The obliquity of the earth's axis in giving southern and northern momentum to currents by thermal effects would, also by the differences of rotational velocities of the latitudes of the earth, give directive influences to the current formed by expansions and contractions, and tend to place the continents opposed to the currents, formed and directed from the open oceanic areas, a pointing somewhat *westward*, as these westerly sides meet more particularly the whirl forces. This matter I will endeavour to develope further in special cases; the present conditions refer to deltas only. I do not claim for this principle more than an influencing cause. It could scarcely be assumed to affect the contour of an igneous system of rocks except in a small degree; but if a surface of land were of loose tertiary matter and rising in an oceanic current, the current might be influential in establishing the contour of such land, and as it continually rose, it would continually act upon the land; the whirls being more set in form as the resistances became in conformity with the least frictional contour.

i. The conditions taken above, as stated in paragraphs *a* to *g*, of the outflow of rivers, are taken for coasts that are shallow relatively to the forces of the currents. If the outflow of a river be into deep water, semicylindrical whirls will be formed, as in projections from a pipe (68 prop.), taken for a unit projectory. In this case alluvial matter will be deposited and eroded nearly equally about the central projection, so that the current will be projected, and form for itself a channel, upon conditions to be taken presently for the conservancy of water courses.

123. Formation of an island in a river or current.

From the conditions offered above we may conclude that a river of small inclination flowing in its own bed will have, to a certain extent, the same disposition to divide by front resistance, and the influence of rolling contact upon its banks, as in its outflow to the ocean;

which will be particularly evident where it passes over a short distance of flat country. In this case also, not very large impediments in the central area would support a cone of impression, and at this position the stream would have a strong tendency to bifurcate, the flowing force being thus deflected, so that a central island would be formed, on the same principle as the deltas before considered, if the banks at this point were not too resistant to restrain the whirl force of the biwhirl present at the point of resistance. On the other hand, as soon as the stream has separated about the island, the separate streams would again form biwhirl systems, whose tendency would be to meet again further down the stream by the minus resistance towards its former channel. The same principles will hold for the formation of an island in an oceanic current.

124. Conservancy of Water Courses.

a. By the above conditions it will be seen that a river flowing through a flat country will have a constant disposition to expand, and will not cut out for itself a central course unless there are present some local causes, geological or otherwise, to produce this effect, that is, as far as the whirl system of its motion directs its waters; this point has been already discussed 91 prop. But in this very same system we find a compensation, in that the higher central velocity directs lateral whirls, by which alluvial matter, upon principles just discussed, will be carried towards the banks by the force of the tangential projection of these whirls. Further, the borders of a stream being relatively quiescent, in proportion as the waters have less velocity, alluvial matter will deposit in lateral parts, and by this means conserve the central course open.

b. The conditions under which a current will be best conserved, upon my theory of whirl force is that in which its bed is of some cylindrical section not less than half cylindrical, except that there may be a small function in equation for gravitation of the water. This will appear to be the best section, by the conditions of the whirl ring cut in horizontal section, 68 prop., as the whirl system would then exist in equilibrium, the equal divergency of the water in the whirl from the central deflection impressing the same force at the bottom of the stream as at the sides. The current would therefore be eroded and have alluvial matter deposited equally, and be thereby maintained intact in its form.

c. If the section of a stream be shallower than a semicircle there will be a constant tendency to divide or to deposit mud in the centre, with deflection of the current to one side or the other, where the whirl system attains greatest tangential force. This may be important in some engineering works in the economy of river embankments, as semicylindrical streams may be formed of less area than is usual with economy of construction, the best form being at the same time acquired thereby, to keep a constant open bed with the smallest friction of resistance to the free gravitation-velocity of the current. In the same manner unrestricted direct open currents of the ocean will be also of semicylindrical section.

125. Active Principles of Biwhirl and Whirl Motions in Oceanic Currents in Superficial Areas, generally.

a. Perhaps the most clear evidence of the direction taken by motive forces in fluids, the action of which, must be as all forces in direct lines, may be found in the great masses of water free from the near resistance of solids contained in our oceans, the *eddies* of which will be the whirls or biwhirls of the systems of fluid motion proposed in this treatise. I have not used the term *eddy* as this term, like that of *vortex* motion previously discussed, has sometimes been applied to other forms of motion than those I wish to consider; in some cases, where the mind could only recognize confusion or opposition, to what it conceived to be the logical principles of motion, and where the active causes were entirely unknown or undefined.

b. The outflow of a river, discussed 121 art., presents a general idea of the disposition of liquids to bifurcate, upon a large scale, when projected upon quiescent or oppositely directed moving matter that offers conic resistance to the flowing force. The most important difference, in this respect, between the projection of rivers into the ocean, and the projection of free oceanic currents, is that in the ocean, from its great depth, we generally lose the effect of that part of the resistance due to accumulations of alluvial matter, carried by the river current, and which is afterwards active indirectly, upon the flowing water to direct it to form deltas or equivalent resistances, by symmetrical cones of resistance; under these conditions oceanic currents have greater freedom, therefore greater radius of motion than those considered for outflow of rivers. The only instance in which the conditions of an oceanic current is nearly equivalent to a fluviatile one, is that wherein a flowing current from any cause is directed towards a coast; in which case, the coast will so far resemble a delta that it will support a cone of impression, and deflect the flowing current to the right and left of its direct course by biwhirl action. Even in this case it may seldom occur actually, through indentation of coast or otherwise, that the current will strike the coast directly normal to its line, so that the cone of impression that may be formed possesses a base symmetrical to the current; but more generally there will be such variation by deflection from a right angle that the resistant water, in front of the coast, will produce an irregular biwhirl, in which one whirl will largely predominate over the other.

c. In oceanic currents moving by rolling contact where the water is deep, the friction of surface motion is very much less, perhaps nearly proportional to the depth, taken as a radius as before proposed, other conditions being equal; therefore in deep water and open oceanic space, larger areas of water are relatively free to form whirl systems by much smaller active impulses of tangential force in moving upon the inertia of lateral masses. Further, oceanic systems of motion, from the extent of the masses finally moved by cumulative efforts, and the momentum these masses afterwards carry, are very persistent in the continuity of any system of motion once induced. This principle applies either to motions of rotation or of translation in direct lines; such motion being subject to less local change, when smaller forces are impressed at intervals of time or locality; which enter cumulatively only, either as accelerating or disturbing influences, in the general composition of their forces of established movements, which these large masses of fluid at the time may possess.

d. If we now consider some of the great oceanic forces of the globe, we have in these, masses of many thousand miles of surface water of great depth, moving with velocities of twenty or more miles a day, the momentum of which it is almost impossible to measure. We may nevertheless take these currents to represent enormous flowing forces, which we may fairly follow into their deflections and observe the effect produced on the inertia of the more quiescent systems of water, of equal or greater mass, placed contiguous to them. We may take for one instance of a mighty current, the united western equatorial drift over the North and South Atlantic as a case of the projection of fluid force on a very large scale, the origin of which is derived from thermal and rotational forces discussed. This current may be conceived to be formed from the combined impulses impressed by aerial and aqueous forces, which together cause the projection of a liquid mass that occupies the greater part of a broad belt, very definable in the Northern and Southern Atlantic Oceans, extending generally over about 25 degrees of latitude, of which the thermal equator in about lat. 2°·5 N. is the central line. These large currents meet only at their western limit, leaving a medial band of about 5 degrees of calm and countercurrents, the cause of which will be hereafter considered. This entire united equatorial current in both northern and southern hemispheres, although it varies slightly at different seasons, we may consider as a constant projection of fluid force which attains an approximately equal velocity in equal latitudes over a large part of the extensive areas taken. We have also in the Pacific Ocean a similar projectile system derived from a similar set of causes.

e. Now taking into consideration the direct momentum of the flow of actual currents, upon principles discussed in the propositions of this treatise, we witness in these equatorial surface motions, powerful flowing forces moving tangentially to extensive free lateral masses of deep water; the effects of which, upon the principles of whirl motions discussed, is to engender rotary systems, whose amplitudes are only limited by the extent of the free area of the contiguous lateral waters. As these whirl systems are the resultants of cumulative action from friction laterally upon local volumes, their persistence will make them clearly definable by the directions of actual currents. The effects of whirl principles of motion, if active, will therefore be observable in all open spaces exterior or lateral to these great equatorial currents, even to the extent of the greatest open oceanic areas of the Northern and Southern Atlantic, Pacific, and Indian Oceans, whose whirls are formed in the greatest free area of *equal* resistance, in circular motive systems, 81 prop., or if under unequal resistances, in deflected rotational systems, 83 prop. In these circular or deflected systems, where there is present any local interference of land to resist the direct impulse, there is generally formed a biwhirl that divides and deflects the waters right and left, according to the position of such resistance, and its adaptability to support or deflect a cone of resistance; such natural systems of whirl action may be now separately considered.

126. Whirl System of the Southern Atlantic Ocean.

a. Perhaps the best conditions for the development of a complete oceanic whirl, of extensive dimensions, occur in the immense bay, if I may so term it, of the Southern Atlantic Ocean, which was before taken for demonstration of rotational movements, 115 art. *h.* Here, between the coasts of Africa and South America, the inflowing southern equatorial current is deflected and directed at its outset by the influence of the equatorial parallel of land reaching along the coast of Africa from Fernando Po to Cape Palmas, where, were it not for the resistance at the equator, caused by the directive action of thermal forces, the aqueous force in whirl circulation would take a circuit onward to this coast and throw the equatorial current in one broad equatorial band direct upon the immense headland on the South American coast, of which Cape St. Roque forms the forward point of resistance. This is possibly the direction taken by the undercurrent, or denser system, which derives its force largely from the impulse of the open Southern Ocean, to be hereafter discussed. The surface equatorial current, which follows directly the thermal effects in this region, keeps near to the equator, moving from east to west, possibly uniting the surface currents with the underflowing whirl proposed, so that the entire system projects its direct momentum upon the same cape of St. Roque. The united current is here split against the cone of resistance that forms in the static water in front of the cape, and by the cumulative compression of the elastic force developed by the resistance, divides the current, three hundred miles before it reaches this headland of St. Roque, 40 prop. *d.* The coast supporting a cone of impression, as in the case shown 77 prop. *e*, causes a general bifurcation of the Atlantic equatorial current at this cape, which deflects the direct force of the current north and south. The northern deflection flowing along the north-eastern coast of South America forming the Guiana coast current, and the southern flowing along the coast of Brazil, forming the Brazil current.

b. If we take the southern equatorial current as it exists at a position of great motive activity, say where it crosses the meridians from 10° to 20° west; it here forms a band of nearly 500 geographical miles of average width. If we consider this as a motive liquid moving at about 20 miles per day, directed by its westward drift upon Cape St. Roque, it would be clear that such an immense stream could not be brought entirely against this cape, or upon any cone of impression we may imagine, directly in front of it. It must therefore, upon principles discussed, have the central portion of the current deflected over such a conoid of impression in a manner that the deflected waters themselves may form the head resistance to the following flowing parts. In this manner the directive momentum of the current by the smallness of divergence from direct impulse that it makes upon the covering of the cone of impression, if I may so express it, will react cumulatively upon its velocity, by the effects of lateral compression; so that the deflected current in composition with these forces will have its velocity increased thereby, after deflection, upon the same conditions as those discussed for the obstruction of a pier of a bridge in a running stream, 39 prop.

c. In this current we find, that the velocity is most increased upon the northern side, from causes to be hereafter discussed. The Southern Atlantic equatorial current as it appears in the excellent physical map of the world by Herman Berghaus of Gotha, which I have followed in these researches, after bifurcating in the ocean opposite Cape St. Roque, takes a grand whirl over forty degrees of latitude and the same of longitude with a small deflection only at the Cape of Good Hope, returning to complete the whirl by the African coast back into the equatorial current from which we at first traced it.

d. This southern whirl current passing along the coast of South America, where it forms the Brazil current, afterwards suffers deflection from release of elastic force at the large bay south of Rio de Janeiro, and there, I have no doubt, forms a smaller whirl system, the cone of impression being shown to exist in about lat. 24° S. and long. 37° W.; part of the deflected current flowing outward past Rio de Janeiro towards Cape Horn, so that this great whirl system of the South Atlantic becomes weakened by these deflections, upon principles of diffusion of force (106 prop.), that it afterwards flows slowly only in what is termed the Southern Connecting Current, onwards to the Cape of Good Hope. This current must nevertheless be otherwise considerably strengthened by the excess of revolution velocity that it carries with it in flowing towards higher southern latitudes, as also by the influence of currents in the Southern Ocean, which I will hereafter consider.

e. If we conceive the southern equatorial current to form a biwhirl whose cone of impression has for its base the extent of land upon the South American coast extending from Aracuti to Quinata, and whose vertex is the small island Fernando Noronha which forms a part of a natural delta to this current. Then the largest area for this southern whirl, upon principles discussed 82 prop., will be that of the greatest circle of equal resistance. This upon a map of the South Atlantic may be found to have its centre in about lat. 20° S., long. 14° W., but from deflection caused by the great width of the equatorial current, the centre of inertia of the waters of this ocean is probably about 3½° further south, or exactly upon the tropic of Capricorn; and by the principles of its projection, if we were to stand at this point, which should be one of constant calm as far as oceanic movement is concerned, then by causes now given, the water should be moving in every direction tangentially to our central position from right to left. The entire rotation of this absolute area being nevertheless extremely slow near this central position.

f. Although the above may fairly represent the absolute conditions, we must not lose sight of the fact that the resistances about the circumference of such a gigantic whirl as here imagined are not equal. Thus if we describe a circle upon a globe from the position given above in lat. 23½° S., long. 14° W., for a centre and with a radius to scale of about 1200 geographical miles we shall be fairly in the current for the entire northern half of the circumference; as the resistances from the southern coast of America from Pernambuco to Rio de Janeiro will be nearly equal, and the coast of Africa from Ichabo back into the equatorial current also. The equatorial current being assumed the motive force will form of itself a resistance on the northern side, as in cases given 81 prop. The resistance will also be nearly equal upon the cone of impression about St. Roque, therefore this part of the circumference of the whirl will be fairly made out by surrounding nearly equal resistances. When the whirl current reaches Rio de Janeiro there is no cause for its discontinuity as a revolution of a conoid of persistion (79 prop.), except that we have here an area of less lateral resistance from the greater distance of land upon the southern side. Therefore a large portion of the tangential whirl force is released at this point, and thrown into the Southern Ocean where the resistance to its impulse deflects part of its volume, as before stated, further southwards along the coast of South America, and another part continues by liquid cohesion in the induced rotary system of the great whirl described; so that this whirl, although weakened, is complete for another fourth part of its course, or back to long. 14° W. Here the diminished force of the same weakened whirl current is approaching the coast of Africa, where it meets again an open area to the circumference of the whirl, in relation to the distance from the centre proposed, in nearing the Cape of Good Hope, upon the African coast, which coast, forms support for another cone of impression, so that again a part of the tangential force is split off, and only a vestige of the original equatorial projection continues in the whirl system of the African branch of the South Atlantic whirl; this part is, however, materially strengthened by other causes yet to be considered.

g. Upon the principles discussed the area taken of the Southern Atlantic would form a kind of basin surrounded by currents which would have varying forces, but would all flow from the position of the centre from right to left. There would therefore be in the central area of this immense basin a mass of water that would be constantly chafed by the tangential forces of the circumscribing currents, which by the natural cohesion of the liquid system must eventually bring the entire mass into revolution upon its centre of inertia as in 50 prop., or in some way break off connection with it near to the borders of the central mass of the system. Further, as the greatest resistances to the tangential force of the moving mass would be constantly on the circumference of the whirl, although the whirl be formed by tangential action in some other parts only, the flowing forces would be constantly influenced to move under deflection towards the central area to continue the circulation (prop. 51).

127. Whirl and biwhirl systems of the Northern Atlantic Ocean—great Northern Atlantic equatorial whirl.

a. Although there is no doubt that the same principles of motion as those discussed above, if true, must hold good in any liquid area, it will be most convenient to follow, for example, the conditions observable in oceanic currents in the Northern Atlantic Ocean, which I shall endeavour to carry further into details, particularly for evidence of whirl systems, that are there developed beyond the great equatorial whirls that I assume to exist on either side of the equatorial currents. In the northern area, surrounded as it is by old civilizations, the conditions of surface motions are better known; and as regards climate and commerce, more important, so that the conditions present may be followed with advantage more carefully and more exactly than elsewhere. In this immense area also, by the irregularities of the bounding surfaces, there are generally interferences with the establishment of simple whirl systems, which lead us to consider principles of projections which are very generally active in other cases.

b. In the North Atlantic Ocean, by the form and position of the land of the north-eastern coast of South America, as before stated, a large part of the southern equatorial current of surface force is deflected into the Northern Atlantic oceanic area as shown by Sir John Herschel. This is probably necessarily so as the southern area is more aqueous and thereby a more free system to maintain impressed force. The northward deflections of these united surface currents is no doubt correctly conceived to be one cause of the higher temperature of the northern hemisphere, particularly where the directions of whirl and tangential forces throw a large part of the surface water upon Western Europe. This communication from South to North Atlantic produces a *longitudinal* aqueous motive system extending from pole to pole by which certain influences of the S. Atlantic Ocean enter materially into the northern system; aiding both in strengthening the N. Atlantic equatorial whirl, and ultimately in producing deflections which are instrumental in the formation of the Gulf Stream. These matters I will separately consider, taking the conditions of the Great Northern Atlantic whirl first.

c. To follow this matter into detail, as before proposed for the S. Atlantic Ocean, we find that if we describe the largest circle possible upon a globe over the Northern Atlantic, the centre of such a circle will be in about the 25th parallel of north latitude and 40° west longitude. If we take a radius of 1500 geographical miles to the scale of the globe, the circle described with this radius would include the northern equatorial current to the south, and the same circumference continued in a westerly direction would touch a point of land near Cayenne (Salut Isle) in South America, and continuing further, would include the large islands eastward of the Caribbean Sea and onward still further, include the island of Bermuda; it would then skirt along the eastern coast of the United States of America, and include the headland of Newfoundland about St. John's; crossing the ocean to the eastward, it would include the Azores, Madeira, and the Canaries, nearly touching G. Canaria, and strike the African continent; now following a southerly direction it would just touch C. Blanco; leaving that cape, still skirting the coast, it would include the Cape de Verde Isles, from which it would return to our starting-point in the northern equatorial current, which is here assumed to be the motive system acting tangentially upon the great northern whirl. I may note that the greatest circle that may be described in the Southern Atlantic to include the equator, is as stated 126 art. *e*, to be in lat. 20° S.; whereas the greatest Northern Atlantic circle is as now shown in about lat. 25° N. The thermal equator over the entire Atlantic system is possibly about 2½° north of the terrestrial equator; so that the equatorial thermal tangential axis of the Northern and Southern Atlantic great whirl systems *corresponds with the thermal equator;* the centres of the greatest circular spaces being equidistant from this. But the Guinea Coast countercurrent, which I have yet to discuss, possibly deflects, by the resistance of this coast, the southern whirl further south to the *Tropic of Capricorn.* So also the south-eastern coast of North America with the Gulf Stream deflects the Northern Atlantic equatorial whirl further *south*, and the active centre of this great whirl is therefore possibly upon the *Tropic of Cancer.* The probable reasons for these centres being upon the southern and northern tropics I will hereafter discuss.

d. If we assume this great mass of circumscribed water to have a centre of momentum in the point taken, lat. 23½° N., long. 40° W., it would be about such a centre that lateral, equatorial, and other currents would constantly tend to rotate the cohesive liquid mass by the impression of tangential forces from the equatorial system; and this centre of inertia of the liquid mass would only be displaced by great excess of force impressed in one direction as shown 83 prop. *d*, so that it would follow generally the conditions discussed for the proposed great whirl system of the S. Atlantic Ocean. Throughout the boundaries of such a system there would be tangential currents constantly drifting in one direction of rotation; which in this case, for the northern area from the western drift of the equatorial current, would be from left to right of the central area. About this rotary system, from the induced revolution of the liquid mass, there would be great uniformity of circumscribing temperatures.

e. In the central area of the great whirl system proposed, by its motive parts being surrounded entirely by horizontal tangential projections, there would be perfect calm. Further, in solid matters carried by the liquid circumscribing currents, the heavier tertiary matters would be constantly drifted by the tangential forces exterior to the great whirl, and any lighter floating matter would be drifted to the interior area as discussed for river whirls. So that at the exterior, islands would be most readily formed by local resistances, as local cones of impression (123 art.), and in the interior we should have floating weeds as actually.

f. In the whole of the above I assume the circumscribing resistances to be equal, so as to produce a perfect whirl system. Where the resistances are evidently unequal the whirl system will only appear persistent proportionally to its freedom of area. But we may, I think, take the proposed conditions of the above area to be the *establishment of motion* about the centre of inertia of its greatest open liquid mass for the N. Atlantic Ocean and consider it subject to the collateral forces that enter only into composition with it. We may do this in the same manner, for instance, as we may assume the moon to revolve by its initial impulse directed by the earth's attraction in a circular orbit by tangential forces and attractions, quite irrespectively of the influence of another attraction, as that of the sun, or of the eccentricity of its orbit; which latter we know are forces that enter into composition, and that modify the motive directions of the primitive system we assume first taken.

128. Secondary whirl systems of the North Atlantic.

a. We may now follow the conditions of acceleration and of resistance to the motive system, proposed above, for the Northern Atlantic Ocean as a whirl system, open on one side and moving upon its axis of inertia in the greatest free area, or rather that of least friction, following the direction of forces active upon it from the effects of impulses moving directly and tangentially, as also of other forces that may act through deflection in or from the direction of its rotation at any angle of impact. Further, considering that such a system of motion suffers resistance from the interferences of all land surface, at or near, its circumference, and also loss of momentum by deflections from whirl circulation by tangential projections into direct lines, where the tangential plane meets an area of greater freedom to throw off water, which formerly impressed its forces by continuity of liquid cohesion simply, in the unit whirl system in its complete form.

b. To follow this matter as it actually exists, it appears extremely probable that if the Northern Atlantic equatorial current alone were active upon the area that I have taken to be circumscribed about an imaginary centre in lat. 23½° N., long. 40° W., and that the Southern Ocean supplied no auxiliary impulse, then this area would be moved nearly as the Southern Atlantic whirl which I have already discussed, and instead of the powerful flowing force of the Gulf Stream, as it is termed, being projected at high velocity we should have a whirl moving at a velocity that would be only the mean between the united velocities of the southern connecting current and that of the Gulf Stream. This would materially weaken the projectile force of the Gulf Stream as it now exists; the Southern Connecting Current gaining, by the greater impulse from equatorial projection assumed, exactly the force that we in this hemisphere should lose.

c. If we now continue to follow the *northern* deflections of the *Southern* Atlantic equatorial current, after this current is divided upon the cone of resistance, before discussed, about St. Roque, where it sends off a considerable deflection into the Northern Atlantic area; we find this current, after division, directed upon the northern area of the South American continent at an angle of divergence of about 25 degrees to its original projection. This directive coast forming the long side of a cone of solid resistance of which, as before stated, the vertex may be represented by the island of Fernando Noronha; and as its angle to the direction of the equatorial current is constantly *greater than that of a free cone of impression* in water (72 Remarks, *b*), the equatorial current flows constantly against this solid plane of resistance without possible deflection into extensive whirl projection. It therefore enters with a large element of its active force, directly into the Caribbean Sea, the resistance of the coast acting constantly in producing a deflection by forming a compression in the water near the coastline, by which the current is greatly accelerated in velocity; the compression acting as a hydrostatic pressure to the flowing force of the following parts, under conditions which have been fully discussed (40 prop.). After skirting the entire coast of the Caribbean Sea the current is drifted forward into the Gulf of Mexico at great velocity.

d. In the large free liquid areas of the Caribbean Sea and the Gulf of Mexico there is no doubt that complete whirl systems would, under ordinary conditions, be formed; but the whirls are here imperfect, as the waters in circumscribing these areas encounter many local obstructions from irregularities of the coast. However, the most important obstruction to the formation of a perfect whirl in the Gulf of Mexico, is in that, where the whirl would fairly complete its northern area, it is deflected inwards by the peninsula of Florida, and in this deflection the current is sent at a salient angle upon the large island of Cuba, which having a north-westerly directed coast causes the current to at once bifurcate obliquely, and drift its largest volume at once into the North Atlantic Ocean, directly towards the axis of the great rotary system that I have just described as being the established whirl of the Northern Atlantic oceanic system, in its greatest free area.

e. If we consider the motive lines of one part of the Southern Atlantic equatorial current in its northern deflection, to offer a plane of resistance to the northern equatorial currents, when they come in contact in nearing the Guiana coast; the northern equatorial current may then be considered to be active entirely upon the great central whirl system I have imagined about a centre in lat. 23½° N. long. 40° W. This being the case the deflected surface water of the south equatorial current, by the directions given to it by coasts, will crowd as it were into the circumference of this great whirl system, after the Mexican current leaves the Gulf; so that the southern current will strengthen the northern system, mostly, where it now impresses its impulses upon the tangential plane of the established whirl system. This impression will evidently occur after the circuit of the Mexican Gulf upon the long extent of the south-eastern coast of the United States of America, where the largest element of the united forces of both currents are impressed, which here form what is termed the *Gulf Stream,* whose force is possibly derived principally from the northern deflection of the Southern Atlantic current.

f. We may take the above probably as the principal cause of the Gulf Stream, but there are no doubt active minor impulses which materially affect the general result. Thus the great Atlantic whirl that I proposed, whose centre is in the great Sargasso Sea, and whose force I have at present only attributed to northern equatorial currents, will be generally strengthened by all currents, and by the plus rotational velocity of deflected currents proceeding from the south. The northern equatorial current itself being materially influenced and deflected by the resistance of Porto Rico and the group of islands westward; thus carrying the deflection of whirl impulse along the West Indian Islands, to meet the outflow of the Gulf Stream proper; much earlier than that just proposed for its impingement upon the great central whirl system, whose circumference would only extend directly in this part to Bermuda.

g. Further, that as there must be generally an approximately equal amount of gravitating fluid, or of force, in all parts of a rotary liquid system, for it to be moderately frictionless under the action of gravitation upon the globe, it is clear that the volume of southern water entering the tangential system of the great Northern Atlantic whirl could not do so entirely, although it would act as an impulse constantly upon it. Therefore the southern deflected current would move against the established northern rotary system as a hydrostatic *pressure*, narrowing the limits of the current by its impulses, but moving thereby with increased velocity towards any more free area open to it. Thus a large portion of the flowing force derived from the southern system that we may assume acted at first as a tangential force under the resistance of the south-eastern coast of North America, to accelerate the great N. Atlantic whirl, would afterwards be thrown off tangentially as direct force as soon as any free oceanic area relieved the hydrostatic pressure from the circumference of this established rotary system.

h. This appears to be actually the case with the Gulf Stream, a certain volume of which may be taken to form a part of the central whirl system in flowing onward in an easterly direction across the North Atlantic in about lat. 40° N. to complete the whirl, but a great part also of its volume, or perhaps all that part derived from the southern equatorial deflection, is possibly thrown off tangentially to the rotary system at the first oceanic area of possible freedom; which in this case occurs, after the united whirl and deflected currents have passed the influence of the land of the United States beyond the Newfoundland coast, as before stated.

i. If we follow the direction of the tangential current that is carried forward in the whirl system of free fluid about the proposed centre in lat. 23½° N., long. 40° W., we find that this moves forward over an area of great local freedom in crossing the Northern Atlantic Ocean eastward in about lat. 40° N. Therefore there is here a constant tendency to throw off the tangential water in passing these free areas, which is held on to the rotary system by the general cohesion of the liquid mass only, so that if by the conditions, there are undercurrents present, the principles of which I will hereafter consider, or any other static water near, then under these conditions, the surface waters will have power by their free velocities to leave the central system, their places being supplied by indraught from below, or within, to complete the tangential whirl system. It is very possible a case of this kind occurs near the western coast of Portugal which supports a secondary cone of impression to the easterly deflected North Atlantic whirl current on its northern side, so that by this means only a portion of the original rotational system continues in the surface whirl towards the African coast, a large portion of the easterly deflected current, carried through the freer part of this rotary system, being split off and deflected round the Bay of Biscay. This deflected part possibly bifurcates again in meeting the general north-eastern drift of the Gulf Stream, with which it again partially reunites, and passes into the main current skirting the British Isles. In its further course, by the continuity of whirl forces, the Gulf current as a diffusional system throws off offset whirls into the English Channel, St. George's Channel, and the North Sea; and further continuing under deflection of the resistance of the coast of Norway, directly into the Arctic circle in Barends Sea, with a whirl deflection into the White Sea, where warm tropical currents may be traced all the year round.

j. Following again the direct Atlantic easterly drift at the point where it is thrown off the rotational system, previously discussed, near Newfoundland, as a great current moving in its central area through a curve from an eastern to a north-eastern direction by the deflections at its borders, we find it throws off whirls over every free space upon which the main current moves in tangential contact. Thus taking now the north-western side of this great north-easterly current, or Gulf Stream, and following its course after it leaves the coast of Newfoundland, we find it produces an immense whirl between the coasts of Labrador and Greenland; further on another to the south of Iceland, subject to deflection to the north of Iceland; and another under similar conditions on the western coast of Spitzbergen, and onward by smaller whirls to the extreme open areas of the Arctic seas; where diffusional whirls complete the direct elements of the surface projection from the great tangential deflection of the Northern Atlantic equatorial whirl system.

k. By following the principle of describing the greatest circles, and applying this to oceanic areas, as before, for the great whirl of Northern Atlantic. If we again strike the largest circles that are possible upon a globe, in oceanic spaces contiguous to this, in the remaining areas of the Northern Atlantic upon principle discussed 82 prop.; the centre of the second large circle would occur in the great bay formed at the mouth of Davis' Straits, which would have a diameter of about 660 nautical miles, its centre being in lat. 55° N. and long. 46° W. Referring to Berghaus' map, before mentioned, I find an eddy centre in lat. 56° N., long. 45° W., but I anticipate this centre would be drawn inwards the amount shown in the map by the influence of the open oceanic area of Davis' Straits. Taking another circle tangential to this second circle in the next northern open area, this may be described upon the globe in about lat. 55° N., long. 25° W., if the circle have a diameter of 900 nautical miles it will touch Ireland and Iceland, and impinge upon the ice pack off the eastern coast of Greenland. This circle would cut into the circumference of the first taken off the Labrador coast. Referring again to Berghaus' map, I find the direction of current lines indicate such a centre, but the volume of the north-westerly directed current deflects this current somewhat further westward. I might carry the principle further, but I merely offer the above cases as examples of a motive principle. I have already demonstrated that such systems may be deflected (83 prop.), but under this deflection they will preserve a certain motive elasticity, which will exert itself constantly to preserve the circular form of the whirl of the greatest area, in an incompressible fluid.

129. Equilibrium of the Atlantic Whirl Systems.

a. The matters discussed above may be considered to be *conditional* principles of motion that we find persistent in oceanic currents, showing that the causes I have discussed for whirl motions are not limited to large or small masses of fluid, but are constant in all cases, although there may be local interferences that materially modify the general directions the currents take by entering into composition with them. The principles of whirl action will nevertheless be clear to us, in observing the general course and direction of oceanic currents, in that where tangential flowing force acts constantly upon a mass, however large, if the friction be not greater in the mass than the flowing force, the fluid mass will be rotated, and at the same time, the tangential flowing force, if most resisted at the exterior circumference, as it is in any rotary system moving by radial velocities about a centre; the circumferential parts will always have a tendency to be thrown off tangentially into any area of less resistance open to them; or to be deflected inwards, into spiral systems, by more rigid external resistances, so that spiral whirls in closed systems are produced by exterior resistances.

b. I have before noticed a concurrent condition which is universal, that where the tangential velocity becomes small, which it does by gradual loss of projectile force before a whirl projection can reach a focal point—that the prescribed area of the interior of the system will be in perfect calm; but if the velocity be very great, and the area prescribed by the whirl not of great extent, then a whirlpool will be formed which is merely a tangential deflection of the gravitative liquid mass affected by the rotary force.

c. It is not imaginable, that such monster whirl systems as occur in the Atlantic and Pacific Oceans, in their greatest free areas north and south of the equator, could be formed by any single effort of projectile force acting for a period of time, such as we may comparatively employ to produce experimental whirls, this would be quite impossible. But as the positions of the liquid masses affected on the globe remain in constant areas, and the thermal and rotational forces continue constantly active as tangential forces, with small variation of direction only with changes of season, upon the parts of the same whirl systems, there must gradually be brought about a loss of resistance from inertia of the water to the flowing tangential force which establishes local whirls. There will also be set up secondary whirl systems surrounding the larger systems to act as friction savers. Therefore the directions of the forces and local accommodations finally become permanent in the whirls, as local systems of rotary motion, and the influences of such rotary systems extend far beyond, and into, other regions distant from the tangential currents by which they were formed; as shown by principles of diffusion of whirl forces, 105 prop.

d. In all deviations of direction or deflection to form whirl systems we must remember that the whirl is formed by the *resistance*, and that this is only the least frictional form of motion to continue a current. Therefore if a fluid be quite free and open into which another like fluid is projected, the projected fluid would, after a certain time, by forming systematic lateral whirls, maintain a *direct* course so far as it was able to overcome fluid resistances.

Further, the static form of head resistance to a direct flowing current, that I term a cone of impression, will be constantly wasted away by every separate or continuous effort of direct projection upon its vertex, and by the deflections from its sides (73 prop. *d*); so that it will be, as it were, driven back into any more free area, and if there be a path bounded by induced rotary motions open for continuous projection, for the lateral fluid to move in the greatest free area, there rolling contact will be ensured, for the projectile fluid to continue its projection with small resistance, as discussed 79 prop.

Fig. 163.—Diagrammatic Whirl Systems of the North and South Atlantic.

e. The principles here offered of oceanic circulation, as cohesive liquid systems, taking the greatest circular area of projection in the Northern and Southern Atlantic Oceans, may be represented as in the above diagram. The current lines of which, by my theory as before discussed, are nearly coincident with such lines as are generally projected on our maps. I have observed that the north-easterly directed superficial current, projected by the causes proposed in the North Atlantic, although deflected by the equatorial whirl, will be established by the constancy of the projection of the current so that it throws off a nearly direct current after leaving the coast of Florida, which moves constantly, as previously discussed, with little variation over a definite area. Under these conditions this current will have its centre constantly in the line of *least general lateral resistance*, and by this means move almost directly into the open northern oceanic area, establishing circulatory systems by leaving all lateral resistances approximately equilateral to it; consistent with such local rotational systems being established in the largest bays and most open areas.

f. It is from the above causes, that the north-easterly directed Atlantic current by the constant direction of its flow, and by the constant action of winds, which I will hereafter consider, reaches in the early autumn far into the open ocean to the south of Novaya Zemlya, and by the same causes that one winter isothermal includes the western coast of Norway in the same temperature zone as a part of England. By these principles it would also be clear that if from any cause this current should have its centre of equilibrium changed, as for instance, supposing the coast of Norway to be blocked with ice to the north, then the current would not be in equilibrium in its present eastern position, impinging obliquely on the coast of Norway, upon which, under the proposed new conditions, it could attain no freedom for its deflected waters, as then the elements of greater resistance, would by forming a new whirl, deflect the axis of the current far to the north-west of its present position; so that instead of whirl systems of very limited circumference impressing direct momentum upon our western coasts and inducing temperate climates from the North Atlantic Ocean—large oceanic whirls would be formed in this area, in front of our coasts, which would bring waters and winds after deflection from higher northern latitudes; so that Iceland, by such a deflection might enjoy the temperature approximately of Great Britain, this last having its temperature proportionally reduced, so that our seas would be loaded with northern ice.

g. I have founded the above paragraph upon the conditions of the open oceanic area to the north of Europe being wholly or partially closed, or a more westerly one opened; but at the same time we may conclude, that so long as this area is entirely oceanic, the impulses that cause the northern current, and the plus revolution velocity that gives this current an easterly drift, would at all times tend to open out this oceanic area, and even to wear away the north-western coasts of any land that might oppose its direct impulse; nevertheless, local changes, as, for instance, an encroachment of the ice pack that commonly rests to the south of Bear Island, would materially lessen the free area for flow of superficial water round the northern coast of Norway, and cause the biwhirl that splits over Bear Island, to deflect more surface water northward into the Spitzbergen whirl, and less towards the White Sea one, thereby changing the centre of equilibrium by throwing it somewhat north-westward, which would materially deteriorate the temperature of northern Europe, even so far as our coasts. Further, as the constant impulse of surface water in this area is *northerly*, by thermal causes, and *easterly*, through deflection by the plus revolution velocity of the globe to waters moving northwards, a resistance that by any cause, as by the presence of land, or ice to the eastward, that overcomes revolution impulse, leaves only the constant northern-directed thermal impulse derived from the original tropical expansions and polar contractions, which would become, by the eastward resistance, nearly inactive upon the polar liquid system after a very limited projection.

h. As the oceanic systems of the earth maintain a nearly constant equilibrium with gravitation at all parts of its surface, the great North-easterly Current or Gulf Stream must of necessity have an accompanying equal countercurrent of water directed back to the area of original projection. This is now reasonably supposed to be partly by a density undercurrent, but as this current must carry with it also a tangential *horizontal* force, the countercurrent may be looked for on the western sides of the oceanic polar areas, from the motive current being easterly. So that in this case we have the Greenland Current carrying forward icebergs from Arctic areas in flowing directly towards the point of projection we have taken off Newfoundland; to complete the circulation of what may possibly be represented as a great circumscribing deflected whirl of this northern oceanic system. This matter I will reconsider by conditions of vertical circulation with which it is compounded. The countercurrent, in like manner, impinging upon and entering the great northern equatorial whirl near Newfoundland, will be again carried by this whirl as an undercurrent from its density to the eastward, and by this means return the plus gravitation water, deflected from the southern equatorial thermal or surface system, back by the African coast into the Southern Atlantic Ocean. Some conditions of the above I will further discuss in components of vertical circulation.

130. Whirl Systems of the Southern Ocean.

a. The impulses derived from thermal causes (112 art.), by which the fluids upon the globe are displaced both north and south in composition with the revolution velocity of different latitudes (114 art.), produce no doubt the greatest superficial effects within tropical areas. We find nevertheless in these areas by the actual circumstances the movements of oceanic currents are under a certain restraint from the situation of continents, the coasts of which deflect the direct momentum of such currents, under conditions already discussed. By this cause we may imagine that the diurnal thermal impulses would be altogether more powerful if they could act cumulatively, for direct projection of the aqueous system in equal open latitudes, as, for instance, that the tropical area were entirely oceanic. This condition of freedom from the interference of land we find nowhere upon a single parallel of latitude upon the surface of the globe, except in the Southern Ocean. Thus in the distribution of land in the tropical area we witness three great barriers to direct projection; those of North and South America, Africa, and the immense group of large islands extending from Sumatra to New Guinea, forming the Malay Archipelago. Further, it is not only the obstructive positions of the land, but the areas also, which may be roughly estimated to occupy the large extent of 60 degrees of longitude in the tropical band, or about one-sixth of the entire circumference of the globe, that may be conceived to cut off space from the extent of a comparatively open oceanic circulation.

b. In the Southern Ocean, taking the parallel of lat. 40° S., we find the land offering one perfect obstruction only, that of South America, and two imperfect, those of Tasmania and New Zealand. These last, as open islands, may be considered to cause deflections of currents rather than to form obstructions. The entire land area in the parallel of latitude 40° S. being only in the relative proportion of about one-twelfth that of the average tropical area, or about one-seventieth part of the circumscribing oceanic area in the same latitude. If we take a still higher latitude, say 50° south, we have here only the small interference of the land about the south of Patagonia, causing a deflection round Cape Horn. So that in this latitude we may consider we have the ocean nearly free to the action of rotational forces, which in one place only, off Cape Horn, is restricted to so narrow a limit as 10 degrees of latitude.

c. Now returning to the conditions of thermal projection from the equatorial area, previously considered, if by equatorial thermal forces, whirls are formed in all the great southward oceans, the Pacific, Atlantic, and Indian, the southern sides of such whirls will be placed within the free area of the Southern Ocean, and it is here as discussed for the South Atlantic (121 art.), that the tangential forces of these systems will be set free, except inasmuch as the whirls are withheld by their cohesion and by gravitative forces as complete circulatory systems. Therefore we should anticipate that the tangential equatorial forces would, through the southern whirl systems, send directly round into the free southern latitudes of the globe, a current whose course would be from west to east, in all the oceans of the southern hemisphere. Further, that this current, acting cumulatively by impulses impressed through ages of time, although by the velocity of impression of tangential whirl forces it should not attain a great velocity, yet by continuity of similarly directed impulses and the absence of direct resistances from land, it would attain in time immense volume-force; so that its momentum would react on the general system of forces as a certain connecting, or *ruling force*, in the cosmic fluid systems of the globe. This we may conceive as an active effect, although there are no doubt other conditions (some of which I will yet consider), which influence the general direction of the currents formed in the manner assumed.

d. If we consider the Southern open ocean of such initial importance as to impress certain functions of a ruling force, upon other motive systems in open contiguous areas, we may then consider how far the general distribution of land, as we find it, may be consistent with such a system, in relation to the extent and forms of oceanic areas, and the obstructive effects the positions of the land areas offer to free oceanic circulation generally. Assuming at the same time that oceanic currents are powerfully erosive forces, whose impulses are largely directed to keep their channels open by wearing the land surface upon which they act, and afterwards by depositing the eroded matter in more quiescent parts.

e. Now taking, as a first condition, the dimensions of oceans in relation to the extent of prolongation of continents from the equator, in a southerly direction by which they act as resistances to the open Southern Oceanic currents, by the coasts opposing such currents by their eastern sides; upon which the tangential whirl forces are assumed to act. We in this find that the greatest prolongation of a continent is that of South America, which extends to

55 degrees south of the equator, or 3300 geographical miles; the Southern Pacific Ocean *eastward* of this, taken at a point central to this land resistance, at about mid distance from the equator to Cape Horn, would be in 27½° S. lat. Here the latitudinal extent of the Pacific Ocean would be about double the southern extension of land, or about 6600 miles from South America to Australia. Now taking the southern prolongation of Africa from the equator as about 34 degrees, or 2040 miles; the width of the Southern Atlantic at the mid easterly land space 17° S. lat., or from Port Seguro, South America, to Great Fish Bay in Africa, this is about 2900 miles, which is nearer in the proportion of 2 to 3; but here we may imagine shading influences from the direct impulses of the Southern Ocean in the great prolongation of South America places the circulatory system of the S. Atlantic much further south, so that it more nearly corresponds with the tropic of Capricorn where the distance between the continents of S. America and Africa is greater. If we, in conclusion, consider the Malay Archipelago in its obstructive effects upon direct currents of the Southern Ocean as somewhat equivalent to a direct prolongation of Asia into Australia and measure this with Australia, as a continent, we have about 38½ degrees of southern extension from the equator, or about 2300 geographical miles. The average mid-width of the Indian Ocean in about 19° S. lat., including Madagascar, is about 4200 miles, this proportion approaching more nearly to that of the Pacific, so that we may altogether assume that the prolongations of continents are somewhat equivalent in proportion to the extent of the oceans to the eastward of the open Southern Ocean.

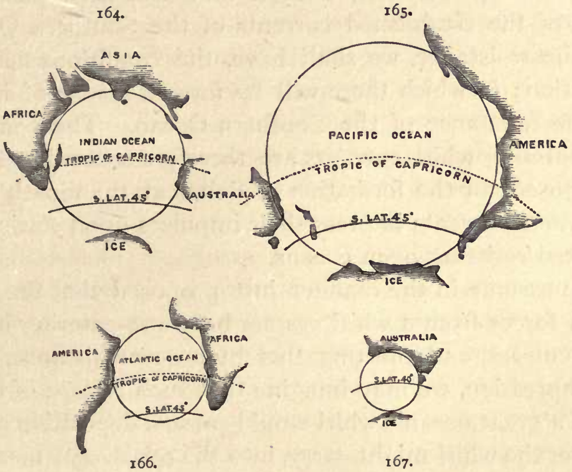
f. If we take this matter in another manner supposing the formation of land areas influenced by oceanic whirl action, and describe the greatest circles possible in the great oceanic areas open to the Southern Ocean, upon principles previously discussed for the N. Atlantic, 121 art., we could thus describe in the Pacific Ocean a circle with a radius of about 3300 geographical miles from a centre upon the tropic of Capricorn in long. 142° W.; starting upon the coast of Patagonia, this would follow very approximately the entire western coast of South and North America as far as the north of California, where the coast strikes out a little from our circle, this circle continued through the N. Pacific would include the Sandwich Isles, which it would pass about 11 degrees to the northward; now turning southward it would touch the isles to the east of New Guinea, which it would leave to the westward, and strike along the eastern coast of Australia, leaving New Zealand the only large land area outside the circle. Continuing the same curvature southward it would fall directly into the deep Antarctic bay of South Victoria, where the circle would meet interference from Antarctic ice, until it again followed approximately the same curvature in continuity past Graham Land outwards to the eastward back to our starting-point near Cape Horn.

g. Following in the Indian Ocean the same principle as the above the largest circle, taking again a point on the tropic of Capricorn as a centre, could be described in about long. 80° E. with a radius of about 1700 geographical miles. With this radius, in describing a circle on leaving the western Australian coast, it would skirt Sumatra, continuing northward to Ceylon, onward it would include the western coast of Madagascar; and continuing south, leave Kerguelen Island to the northward, completing its curvature in the open ocean back to Australia, our starting-point.

h. The Southern Atlantic I have already considered. The largest circle in this ocean could be described again upon the tropic of Capricorn in about long. 12° 30′ E. with a radius of 1400 geographical miles, the circumference of which would follow very fairly the influence of land resistance in this immense bay.

i. The three circles above described would include the southern sides of nearly the entire Southern Ocean, except that part which is opposite the immense bay formed by the southern coast of Australia, in which, upon the parallel of lat. 40° S. and long. 127° W., another large circle could be described with a radius of 900 geographical miles.

j. I do not for a moment assume that the great circulatory systems here proposed to take some elements of direction from the open Southern Ocean, do not meet with many interferences and deflections quite irrespectively of plutonic forces by which the land areas were elevated. Thus the thermal and revolution forces which maintain the superficial equatorial circulation proposed, would in all cases be somewhat ellipsoidal, the ellipsoids being generally prolonged from the equator in a south-westerly direction into the southern hemisphere, and a north-westerly in the northern, by the influence of the rotation projection of the tropical upon the lateral waters. I nevertheless anticipate there will be evidence of such elements of whirl force as may be impressed by the eroding action of oceanic currents upon coast lines. This we may find not only in the delta-like pointing of continents towards the open Southern Ocean before proposed, but in the circumscribing forms of land witnessed in the hollow eastern and western sides of all continents; or where such hollows are not present then by groups of islands which complete the like concave forms, or induce upon oceanic currents effects which are equivalent. Further, it is not at all presumable that a few millions of years would produce the immense effects here proposed, but in the infinite past there is time enough for all changes, if causes are active to produce constant effects all in one direction as proposed. The positions of land relative to the extent of oceanic areas discussed above in paragraphs *f*, *g*, *h*, and *i*, are shown in the above diagrams, the projections being sketched from the oceanic centres given as the contiguous lands appear upon a globe.



Figs. 164–7.—Southern Equatorial Oceans projected from Centres given § *f*, *g*, *h*, and *i*.

131. Secondary biwhirl systems in open oceans.

a. I have already discussed causes that influence the shaping of delta-like continents by land resistance to flowing forces in the currents of the great circumscribing Southern Ocean 122 art. *h*, the principles of which are further assured by the evidence of actual currents and the circular form of southern equatorial oceans. Now assuming these continental deltas, by reaction, to deflect also the currents into whirl systems which are somewhat proportional in dimensions to the prolongation of the continent southward; then by these whirl deflections taking a large element of their motive energy directly from the open Southern Ocean, they will project their waters upon the western sides of such continents. In this case the whirl projections will be easterly to the oceanic areas, and their whirls, if continued in a complete circuit, will re-enter the Southern Ocean upon the *western sides* of the southern equatorial oceans. In such immense systems there will be considerable elements of freedom which will cause them upon re-entering to almost resemble direct projections, so that they will here encounter the resistance of the established currents of the Southern Ocean, and through the resistance, we shall have, the conditions necessary to biwhirl action; in which there will be formed a cone of impression against the resistance of the Southern Ocean. The conditions of these re-entering whirl currents are therefore somewhat similar to those proposed for the formation of deltas at the mouth of a river 122 art., where currents impress their impulses upon static or otherway directed resisting open oceans.

b. If we assume in the manner just proposed that the release of tangential forces from a whirl system before re-entering its original projectile current, and meeting thereby a new resistance, to form a cone of impression, we may imagine that such release of tangential force from a great oceanic whirl would cause a deposition of any tertiary matter the whirl might carry, into the relatively quiescent cone of impression formed by biwhirl action, as proposed for deltas; and this re-entering whirl being in the open ocean, such tertiary matter might by these conditions form an island. Thus in the Southern Pacific, such gigantic biwhirl action, as would be here induced, might be active in aiding the formation of the island of New Zealand, which would here take about the position of the cone of impression to the re-entering whirl of the ocean. The same set of causes would also be instrumental in forming the island of Madagascar from the re-entering whirls of the Indian Ocean; and in like manner, the Falkland Isles, from the more restricted oceanic areas of the South Atlantic. These land areas being, upon principles discussed, all at the south-western corners of the great southern equatorial oceanic systems. We have also evidence of like action in minor southern oceanic whirl-systems, although these are within the northern hemispheres in which the islands are similarly placed. Thus, in the Bay of Bengal, in relation to the large island of Ceylon which is at the south-western corner of its whirl system in this bay; and in the Arabian Sea, Socotra Island. Thus, taken as a general principle, the Southern Oceanic areas have islands situated uniformly near their south-western corners, where they re-enter the more open oceanic space which I assume in a certain way to *rule* the directions of their circulation. There will necessarily be influencing causes, not here considered, producing local differences of which I may instance *magnitude* only of some of these islands. Thus we should anticipate the Pacific whirl system from its magnitude would develope the largest cone of impression or island, and impress its force the deepest into the Southern Ocean; the Indian Ocean would develope the next largest from the extent of opening of this area to the Southern Ocean, and we should assume the Southern Atlantic to be a relatively cramped system in relation to forces in the Southern Ocean, by the shading of the direct impulse by South America from the full impulse of its currents, so that here we have only the small resultant of the Falkland Isles. This matter is otherwise complicated with details I cannot now follow.

c. I hope at some future period to have an opportunity of discussing volcanic conditions from my own point of view, in which volcanic forces appear to me to follow certain laws by which the system of the globe, past and present, appears to be entirely consistent with the distribution of land and water upon its surface. Being unable to discuss this matter in a treatise upon fluids I consider this subject so far incomplete.

132. Conditions of countercurrents.

a. We may take countercurrents to represent the opposite sides of whirls or of continuous series of whirls to that from which they obtain their impulsion. Examples of these countercurrents are found in 82, 83 props. for circulatory systems, and 88, 89 for parallel systems. In such systems there will always be an axis or axes of inertiæ where the forces of impulsion and of resistance are in equilibrium, such spaces in oceanic systems being quiescent.

b. If, for the sake of division of active forces, we conceive the intertropical area to be that particularly of thermal expansions and greatest revolution velocities. Then the circumpolar areas extending to the tropics may be taken to be the areas of active contractions as they are also those of least revolution velocity. Therefore, by causes previously discussed of the action of thermal and revolution forces, the tropics will be as lineal axes of inertiæ, if I may so express it, so far as the disposition or division of active forces is concerned. In this case, supposing a whirl system formed by the tropical ocean, the thermal surface forces moving into revolution velocities would act as a broad band drawn across one side of a flat wheel free on its axis. The band extending from its axle to its periphery in one tangential direction, the impulse of the lineal parts of such a band being as the radial distances of its parts from the centre of the wheel (or whirl).

c. It is most probably from observation, as well as from consistency with the direct action of the sun's rays, and the formation of the globe, that the axes of inertiæ to the equatorial oceanic systems in equilibrium are placed directly upon the *tropics.* This is also consistent with this position being that on which the centres of the greatest oceanic circles can be drawn upon the globe, as described for the southern equatorial oceans, 130 art. *f*, *g*, *h*, and for the Northern Atlantic, 127 art. *c.* The same principles hold also in the Northern Pacific, which I have not discussed, but this area meets nearly the conditions of an open ocean, in which the equatorial system is deflected somewhat southward by the influence of the great southern whirl; so that the equatorial currents of the Pacific Ocean actually fall 10 to 20 degrees north of the equator. The southern equatorial current having its centre nearly upon the equator itself. Thus taking the Northern Pacific at its widest latitude in about long. 180°, and describing the largest circle touching the equator and the north of Behring Sea, the centre would fall in about 30° instead of 23°, that is, 7° from where its position would be were it not for the deflection of its superficial currents, by the influence of the great southern whirl (130 art.). This is neglecting the influences of the Aleutian Island, the projection of which tends to the division of the Northern Pacific into two whirl systems, the complex details of which I cannot now follow.

d. In the previous discussion as regards equatorial currents I have taken the conditions of the outer or polar sides of these currents only, as best showing rotation on areas of lateral resistance and inflection of these currents in whirls, the principles of which I wish most particularly to ensure. The same systems of forces are also evident within intertropical regions where we find the revolution velocity diminished in its force by circumferential *differences.* This decrease being as the equatorial cosine of surface declination; the difference of thermal force acting in like ratio. We have, therefore, intertropical countercurrents as we have extropical ones, which are active upon equatorial lateral spaces of intermediate calm. For these there is also the influence of vertical circulation which I will subsequently discuss. The areas of equatorial countercurrents being entirely oceanic the conditions are much less complicated than those just taken, where we have interference of land. These currents form of themselves moderately parallel bands in relation to the interspaces of ocean about the thermal equator; they therefore meet the conditions of cohesive parallel fluid systems; motive upon principles discussed in 93 prop., which establishes the necessity that to every current there *must* be a countercurrent. This being also dependent upon the conditions given 88, 89, and 90 props., and of deflected whirl systems, 83 prop.

CHAPTER XII.

WHIRL AND BIWHIRL SYSTEMS IN HORIZONTAL AERIAL CIRCULATION—ELLIPSOIDAL SYSTEMS FORMED BY COMPOSITION OF THERMAL FORCES INTO SURFACE REVOLUTION—EQUATORIAL AND POLAR PROJECTIONS.

133. General conditions. Flexibility of aerial systems.

a. In considering the conditions of liquid whirl systems I pointed out that these systems of incompressible fluid maintained circular areas of motion with considerable persistence (arts. 126, 127), acting with the energy of their cohesive and elastic properties to conserve this form of motion. In aerial systems the similar directive motive forces into proportional resistances produce also *circular* areas of whirl projections, but from the flexibility of aerial systems there is less persistence of this form under inequality of resistance; the entire aerial fluid mass being in all parts compressible into less or extensible into greater volume, by the impression of exterior forces. Further local resistances, which by their positions would impede the free formation of a whirl system in a certain area in water, would by the flexibility of elastic accommodation in air complete the whirl by *deflection*, to any moderate extent, in deformation of its direct motive circular or spiral lines, and this would occur without much loss of general motive energy, which would be largely conserved under the balance of differences of local compressions in the entire elastic system. In this manner the aerial system being flexible, its whirls by pressure or deflection may continue active in *ellipsoidal areas*, under lateral pressures, or in other limiting areas under restraint, with much less loss of initial directive momentum than that which would occur in a liquid system under similar conditions; the liquid being able only to assume such elliptical or other deformation from symmetrical circular whirl projections, in a highly frictional manner, and by loss of directive momentum.

b. The extent of any rotary or whirl system in an aerial fluid, as in an aqueous one, is derived from the continuity of impression of tangential forces, and subject to deflections by resistances as stated above, the forms and dimensions of aerial whirls or other circulatory systems, induced by continuity of direction of aerial tangential currents, may be therefore materially different from the oceanic systems upon which they may repose, and from which, in some cases, they may derive their impulses, as the aerial currents can evidently move over mixed, solid, and liquid surfaces which the aqueous cannot. The solid land will, nevertheless, be the most frictional and resistant, and will generally engender secondary systems of local whirls of smaller area, or diffusional systems, by the immediate action of the resistances, that produce at the time local deflections. Whereas, over the less frictional open oceanic areas, the aerial whirls induced find modes of accommodation for more extensive regular systems of forces which, after establishment, have a tendency very generally to circumscribe the entire oceanic area, as the plane of least resistance, if the forces are in any way directed tangentially to quiescent parts, the actual conditions of which, as they occur upon the globe, I will endeavour hereafter to show.

c. By the flexibility of aerial systems they are able to move more directly to the influences of thermal forces, in composition with latitude velocities, than is possible in the more cohesive aqueous systems. Further, the aerial systems are not circumscribed anywhere by *perfect* resistances, as we find in the case of the aqueous at every coast-line, therefore aerial systems form generally *ellipsoidal whirls* upon the surface of the globe upon conditions discussed, 115 art. *a.*

134. Influences of coasts upon aerial systems.

a. Accepting the conclusions given, 119 art., for the motive causes under which horizontal aerial currents, moved by the adhesion and carrying force of the surface of the ocean, follow powerfully the directions of oceanic currents, as also that the momentum and adhesion of the air moved forcibly by exterior force, has a great influence in directing ocean currents, these forces being frequently derived from the same set of causes, therefore consistent, we can imagine that in the air, from the density of its lower stratum, that the same resistances of the land which oppose the continuity of oceanic currents will in like manner, when the land is elevated above the oceanic level, oppose the similarly directed aerial currents, and deflect them also similarly, although not entirely. Therefore aerial currents will naturally follow certain conditions that I have discussed for oceanic currents. This is further assured by the rationale of my experiments, which abundantly show that air and water move in systems of similar deflections, under like or equivalent resistances that may be active upon their masses, under all the conditions previously discussed.

b. Under the same conditions, the denser lower air resting upon the earth may be to the resistances of elevated surrounding parts of land as though it were inclosed in a containing vessel, as discussed for the oceanic systems in the great oceans; and under this condition, resistance to the direct thermal and rotational influences which produce circumscribing equatorial currents, will cause deflections from elevations of land as previously shown for oceanic systems; and therefore by like circumstances most of the conditions of rotary systems must be induced, as were discussed for aqueous areas in the last chapter, subject in this case for air, that the circumscribing resistances cannot produce anywhere *perfect* resistance, by the presence of such oceanic coasts, as before stated.

c. If we imagine the cumulative action on one part of a system to continue its projection over other parts that possess less propulsive force or even offer some resistance, the aerial fluid may be imagined to be projected over the frictional locality, or it may be drawn over it by the traction of the propulsion in a forward part, by the general cohesion of the fluid system and minus pressure produced in front, 35 prop. By cumulative action also, constant forces in the currents of the ocean will propel the superimposed currents of air, or the air, the ocean, acting reciprocally as before proposed, the one possessing the least velocity deriving directive momentum from the other. In this case the extent of projection in the liquid or the air will be proportional to the extent of the area of freedom from resistance from solids or opposing currents, so that a constant current of water may aid in directing a much more extensive current of superimposed air. Further, the resistance of a coast is perfect to the water, but the induced momentum in the air may carry directive force entirely over the coast and project the current inland, as before stated, and the continuity of the backward force of the motive system, both aerial and liquid, may continue to propel the air inland until it encounters resistance in general equation with the backward forces of propulsion.

d. If we imagine a motive system consisting of the entire air resting above an extensive liquid, the surface of which is moving at the same rate as the air—a quite possible condition—the air will form a gravitating system whose mass momentum will represent, as before shown, a force of about 2000 lbs. per superficial foot over the earth's surface. This moving mass may be propelled forward by thermal or other causes at 20 miles or more per hour in streams many miles in width, and in this manner represent a force scarcely calculable. Such direct momentum induced in an extensive aerial system cannot possibly follow all the minor surface motions of the ocean; for instance, the deflections caused by a rugged coast, but will carry forward its direct impulse almost entirely over the coast lines, as before stated.

e. Where the aerial momentum carries the wind force directly over a coast after the wind has overcome this resistance, it will be, if *impulsive*, a free projection that may be strengthened by the following parts being projected in like manner, or, if *constant*, it may by its direct momentum draw after it backward parts, as before stated, so that as a general resultant a continuous system of motion will be induced, by the cumulative action of forces in local parts. This will be particularly apparent near coasts where directive oceanic currents are known to prevail, as for instance on the western coasts of Ireland, to south-westerly winds, which are found to prevail inland as well as upon the coasts.

135. Influences of aqueous areas upon the direction of aerial forces.

a. The reciprocal action of thermal forces in moving fluids upon the globe from and towards the tropics and polar areas (112 art. *f*, *g*) will be naturally subject to such resistances as may be offered at the surface of the globe. This will be particularly evident in aerial fluids, which increase in density in proportion to the compression of air above any stratum, by which half the gravitating atmosphere remains within 3·6 miles of sea-level (art. 116, *b*), a less altitude than some of our highest mountains. Therefore it will be quite clear that land, independently of its irregularity and frictional surface, will offer in many cases nearly absolute obstruction to lower denser aerial currents.

b. I have shown (art. 112) that although the force of expansion of fluids by heat at the tropics must be in equilibrium with the force of contraction by radiation in circumpolar areas, particularly for vapours, for the atmosphere to remain at the same average pressure; that the expansive force of the tropical system will be capable of the greatest penetration from its diurnal impulsive character under the influence of the revolution of the globe (112 art. *h*). Further that by the conditions of fluid forces moving toward equilibrium through the paths of least resistance only (art. 112, *f*), there will necessarily be formed in every natural aerial system, through the irregularities of the surface of the globe, *set courses* in which fluids will move with the least resistance; and as the parts of the globe remain permanent these may be discoverable by observation.

c. If we now consider the uniformly level surface of the ocean, neglecting altogether the minor irregularities of a few feet in surface waves, we find this evidently a less frictional surface than the land which rises with various inclinations. Therefore the same land, as discussed in the previous chapter, which offers *perfect* resistance to motive waters, will offer *partial* resistance to motive air; and so far as this resistance is active, aqueous and aerial systems moved by thermal expansive forces from tropical areas will be induced to move in like directions upon the surface of the globe.

d. Upon the above conditions where the thermal forces, active in expansions within the tropics, are assumed the superior or *ruling forces*, by their impulsive character over the contractile forces in circumpolar areas, these last being approximately uniform, we find upon conditions just discussed (112 art. *h*), *that the superior force will command the course of least resistance.* In this case therefore the tropical *thermal projections take the oceanic area* as being the least frictional to their impulses, and this area becomes the motive region to inflowing polar currents; which therefore, to complete circulatory systems, must have their countercurrents necessarily over land, in the area which will be, *except the oceanic surface*, the least frictional to them.

e. Upon the above principles the aerial system of the globe may be considered as a thermal projectile system from tropical to circumpolar areas over open oceans, wherein the projectile forces occupy the whole or the larger part of the aqueous area, taking one direction; the impulse of such currents as are formed continuing active by the momentum they carry over and beyond frictional resistances. In this manner the superoceanic currents at all times form the means of supply of air to polar areas; whereas the return or counter currents, from the oceanic space being already occupied by the equatorial impulsive projections, almost universally move over the near continental areas. In this manner also the condensations of air in winter over the great continental areas of radiation, of which, in the north, central Siberia forms the centre, attain supply through the Atlantic currents where the atmosphere moves with the greatest freedom, therefore in the greatest continuity. And it is to the influence of the indraught of such superoceanic currents toward circumpolar regions that the temperature of North-western Europe is so high at the decline of the year. Whereas by the same principles of continuity of induced motions in the directive momentum of the fluids, *the established currents resist the return in this same direction with the return of spring, so that the expansions by the sun's heat over the former areas of contraction are with greater facility moved over land areas where there is no aerial direction in opposition.* Except in the advanced spring, when the northern thermal expansions quite overpower all resistances, by the increase of altitude of the sun, which moves the air by its expansion in a south-westerly direction over nearly all northern areas.

f. The courses that aerial currents take are not difficult to be observed, by the condition of the air at any point, in the amount of vapour the wind carries, as we find such aerial currents moving over the great oceans are saturated, or approximately so, by the evaporation at the oceanic surface, and when drifted on land are always moist, and ready upon any decrease of temperature to produce rain or snow. Whereas currents deflected from circumpolar land, particularly from the higher latitudes, after a certain distance have the moisture that they previously contained already largely reduced by condensation. The direction of these currents being clearly indicated by a line of dry air throughout their courses in the whole area of their return to the equator, except within nearly the extreme polar limits.

g. The directions of aerial currents into polar regions are equally clear by the warmth they carry with them, as also by the volume of vapour under constant condensation in which the vapour system parts with its latent heat and further warms the air by its condensation to water in rain, and still more so, if condensed to snow.

h. By the above conditions we may conceive the causes of a north-easterly direction being given to equatorial aerial fluids into the northern hemisphere over oceanic areas; but the resistances that turn these currents from polar areas, redirecting them towards the equator, may not be so clearly defined locally, as these will necessarily be over land areas, assuming the oceanic to be occupied by equatorial projections as before. In this direction of their course they must follow the influences of resistances which present to them the least friction in such manner as they may complete the circulation, over the continental area.

i. For the condition of equatorial projections being the ruling forces there is one cause that is permanent, besides the diurnal impulsive character of the projection, which is, that the thermal equatorial zone represents an area of the entire surface of the globe between the tropics; whereas the areas of condensation, that is, the polar areas, are relatively small, by the space between any two meridians constantly decreasing from equator to pole. Therefore equatorial fluids directed to polar areas have their amplitudes of motion restricted in such areas, and thereby produce a condensation of the elastic system, sufficient to form a resistance and cause the deflection necessary to complete the circulatory systems.

j. Where the surface is aqueous or covered with snow, from contiguity of the ocean, the polar currents are intensely cold, carrying forward fogs, that the sun's heat loses its force in partly dissipating before its rays can reach the earth's surface. This is the case in return currents from the extreme north-eastern limits of Asia and of Northern America, the directions of which I will hereafter show.

136. Cumulative action of directive forces upon the air observable under actual condition.

a. From the cumulative action of separate forces in air which act at regular intervals, as previously discussed for oceanic systems of motion, this action becomes finally persistent in areas where it can be propagated with the least resistance. Thus in the constant annual changes of temperature or of expansive action in northern areas, we find certain tracts, particularly over the oceans, are established, along which currents flow in one direction for a much longer period than the causes that produce them are active. This is partly ensured in that flowing fluids under resistance have in flowing in one direction, power to engender very frictionless forms of accommodation to their motions by whirls, and by the momentum of these as well as by the currents, to offer considerable resistance to any system of *reversal* of their direction as before mentioned, so that when these reversals are possible they are generally brought about, not by inducing at once an opposite direction, but by *some form of deflection*, by which the induced motion continues its motive lines, although through a certain circuit in the fluid system.

b. Upon the above principles, aerial and aqueous fluids generally have power to maintain motive evidences of forces impressed upon them a long period before, so that they do not readily come to rest in equilibrium with the forces that are immediately impressed upon them. This principle is particularly evident where the motive system and its resistance is entirely fluid, as in air moving over oceanic surface, the effect of which is observable in such instances as the maintenance of open polar oceans by currents whose temperature plainly points to a motive source very distant in tropical latitudes, which continues active above the resisting influences and effects of seasonal obliquity.

c. Over land surfaces, aerial currents present somewhat opposite conditions, from the effects of radiation. These surfaces in large continents have a tendency by the constant local intermittent efforts of expansion and contraction in the air, diurnally caused by the alternate action of the sun's and the earth's radiation, to bring about everywhere over continental areas where the sun's force is active, a state of local equilibrium. This principle of oscillation of temperatures, or of expansive forces, forwards and backwards over the point of equilibrium in inland areas, resembles the vibration that we apply to scale beams, to produce the equilibrium of rest, when such are in motions of great amplitude. In like manner these terrestrial areas if isolated from oceanic influences rest diurnally on an average in equilibrium, in consonance approximately with the sun's influence upon the surface exposed and the earth's radiation. The same sun's rays which according to season act obliquely to the earth's surface cover a larger area, as before stated, in inverse proportion of the radius to the sine of the obliquity; omitting conditions of differences of diathermacy of the air in the obliquity of the ray path, clouds, and other minor conditions.

d. If we conceive the principles established, that I have offered for constancy of expansile aerial and vapour forces over the equatorial regions, and contractile forces over polar regions, and of such systems engendering counter currents, particularly for the return current of dry air deprived of its vapour by condensations; we may conceive that there will be established by cumulative efforts of diurnal and seasonal influences, certain conditions of continuity of motion found only possible in a circuit, in which, as before discussed, the aerial matter will move in the planes of least resistance, wherein the superior force of original projection takes the area of least resistance for the establishment of its currents. Such circuits in the isolation of interior parts meet the conditions of whirls engendered or moved by tangential forces of exterior currents, and where the equatorial whirl projection takes as actually the oceanic area the part of the whirl that forms the countercurrent may become necessarily directed overland. Further by the continuity of the whirl motions over oceanic surface the overland aerial current receives constantly directive impulse and becomes in certain cases as persistent in locality as the oceanic one.

137. Diffusion of aerial forces.

a. Where aerial forces have not sufficient momentum to overcome all resistances in order to form a circulatory whirl system, their forces become in overland projection oftentimes diffusional. In this manner we observe that in the central areas of a continent in the return to early spring, that by the expansive force of the sun's heat, the air has a powerful force outwardly in every direction, the result of which is to retard the impulsive projection over oceanic areas, and to accelerate the parts inflowing towards the equator. In this case the thermal oceanic currents are nearly cut off for a time from the internal circulatory system, and meet immediately more powerful resistances where air is expanding outwards in every direction towards continental boundaries; so that established currents move henceforth under pressures from this resistance, forming sometimes closer systems, but more frequently systems of diffusional whirls (105 prop.), which are developed proportionally, as the elasticity of the air permits accommodation, by increase or decrease of volume, under the resistances it encounters.

b. Where the land resistances are constant for a great distance, from the extent of continental area, so that oceanic influences are shut off, such areas become regions of great stillness, where radiation forces of the sun and earth act with greatest intensity, producing, according to the circumstances, hot or cold districts.

c. This diffusional motion of air is a very common form of projection for winds, even in close areas, and is very observable with a little care in many cases. From my window as I sit writing I can observe a large number of tall trees at different distances. The wind, although it is in its general direction southerly, is veering and backing most unequally as the restless weather-cock indicates, so that when distant trees are in motion, near ones are quite still, and *vice versâ.* I can, even after watching for a few minutes, anticipate which of the trees will move next. One particular tree, a young willow of about 40 feet in height, appears to be blown periodically at intervals of about 50 seconds apart. I plumb this tree by a blind line that hangs in the centre of my window. I find that when it is blown, it bends for about 12 seconds to the westward of south, then rests apparently quiet for about 35 seconds, and then again for about two or three seconds it blows to the eastward of south; the moving parts appear to describe ellipses.

d. It is quite natural that by feelings upon our small bodies we should take it for granted that winds blow directly, but whenever the motion of the wind is made visible from any cause, as in the carrying of autumn leaves or dust, it becomes at once clear that winds move very generally in curves, often in very small visible whirls. When such whirls have broken up the aerial resistance, there will be direct central currents, and the diffusional whirls will exist at the front and sides of such currents only. But this principle of motion will be fully developed at every entering or displacing aerial current where a lineal current of projection is not established, however extensive the area, the whirls themselves being proportional to the force of projection, although composed very generally of an almost infinite series of smaller internal whirl systems (109 remarks, *f*).

138. Comparisons of actual areas of resistance upon the surface of the globe to systems of aerial forces.

a. Following the general principles above discussed, if we take the conditions of free oceanic areas and the forms of continents that are actual upon the surface of the globe, we find that we have in the northern hemisphere two large, although unequal oceanic areas, which will be the natural areas of thermal projection in this hemisphere, from causes discussed, offering the greatest facility of projection of aerial currents, that of the North Pacific and that of the North Atlantic. Over these oceans inwards towards the thermal equator in the northern tropics we have the inflow of the lower aerial currents in a southwesterly course in the well-known trade-winds into a considerable part of the tropical area, most particularly upon the eastern sides of these oceans. This inflow from the opposite resistance of like inflow at the equator from the southern hemisphere, and from the excess of rotational velocity in the equatorial regions (115 art.) deflects the inflowing aerial currents to a westerly course, so that we have near each side of the thermal equator, over a large portion of the great northern oceans, two zones of nearly uniform western directed aerial drift, or easterly wind, which we may take by itself to represent a considerable force as active upon the more quiescent contiguous circumpolar parts of the aerial system of the globe.

b. If we follow the general positions and configurations of the land which may act as resistances in relation to the less frictional super-oceanic areas of the North Atlantic and North Pacific oceans to the western drift of the aerial currents considered above; we find that the terrestrial continental land systems in opposite areas of the northern hemisphere, are very approximately alike upon their eastern sides, or those which directly oppose the set of the westerly directed equatorial aerial currents over either ocean.

c. On the northern side of the equator we may further observe that the land areas which form the eastern sides of the continents to the North Atlantic and North Pacific oceans, being nearly opposite upon the globe, they thereby divide the entire northern hemisphere into two nearly equal systems, as far as the resistances of land surfaces, open to direct equatorial superoceanic currents in this hemisphere are concerned. Thus taking the Northern Atlantic western boundary from the coast of Nicaragua, or from about 10° N. lat. and 84° W. long., to Scoresby Land in Greenland, or from about 70° N. lat. 22° W. long., and Cochin China in about 10° N. lat. and 108° E. long. to the Gulf of Kresta, in about 66° N. lat. and 108° E. long., which is the furthest land in the direction taken, and which might, if we neglect the influence of the narrow promontory of Tchuktchis as active upon the momentum of wind, be carried through to Cape North in the Arctic Ocean in 69° N. lat. at an inclination to the polar axis approximately equal, but if we take the general direction of the land area into the Sea of Okhotsk, this direction will be much more nearly coincident with that of the North Atlantic western coasts, particularly if we recognize the influence of Iceland upon the general equilibrium of land distribution, and possibly the superior elevation of the interior of Greenland. In this manner we may observe that so far as the land areas are situated in relation to equatorial currents under thermal and revolution influences, that the deflections of direct flowing aerial forces impinging upon these areas from any cause will be similar in the Pacific and Atlantic systems; and that if the general principles of motion in the air support a system of equal deflection of impulses from land, such a system will divide the northern hemisphere into two not very unequal aerial systems of projection and deflection, the further conditions of which I will presently consider.

d. If we examine the areas of resistance in the southern hemisphere we find the land occupies much less area, and that the oceanic systems are divided into three parts which are comparable in magnitude with the northern oceans, so that by the theory of the ocean being less frictional to aerial motions, we should here expect to have three aerial whirl systems established which would be comparable with the northern systems. We have, however, in this area the Southern Ocean uniting the whole liquid system, thereby rendering the general principles that I shall endeavour to define for the northern area, more open to compensations. Further, from the motive continuity or cumulative action of the aerial forces over this larger open southern system, the whirls produced are more free from local resistances, and therefore take more nearly circular forms.

e. To classify general conditions as much as possible I may observe that the entire configuration of the Southern Atlantic Ocean is very comparable with the Northern in size and form, and the same may be observed of the Southern and Northern Pacific, both oceans being open to each other at the equator; so that if the whole area of the Indian Ocean were converted into land from the equator to the south pole, there would be a general equilibrium in the terrestrial and aqueous systems of both hemispheres; but as it is the Indian Ocean may be conceived to form an exception to this general condition of form, that holds in the other four great oceanic areas.

f. An active division of the Pacific and Atlantic Oceans equatorially may, in a certain way, be assumed to be produced by natural thermal effects, together with the action of revolutional velocities upon the current systems induced thereby, which render the northern and southern hemispheres motively separate systems. There is, nevertheless, even in this case, undoubtedly a reciprocity of action in the systems of forces both in the Atlantic and Pacific Oceans on either side of the equator, wherein we have *ruling systems* of forces and deflecting resistances, the particulars of which I will now separately consider.

139. Aerial whirl system of the Northern Atlantic.

a. In the deflection of southern equatorial *oceanic* currents northward we witnessed the powerful directive influence of the northwestern coast of South America (127 art.). This same coast-line we find is also especially adapted to deflect an aerial current impinging against it, by the position of the land towards the sea, backed as it is internally by the S. Francisco range of mountains, which bifurcate towards the sea near Aracuti and Quinta; thus presenting a strong opposing front to the continuity of the westerly impulse of the southern equatorial aerial currents. We may assume a cone of impression to be formed about the same point of St. Roque, where the southern equatorial aerial currents possibly divide, in a similar manner to that previously discussed for the aqueous system. By this division a portion of the southern equatorial aerial impulse is deflected into the northern latitudes, which may partly cause the high temperature in the North Atlantic area. Henceforth from the southern area, the northern deflection, by the same continuity of inland resistances in the mountain range west of North America, encounters an equally opposing front of solid resistance extending right throughout the entire continent into the Arctic regions. Thus after leaving the South American coast off Venezuela and Sierra Nevada which shades the Gulf of Darien the aerial currents following latitude lines fall directly upon the mountains of Veragua, where the internal curvature of these mountains deflect the same currents north-westwardly upon the high lands of Honduras, after which we have the continuity of a mountainous district throughout the great American equatorial isthmus, which, except at Honduras, is nowhere so low as 3000 feet above sea level, but generally over 5000 feet.. This range of mountainous country, extending into the Arctic Ocean by the western coast of North America, offers an almost perfect barrier to the lower denser aerial currents within the altitude of this mountain range, below which it is probable, a system of aerial deflection is formed which includes currents within 5000 or 6000 feet over the entire eastern parts of North America, these currents generally taking a somewhat northeasterly direction.

b. We may also imagine as secondary to the above system, and independently, as it were, of this great range of total resistance by mountainous land, within which half the mass in altitude of the moving atmosphere present may be deflected from its western drift (116 art. *b.*) the lower strata of the air encountering the considerable partial resistance by land surfaces in lower currents, from obstructions due to the irregularities of inclination of surface and by local resistance of rocks, trees, and every projecting object thereon; by the effects of which it will be seen, that it is over the oceanic areas only, as before stated, that we must look for the establishment of the most constant lower currents, that are palpable to us in our position upon the surface of the earth; and here the condition of aerial projection will naturally most nearly approach the direction of the oceanic systems previously discussed.

c. The continuity of induced motion being best supported upon the lower oceanic surface where the aerial fluids are more dense, together with the influence of motion of the liquid plane itself, on which these denser aerial fluids rest, represent powerful forces relative to the entire inertia of the mass of air and vapour above to deflect and project this in the same directions as previously proposed for the aqueous drift. So that it becomes probable that very large elements of the lower strata of western aerial drift follow closely the oceanic currents, making whirls of larger extent only over the near coasts by their freer tangential forces and less perfect resistances, there being no perfect resistance by the coast to aerial motions. Therefore these lower currents from all causes issue under equatorial deflections with a large element of their original momentum into northern areas approximately parallel with, and not far from the Gulf-stream.

d. If we assume the Gulf-stream a starting-point to easterly aerial drift and continue the projection in whirls as previously discussed for oceanic systems from this stream, we shall have a general flow of the aerial currents into temperate latitudes in a northeasterly direction tangential to the great free oceanic spaces; much of the aerial directive impulse being derived, as before pointed out for the aqueous system, *directly from the minus surface revolution* velocity of the earth, where currents are drifting through deflection towards northern latitudes; so that the set of the aerial, as well as the oceanic currents, will by like causes be across the Northern Atlantic in a north-easterly course towards the coast of Norway. These currents being largely influenced by the facility of passage, and the momentum they carry to move forward to the greatest free open space, which will be by continuity of projection into the oceanic area that surrounds Novaya Zemlya. This direction being also influenced for the continuity of direct momentum in moving by deflection from the resistance offered by the floating ice and oppositely directed currents on the eastern coast of Greenland.

e. Taking the cold of the Arctic regions to form one element of directive force by minus pressure from condensations, these regions forming also resistance to the direct continuity of momentum of aerial fluids, upon principles already discussed; we might conceive that the motive air maintained by the directive influences, and minus friction of the North Atlantic oceanic system just considered, would as the condensations drew in aerial and vaporous currents, cause the polar resistance to deflect the north-easterly directed indraught, and thus cause the flowing current to move tangentially to the more static air resting over the entire continent of Europe and Asia, of which Novaya Zemlya forms the central northern point. In this manner the Atlantic projection would be influential in producing an immense circumscribing whirl, which, by the propulsion impressed by the oceanic surface, would circumscribe almost the entire area of Europe and a large portion of Northern Asia. At about Novaya Zemlya, by the presence of a possibly constantly frozen ocean, we lose evidence of further equatorial projection by isothermals of high temperature, but at the same time we are clearly certain that such currents as traverse the Atlantic with excess of easterly momentum, and which evidently enter the Arctic Ocean, must necessarily have established elsewhere counter currents to complete the circulation, as a projection of aerial matter upon the surface of the earth could not be projected indefinitely in one direction without destroying the equilibrium that evidently exists. Now, for evidence of such counter currents, we must look upon the terrestrial surface for the exactly opposite effects to those we witness in the thermal projections. As the tropical projections are formed of *warm moist air*, the polar counter currents must be looked for in *cold dry air;* and the direction taken will be evident by the surface conditions of the land districts over which air passes in its effects upon vegetation. Such dry districts will therefore complete the ellipsoidal courses in which equatorial aerial whirls will be necessarily deflected by causes already discussed (115 art. *a*).

f. In this matter if we follow the evidence of facts, it appears to be quite possible that the powerful equatorial drift of air partially following the oceanic currents previously discussed, carried northward from the tropics after being deflected by the south-eastern coast of North America, henceforth takes a sweep in a north-easterly direction entirely through the Northern Atlantic Ocean, constantly gaining eastward momentum by minus velocity of the earth until it enters the Arctic circle as far as Novaya Zemlya, assuming it is here deflected by resistance to its direct momentum on its northern side, it then enters Europe and Asia about the White Sea and Kara Sea, now attaining an entirely eastern drift, crossing Northern Siberia; hence continuing by the induced whirl momentum after a broad Arctic sweep, as a compensating current, by a well-marked course, discoverable by the dryness of the deflected air proceeding from the polar regions, over the areas entirely desert, of Kesil Koom, The Salt Desert, Deserts of Arabia, and completing the whirl through the great deserts of Sahara back to the equatorial currents where its presence becomes again visible in the dry trade-winds blowing off the south-western coasts of Africa about Cape de Verde Isles. In this general system briefly sketched, local deflections and causes of minor irregularities are entirely neglected, also the incidence of oceanic counter systems. The ellipsoidal path of dry air is very clearly shown by the rainless districts of Dr. A. K. Johnston's map of meteorology, plate 15 of his School Physical Atlas.

140. Aerial whirl system of the North Pacific Ocean.

a. In the aerial system of the North Pacific Ocean we have not, as in the North Atlantic, land so situated at the western side of the ocean that the *southern* trades can be deflected northward into this system, except in so far as this may occur from the south-eastern direction of the northward coast of the large island of Papua, extending as it does obliquely 10 degrees southward from the equator, and for such distance taking a position and inclination to the western direction of equatorial currents very equivalent to the north-eastern coast of South America. As this south-easterly directed coast extends further into the southern hemisphere than Cape St. Roque into the South Atlantic, it would no doubt be more active throughout the whole year in deflecting aerial currents, if this land were continuous like that of America, and were of equal altitude inland; but Papua, being an island, there is less perfect or continuous resistance from dis-continuity of coast-line, and it therefore acts with less constant force in deflecting southern equatorial currents, although there is no doubt it acts by its inclination to the equator in such a manner as to direct some part of the southern trade-winds northward, rendering thereby aerial currents of the northern hemisphere under the influence of this deflection somewhat stronger, and making the northern Pacific climate warmer than it would be if this land had other direction of its northerly coast-line.

b. If we now follow the conditions of active equatorial forces in the northern area of trade-winds in the North Pacific Ocean, we have here, in the broad extent of this ocean, a much larger area of equatorial projection without interference of land resistance than in the North Atlantic. We have also in this area similar conditions of resistance in the Malay Archipelago to those of the Isthmus of Panama to the North Atlantic system, although these resistances are evidently less complete, the sea being even quite open in the Straits of Malacca. Therefore the general conditions of land resistance to aerial currents, although great, is less perfect here than in the Atlantic area just discussed.

c. We have further similar conditions to the westward, both in the North Atlantic and North Pacific, in a series of islands which act as breakwaters to the aqueous equatorial drift, which may act also in a certain sense possibly as *breakairs* to the lower aerial system. This similitude occurs in the presence of the West Indian Islands, backed by the American continent, in the one case, and the Philippines, backed by Borneo and the promontory of Cochin China, in the other. In so far as these inner systems of resistance are active, the resistance is greater in the Pacific area than in the Atlantic.

d. I think probably upon the whole, that considering the northern position of equatorial currents and the greater freedom of the Pacific to impress a constant momentum by the greater extent of this ocean than in the smaller equatorial area of the Atlantic, acting upon the altogether less perfect resistance of the Western Pacific boundaries, that the two aerial systems of the Atlantic and Pacific Oceans may maintain their projectile deflection currents northward with nearly equal force in so far as the equatorial impulse in their currents is concerned.

e. After the North Pacific aerial currents are deflected from equatorial impulse by the Malay Arcipelago, upon principles discussed for the Atlantic area, into the China Sea there is a moderately frictionless path open through the seas of Japan, Okhotsk, and Behring Sea over Northern Alaska into the Arctic Ocean by Melville Sound, returning to complete the whirl through Baffin's Bay, Davis Straits, over the lower parts of Labrador and Canada, and back into the Pacific equatorial system, thus completing the same form of immense whirl as that proposed for the Atlantic aerial system.

f. It appears to me extremely probable that at some not very distant geological period the whole of the Eastern Archipelago was an united system of land offering perfect resistance to the impulse of the western aqueous and aerial drift of the Pacific. I conceive this by the perfect systematic direction of the water-way between Behring's Sea and Davis Straits. Under such conditions of perfect resistance, in comparison with the Asiatic continent, the aerial and aqueous deflections would be immensely greater than at present, as currents would follow, generally, in the direction just pointed out instead of the more cramped inner system of the western coast of North America. Under these assumed former conditions, by the perfect deflection from the Western Pacific, warm currents would flow into the Arctic regions, an open sea would be maintained through to Davis Straits, and the Arctic islands of North America, and even part of Greenland would enjoy the temperate climate of Bear Island off Spitzbergen, in the Arctic circle; where cereals in favourable seasons are said to ripen grain on southern slopes.

141. Equivalence of the North Atlantic and North Pacific aerial systems.

a. The deflection of aerial currents is evidently, by the conditions proposed in previous paragraphs, under much less restraint than oceanic deflection, which at every coast-line becomes absolute; so that we find the aerial impulse, although following by preference an open oceanic area, may be continued beyond a coast, that aerial whirl systems generally will be naturally of larger dimensions and may often entirely circumscribe aqueous ones. We also find that the open area of the ocean, which may present by the direction of its coast-lines a less frictional path for an oceanic whirl, may not be the least frictional for a superimposed aerial one, if in this there is much greater deflection of direct momentum. The least frictional course in this instance will be in preference by deflection to greater curvature for the aerial momentum to carry the wind over the coast-line as before stated, and thus direct the aerial impulse inland as we may readily observe to actually occur within limited local conditions by winds over our coasts.

b. Under the whole of the conditions given above, in the Northern Atlantic and Pacific aerial whirls, which I have taken for illustration, although the generative thermal forces induce whirl forms by local resistances and the necessities for a moderately frictionless projection of the equatorial current, these whirls as circulatory systems are nevertheless motive under the enormous restraint of the differences of *latitude velocity*, which varies from about 1000 miles an hour at the equator to zero at the poles, giving to the winds under latitudinal displacement, possibly as Sir J. Herschel has estimated, two-thirds of their force; so that the deflected thermal impulse carried northward is constantly gaining eastward impulse relatively to the motive surface of the earth, until as before proposed, it becomes nearly a due east wind; whereas, by the continuity of the same system of forces, the northern or overland supply current is carried constantly westward by the plus rotational velocity until it reaches the equator, where its impulse becomes nearly due west. In this manner the whirls just described of the North Atlantic by circumscribing the immense area pointed out of Europe and Asia, and that of the North Pacific, Northern America, form thereby two ellipsoidal systems over the geographical position of the northern hemisphere, in which we may represent directions of forces somewhat in the manner of the orbit of a planet moving about its primary in an elliptical path, although the primary assumed, in this case, is not a solid but only a position in space, which is merely represented by a quiescent focus. Such a focus as here proposed for the Northern Atlantic aerial system would be situated at a yearly average possibly in about 33° N. lat. and 50° W. long., and in the Pacific aerial system possibly in about 30° N. lat. and 160° E. long. The last position being much less easily definable by causes given irrespectively of seasonal influence; from the difficulty of estimating the imperfection of the resistance in the open channels between the islands of the Malay Archipelago, and the uncertain influences of currents in the more open oceanic area south of the equator. In either case these foci would follow the influence of the seasonal altitude of the sun. I have conceived the following diagram to represent the direction of forces active in the aerial system upon principles discussed for the general direction of circumscribing areas in the Northern Atlantic and Pacific systems, subject to local deflections, from elevation of land, and other conditional circumstances, at about the summer solstice. In this sketch I assume other forces may be active or may enter into composition, therefore it represents only my theoretical ideas of cosmical aerial motion.

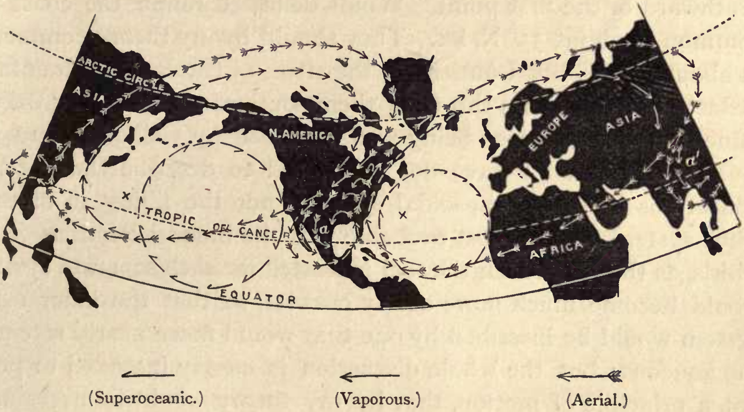


Fig. 168.—Diagram—Theoretical Aerial Systems in the Northern Hemisphere.

c. In the above diagram the arrow with one barb represents the influence of primitive oceanic whirls, directing the superimposed aerial systems by carrying force.

The arrow with two barbs represents the vapour system, in its circuit of greatest activity, falling generally somewhat short of the aerial projection, but subject to certain conditions of vertically directed forces to be hereafter considered.

The feathered arrows represent the active extent of the great superimposed aerial whirl systems of the Atlantic and Pacific Oceans which circumscribe inner systems, although not by definite lines.

The crosses represent assumed foci of the ellipsoidal whirl systems.

a a', Regions not entering into the great ellipsoidal whirl systems where possibly biwhirls throw off a portion of the tangential force from the ellipsoidal systems. Regions of the origin of storms and hurricanes to be hereafter considered.

In making this diagram the large group of islands to the north of Hudson's Bay were inadvertently omitted, as also the arctic continent north of N. Lincoln, this omission will, however, in no way interfere with the plan of the systems proposed.

d. By observation of the isothermal lines given in Dr. A. K. Johnston's climatological chart, the entrance of the return path of the Pacific whirl, here proposed, would appear to be in about 42° N. lat. upon the western coast of America, between Fort George and Fort Ross. The oceanic bifurcation does not occur until 30° N. lat. or 12° southward of the first point. Winds deflected round the coast are common in about 35° N. lat. They should by my theory commence in about 31° N., but I anticipate the chain of the Rocky Mountains deflects these currents throwing them southwards and upwards, the same directive influence being also produced by California and the Gulf of Mexico. I have not attempted to describe the exterior deflections of these ellipsoidal systems, nor the filling in of their whirl systems. It is most probable that the inner deflections of the whirls, as they approach the foci indicated for each separate system, would become much more nearly circular, so that the inner aerial system would be inscribed by one that would more nearly resemble the aqueous; but the whole discussion is merely intended to point out a principle of motion, that by my theory would be necessarily induced, in an expansile and contractile aerial system, in relation to actual areas of friction and deflection under the impulses of different latitude velocities of the earth; as also by the functions of motive tangential forces, active upon lateral aerial fluid masses upon principles discussed in this treatise, under conditions showing that fluids cannot slip or glide upon each other without immense friction, and therefore must, if free, move by some system of rotation. In the advancing summer, by the expansion of air over continental areas, the superoceanic circulation is more restricted, and the countercurrents are deflected by resistance to closer systems, returning in areas more nearly contiguous to their oceanic projections, some conditions of which I will consider further on.

e. The deflection of aerial rotary systems immediately from the equator, by reasons I will hereafter discuss, will be at a greater elevation than the inflowing cooler denser currents, so that the plane of the whirls will be *oblique*, as I will hereafter endeavour to show.

There will be also internal bifurcations; but possibly at a few thousand feet of elevation, where freedom is more equal throughout the system, and where the earth's power of condensing heat has less influence, the currents will proceed in nearly horizontal planes on average very nearly in the directions shown in the engraving, for the circumscribing systems. There will necessarily be also northern foci to the ellipsoidal systems, which by the circuit of forces will be centres of relative repose. Assuming no seasonal force changes the position of such foci, they would receive no part of the equatorial currents, and would therefore become very cold spots on the surface of the earth, particularly as vapour condensation would be absent, the vapour currents being drifted only round such foci at a distance.

f. On the other hand near the equatorial foci would be areas of intense heat, but these being actually oceanic, the heat would largely disappear in vapour expansions at the liquid surface; protection from sun's direct rays being also produced by the formation of cloud above, therefore the heat effects would not be measurable by our thermometers at the surface of the globe, or outwardly in any manner except that it would generate overflowing vapour forces in rains and storms, which would appear in the near or distant parts of the rotatorial system thrown tangentially some distance from the relatively quiescent foci.

142. Indian oceanic aerial system north of the equator.

a. The portion of the Indian oceanic aerial system north of the equator, will, on a small scale, resemble that of the Pacific Ocean, but the former being more directly under the influence of land surface to the north, will be more particularly affected by the influence of the seasonal obliquity of the earth. Thus in our summer, when the sun's rays are nearly or quite vertical to the tropic of Cancer, the greatest heat will be distributed almost entirely over the land areas of Arabia, India, and a part of China; at such times, from the land retaining the heat forces immediately at its surface, this area becomes as the thermal equatorial zone to the Indian oceanic system. Therefore during our summer the circulation of aerial currents in the Indian Ocean meet certain conditions of circulation in the Southern Ocean which I have yet to discuss. But in the southern summer when the thermal equator of this district is entirely over the oceanic area to the south of the Indian Ocean, the conditions that hold in the North Pacific are more nearly coincident, and the south-eastern trade-winds in the Indian Ocean move over purely aqueous areas, with direct momentum upon the high tablelands of Ajan, Somauli, and Hadramaut. Under these conditions the northern equatorial currents, by inland resistances, are possibly deflected through the back of the Indian Peninsula, by the valley of the Indus to that of the Ganges, returning to the Indian Ocean, by forming a similar although closer whirl system, than that of the North Atlantic or North Pacific Oceans before discussed (140 art.).

b. By the above conditions the Northern Hemisphere possesses two great circulatory systems in the northern summer, those of the Atlantic and Pacific, but in the northern winter, from the movement in position of the thermal equator further south, the Indian oceanic aerial system becomes also one of the northern systems, and it is probably to the effects of this change we owe some of the stormy conditions of certain districts of India.

143. Aerial systems of the Southern Hemisphere.

a. My information will not permit me to offer more than a few observations upon this subject. But there is no doubt, I think, that the southern hemisphere being for the most part oceanic, that the aerial system reposing thereon will be more influenced by the oceanic circulation than is the case in the northern hemisphere. And here, in the first place, as regards the great open oceans, the Pacific, Atlantic, and Indian, that the same system of forces will prevail as I have defined for the Northern Pacific and Atlantic systems. This will be certain, so far as the influence of the superficial oceanic carrying action is concerned, and in this the oceanic systems north and south very closely agree in the forms of whirl projections; which is observable in maps of oceanic currents, as in that of Dr. A. K. Johnston (Hydrology), wherein the *oppositely* directed similar biwhirl systems from equatorial projection may be clearly followed by current lines correct to theory, but subject to the differences of resistance from the land. Where the systems of the northern and southern areas do not agree is towards the polar areas, where in the one case we have the prevalence of land, and in the other water.

b. The influence of the extent of ocean in the higher southern latitudes will be to permit forces derived from thermal causes to act cumulatively in directing the oceanic surface; so that in the south the polar forces will more largely prevail than these forces in the north, the equatorial forces being proportionately less, as previously discussed for oceanic systems, 130 art. *a*, *b*. In the south also a large element of the circulatory aerial systems of the southern oceans appears to be derived from the Antarctic drift due to vertical circulation, that I have yet to discuss, which sets in especially upon the western coasts of South America, Africa, and Australia, the aerial systems being nearly superimposed upon the oceanic. This condition may at least be assumed to prevail upon the immense area of the Pacific Ocean; the thermal equator forming always one limit of the aerial circulation where tangential forces are most active upon the system. These tangential forces also tend in all cases to throw the aerial fluids for some distance inland of oceanic boundaries, the evidence of which is found in the amount of rainfall and vapour near the coasts.

c. The South Atlantic aerial system following the impulse of the southern equatorial current is possibly for the most part deflected to the northward past Cape St. Roque, where the oceanic system bifurcates; the greatest aerial impulse of thermal forces being carried to the westward as far as the mouth of the Amazon. The cone of resistance to this system forming its base upon the mountains of Acarai; the southern tropical aerial whirl being thrown along the valley of the Amazon, to the east of Sierra Nevada into the valley of La Plata, completing an ellipsoidal course in the south Atlantic as far as the 40th parallel, by again turning inward to the Cape of Good Hope, in every way similar to the ellipsoidal systems discussed for northern areas, although not of equal extent. The whirl gaining force from the open currents of the Southern Ocean by forming a projection similar to the oceanic.

d. The Indian superoceanic aerial system is much more complicated by the mountainous interior of Africa lying between 30° and 40° W. long., but here there is evidence of the influence of the tropical impulse over the less frictional area of the Indian Ocean upon the mountainous interior which deflects the vaporous air to form the immense lakes of Albert Nyanza, Victoria Nyanza, Tanganyika, and Nyassa, in continuing, as before, an ellipsoidal system which completes its circumference in the Indian Ocean and the western coast of Australia back into the equatorial zone.

e. In assuming the deflections of tropical aerial western drift to be caused by the elevated lands of Central, North, and South

America to the west of the thermal zone, we witness by biwhirl deflection, similar conditions in the fluviatile system on both sides of this resistance. Thus the Amazon passing with its immense valley to the west of South America in a south-easterly course meeting the valley of La Plata whose direction is south-easterly, resembles the Mississippi as it continues in the Missouri in a north-westerly course to the west of North America, meeting Hudson's Bay and Straits in a north-easterly direction, or in taking a nearer circuit in the Mississippi proper into the valley of the St. Lawrence. These fluvial valleys I conceive evidently indicate the courses of vaporous projection from tropical impulses.

f. It might possibly be imagined that if such systems as those discussed above were entirely cut off from further volcanic influences, that in an immensity of time by erosion from rainfall, the eastern tropical coasts of both North and South America might be detached as islands, which would then find equivalence in the present conditions of Papua, and the Japan islands in the Pacific system, and of Madagascar in the Indian oceanic system. Under such conditions the Isthmus of Panama receiving a more powerful impulse from the more open equatorial oceanic and aerial currents would possibly be wasted away, and conditions similar to those we witness in the Eastern Archipelago be established.

g. For the countercurrents that return by the easterly oceanic coasts to complete the atmospheric whirls, as these are projections from higher latitudes, they will be necessarily marked in their courses by the dryness of the air, as before proposed for the northern systems. But as the southern land areas are of less extent, so the aerial systems will not be marked by the same dryness as we have in the northern systems, particularly as in the Asiatic countercurrent (141 art.). The southern systems would nevertheless, as whirl systems, be marked by dry air upon the inflowing sides of the whirls; this is found in that the aerial southern whirl systems which act as supply currents to the south-east trades leave rainless districts, where these currents are passing northward towards the equator, with the eastern impulse derived from whirl circulation, which throws such currents to the eastward of the great southern oceans. Thus we witness rainless districts to the westward of South America in Peru, also inland of the western coast of Africa, and of the western coast of Australia. It being further seen that it is exceptional for great rivers to flow out from the western coasts of large continents unless under very exceptional internal geographical circumstances not here considered.

144. Whirl areas complementary to the Ellipsoidal aerial systems of the northern hemisphere.

a. If we take the two great aerial systems proposed, of which the diagram, Fig. 168, is an illustration. There will be seen intervening spaces westward of the equatorial foci of greatest activity of these systems just exterior to the tropics; and as the motive fluids in these regions present frictional surfaces by difference of velocity of parts, they must find accommodation for such motions as in other cases considered, and systems of direction of forces must be established here as elsewhere. These spaces are shown at *a*, *a′* on the diagram (Fig. 168), proposed to illustrate the summer condition in this hemisphere. It is most probable that the interspaces between the great Atlantic and Pacific systems are filled up by large offset variable whirls projected from the return side of the great ellipsoidal systems. If we take the interspace system marked *a* shown in the centre of the diagram (Fig. 168), we find from this, that the Pacific whirl after crossing North America meets and compounds with a like whirl from the Atlantic system, the direction of curvatures of the spirals formed being alike in both cases. This whirl system may be as shown in the diagram below, which is intended to represent, in principle, the meeting of the ellipsoidal systems about the Gulf of Mexico drawn south upwards.

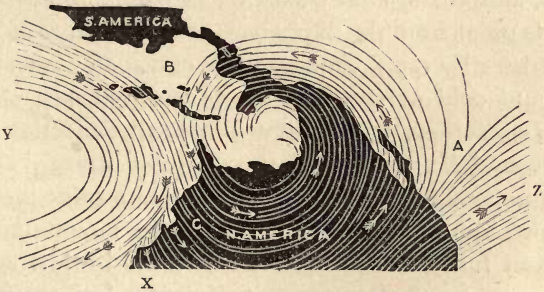


Fig. 169.—Diagram—Interspace Whirl, North Atlantic and Pacific Interspace System, south upwards.

b. In this diagram the Atlantic whirl system is shown to the left of the Fig. at Y, and a part of the Pacific return whirl system at the lower corner to the right about Z. The part of this return whirl shown at A, has its tangential force partially released, which causes it to bifurcate and throw off a whirl in the interspace of the great ellipsoidal systems about this part which acts as the cone of impression to such biwhirl by the resistance of the inertia of compressed aerial matter in front; this point A, by conditions here given not being a part of any rotary system. The biwhirl deflections are curved from A to B and onwards to C, where the tangential forces of the Atlantic ellipsoidal system unite and form another biwhirl as shown at the bottom of the figure at X. The ellipsoidal projections move as here proposed at their circumference by whirls of rolling contact upon the contiguous parts, such whirls nearly filling by deflection the interspace between the Pacific and Atlantic whirl systems. In drawing this whirl on wood which I intended only as a diagram, I neglected the evident influence of rotation of the globe upon its system. It should really take a plane of obliquity in a direction parallel to the great ellipsoidal whirls, and there should be a secondary biwhirl to the right of the point B. In this case the Atlantic whirl system would extend further into the Gulf of Mexico, entering the land near the mouth of the Mississippi river. From the Pacific coast current, represented to the right of the figure, being subject to diffusional influences by the form of the coast and mountains inland, the diagram as regards actual conditions is purely theoretical and given to define a principle only.

c. In the case of the formation of a large interspace whirl, as here shown, the whirl systems would unite the opposing directions of forces at B, and the system would be thrown into unstable equilibrium at this point from the differences of temperature of the uniting whirls, and here by rapid condensation of vapour upon contact of the warm and cold currents, as far as the influence of the whirl systems is concerned, and except the local deflection from any cause, this would be a region of the origin of storms. It must be distinctly observed that the above represents a principle of motion only in which the deflections caused by the mountainous district are not taken fully into consideration. This biwhirl system proposed appears nevertheless to be sufficiently active over all local interferences to throw in dry Arctic currents over a part of Mexico and render this a rainless district.

d. Exactly the same conditions must occur if my theory is correct, at the meeting of the Pacific and Atlantic great whirl systems in the China seas, where the Indian oceanic northern whirl is inactive in the northern summer, but here again we have great complexity of local interferences which would require special study to define anything more than the general principles, which would be possibly only similar to those given for the Mexican area.

e. Under any conditions, as in the interspaces between the great ellipsoidal whirl systems proposed of the Pacific and North Atlantic Oceans, the interspace whirls would be formed of aerial matter thrown off or set in revolution by the one system or the other, according to seasonal obliquity, and the influence of distribution of elevated land resistances; the entire region of such interspace whirls being one of great climatic changes. In what we may term the *Gulf of Mexico whirl system*, the tangential forces from the North Atlantic whirl would throw in heat currents. Whereas the Pacific whirl system would throw in cold currents, and there would be induced from the change of the position of the heat zone by seasonal obliquity, displacement of the great ellipsoidal systems, which would cause them to seek accommodation according to the conditions of local resistance from the distribution and elevation of land, so that interior whirls or diffusional currents could only be traced with great difficulty to systematic causes.

f. I have shown one interspace whirl as representing a principle, but from interfering causes, such as mountainous districts, and general irregularities of surface, I have no doubt that this whirl is split up into many others, and that the system of projection over a large portion of the area becomes entirely diffusional, except possibly at great altitudes. Further, the seasonal obliquity, which entirely displaces the great ellipsoidal systems, changes also their general configuration. In this case by the conditions of a whirl taking the greatest circular area (82 prop.), when in the northern winter the heat zone is southward, the Atlantic whirl finds greater freedom by longitudinal contraction, and the Pacific whirl by longitudinal extension, which places the Mexican Gulf interspace system more nearly under the cooling influence of the Pacific system.

g. There is also a further condition, that by longitudinal contraction of the North Atlantic aerial system in winter, the Alleghany Mountains commence to block the tangential deflection of the North Atlantic whirl, which blocking gives greater freedom to the Pacific projection. It is probably to this cause partly that north-eastern America owes its intensely cold winters.

h. The eastern complementary system, in which I have shown the complementary whirl at *a'* to the right of the engraving (Fig. 168) on page 401 is shown too far westward, as in this position the aerial currents would encounter the immense resistance of the elevated district of Thibet, of over 5000 feet above the sea-level, independently of that of the mountains rising still higher in this district. In this case it is most probable the complementary whirl of this system enters to the westward of the sea of Okhotsk, passing through the lower lands in a south-westerly course to the Yellow Sea.

i. When the sun is south of the equator, the great ellipsoidal systems, from the greater distance of their motive projections between the thermal equator and the north pole causes these systems to become more nearly circular. In this case the complementary whirls between the great systems shown *a, a'* Fig. 168 become necessarily larger, and it is possible that the north-western monsoons of India are partly engendered by the complementary whirl between the eastern meeting of the Pacific and Atlantic systems, as well as by the condition of the Indian Ocean now forming a separate northern whirl system.

145. Polar interspace whirls to the north of the Atlantic and Pacific whirl systems.

a. The causes discussed which produce complementary whirls over Mexico and China by the meeting of the great ellipsoidal aerial whirl systems of the northern hemisphere will also produce at the northern meeting of the same ellipsoidal systems like conditions of interspace whirls, the positions of which would be over North Greenland and North-eastern Siberia. We may assume these northern systems to be much influenced in the North Atlantic area by the extent of open oceanic space that may be at the time free from ice. The whirl system will therefore by this cause be pushed back northward in summer and be brought forward in winter. But as both thermal projections and condensations always become weak within the area of constantly low temperature, such spots will be relatively quiescent, and radiational forces be therefore more active in producing intense cold. Further the meetings of the ellipsoidal systems in polar areas, considered above, would be altogether of a less tempestuous character than the equatorial meetings, as the sudden condensations of vapour forces would be absent after a certain extent of projection over the cold region. The area of interspace being more close, and in all probability more regularly constant although weaker. This rotary system would be established, subject only to displacement by certain seasonal influences.

b. The direct impingement of the two ellipsoidal systems in the northern hemisphere suffering restraint at about the 80th parallel, would engender by tangential forces over the entire polar region a constant rotation of this area, producing a constant circumscribing westerly wind around the north pole. Such a system would produce a considerable minus barometrical pressure by tangential action near the pole, and if held constantly without interfering causes would render the inner polar region one of such intense cold in winter as we have no conception of on the globe, as equatorial currents would thereby be entirely shut off, and the polar area be subject to radiation forces at all times in a bright and almost perfectly dry atmosphere.

c. The like conditions occur in the southern polar regions, but in this case as the circumscribing area is less frictional, the tangential forces are therefore more active, and this region becomes one of lower pressure and I have no doubt of more intense cold.

146. Interior whirl systems caused by indraught of aerial currents over continental areas.

a. Following the condition of indraught previously briefly mentioned, art. 134, we have, besides the general agencies of the radiation of heat from the sun and the earth, in the equatorial and polar systems discussed, in composition with those forces, local air and vapour systems over every large terrestrial and aqueous area, where similar agencies to those previously considered are active, according to the powers of radiation the selected area may possess. In such areas the radiational forces will be very different in the separate cases, whether the surface be largely covered by land or by water, the land being a very bad conductor of heat although a very rapid radiant; whereas the liquid areas are to a certain extent absorbent of heat rays; further by the mobility of the liquid system and its general constant motion, the radiation heat-forces are distributed in a short space of time with considerable uniformity (136 art.).

b. If we take the conditions of large continental areas, we find that radiational forces materially influence the general directive motions of aerial currents. The most striking case of this kind is that of the large continental area of Europe and Asia already mentioned. In this immense area the difference of radiational forces active in expansions and contractions of the air, induced by changes of obliquity of the earth's axis, and thereby by the sun's influence at different seasons of the year, represent in these expansions and contractions through the changes of temperature, immense forces, active in the displacement of a large quantity of aerial gravitating matter, affecting the general equilibrium of elastic forces in the universal system of the atmosphere. These forces, at periods of greatest motive intensity, overpower locally established average surrounding aerial conditions, and naturally deflect the direct currents of the circumscribing systems I have just discussed as equatorial and polar systems. Thus we may find in the central area of the circumscribing whirl system proposed for Europe and Asia, that air is being drawn in over this entire area, by contraction from the superior heat radiation of the earth, at the decline of the year, which indraught continues in central Siberia until January, or until we have the weight of atmosphere represented by 30·4 inches of mercury per area; whereas at the return of spring, by increased elasticity in the same area, the air is pressed outwards, until in June and July we have only 29·5, or about 62 *lbs. per square foot* less weight of aerial matter above this central radiation area. This is quite irrespective of the vapour system, which upon production, replaces the air and constitutes a very large portion of the elastic gravitating mass remaining in the warmer season; and a very small part in the colder, so that really the absolute amount of displaced air is much greater than that given by index of pressure.

c. These conditions entail the establishment of means of supply and of outflow by currents, or other modes of direction of the atmosphere to and from the central areas; exactly the same principles holding as previously discussed for expansile and contractile forces in equatorial and polar systems. For instance, the current lines will move over the areas of least resistance which are found to be generally over oceanic or aqueous areas, or if over land, they cross such areas as present the least resistance through the greater uniformity of the surface plane. Referring to the distribution of land and water as shown on a terrestrial globe, the greatest land area in Europe and Asia will be found between the parallels of latitude 50° and 60° north, extending say from 15° to 140° E. long. The centre of greatest solid area being in about 50° N. lat. and 90° E. long.

d. As similar conditions hold largely for both contractions and expansions of aerial systems, it will be convenient for simplicity to take one of these only, and the most convenient will be that of *indraught*, which takes place in any continental area at the decline of the year, or after the sun has passed the summer solstice at the locality taken.

e. The amount of supply to any internal area of contraction is proportional to the forces of minus pressure over the extent of the area taken; such minus pressure giving directive impulse to the area of plus pressure upon all sides. But there being, as I have suggested, in most areas exterior currents already established, the indraught will be taken with greater facility from any exterior projectile aerial system whose impulse is most directed towards the axis of indraught.

f. In such supply currents, although the forces act directly as all compressions and condensations do, they must nevertheless in moving in currents upon local resistances take whirl forms, or make rolling contact upon the lateral inert fluid masses, as a necessity of fluid projection in any form whatever, upon principles already discussed.

g. If we follow the same direction as previously proposed for aerial currents illustrated in Fig. 168, page 401; the most direct course to the Asiatic interior would be by the Arctic seas when such seas were open. This therefore points the direction of currents in autumn when the thermal forces are most active in the northern hemisphere, but the supply currents of the interior of Asia are required in the winter also when these arctic areas are closed by ice, and the ellipsoidal system becomes more circular and somewhat displaced southward.

h. In the general circumscribing aerial ellipsoidal system illustrated, Fig. 168, I could take no conditions of internal circulation; it is nevertheless quite certain that such exists if only under the directive influences of oceanic currents, and it is quite clear also that an internal radiational district of contraction would attain supply from such an internal motive system with equal or greater facility than from an external one whose forces would be directed tangentially more powerfully from the central position taken. In this manner the aerial forces assumed to accompany the great North Atlantic equatorial *oceanic* whirl would carry impulses more directly into the interior of Europe and Asia than that of the extreme ellipsoidal system that I have proposed to extend its direct circulation as far north as the Arctic Ocean.

i. Accepting these premises, if we now take into consideration the direction of forces towards this interior Asiatic area, we find that the general aerial direction is ruled largely by the eastern rotational impulse of the globe in its minus velocity to deflected equatorial currents known to exist over central oceanic currents of the N. Atlantic; assisted by the carrying influences of the oceanic system whose direct impulse is towards the coast of Spain; so that it would appear suggestive, assuming tangential forces thrown off the equatorial whirls, that the Mediterranean area following the track of the Black and Caspian Seas, would be one most open for a supply current to the contractile area, of the entire inner circulatory system towards the interior area of Europe and Asia at the decline of the year. But this as all other courses would be subject to certain conditional forms of resistance that such currents may encounter from the land areas over which the current must pass. When we examine the conditions to which the direction of forces proposed above would be subject, we find that in this proposed course, currents would encounter considerable land resistance, which would materially deflect the direction of impulse which the favourable position of the liquid areas would otherwise offer. In this case in the easterly direction overland in the Mediterranean area, we should have first the mountain ranges of Spain and Morocco, leaving open only the narrow straits of Gibraltar southward for free currents of low dense air at an approach to oceanic level, and as such space would have also to include local accommodations, it would therefore be nearly inactive, or very frictional for a system of indraught, were there other conditions more favourable present. Further eastward to the interior of our contractile area we should have causes for deflection by the Apennines of Italy, and by the mountainous area of Greece; and finally further eastwards by the entire Caucasian range. Therefore this Mediterranean course, that the direction of open seas appears at first to point to, would by the situation and altitudes of land resistances be a very frictional one. Considering the actual positions of liquid areas, this may possibly at some distant cold geological period have been the direction of inflow, but it could not possibly be so now.

j. If we return to the directive impulses of the aerial system proposed for the North Atlantic, we find that from the tangential force of the great whirl proposed for the aqueous system of this ocean directing its currents somewhat northward of the coast of Spain; that the point of Cape Finistère supports a cone of impression, which, as before discussed, deflects, the flowing forces right and left. Supposing this conic area to be one of pressure also to the aerial system, under the influence of the oceanic carrying force to the atmosphere above, then so far as this carrying influence of the oceanic surface could direct it, by aerial deflections northward of this conic area over the directing oceanic lines, the assumed currents would gain easterly momentum by the minus latitude velocity of the earth. So that in the northerly direction possibly, there would be the most direct impulse to establish supply currents to the internal area proposed.

k. We may now consider the conditions we have taken of the direction of the oceanic system which by the influence of central momentum of the N. Atlantic, may be assumed to project a tangential current in a north-easterly direction, that is, towards the Baltic. There appears to be present many conditions which favour this course of aerial supply by a moderately frictionless course in the autumn and winter when it is required by aerial conditions of condensation from terrestrial radiation.

l. This course, in the first place, is only a little more northerly than the aerial deflection I assume over Cape Finistère in Spain, so that a direct impulse is given by the winds of the North Atlantic equatorial system to move in a more northerly direction. Therefore in this direction a moderately frictionless course is opened by the English Channel, and the low-lying countries of Holland, and Northern Prussia, as the most free area; deflecting possibly a part of the indraught as far north as the Gulf of Finland, Lake Ladoga, and Lake Onega. Here passing by moderately level ground as far as the Ural Mountains, which are really to this course the only serious land impediment. But these mountains are for the most part undulatory with many easy passes, so that the greatest resistance that they effect is only such as will cause lower supply currents to be deflected upward and therefore supply largely by a system of overflow. The above therefore seems altogether the most probable means of supply currents to the large interior areas of Europe and Asia under aerial contraction, and this course appears to be consistent with the excellent maps of Mr. A. Buchan in his important paper previously referred to. A deflection of the surface currents being observable in these maps in front of the Ural Mountains.

m. In the Baltic system, assuming the causes given sufficient to maintain the currents proposed, these currents could only enter the area of contraction by lateral whirls making rolling contact upon the contiguous aerial fluids present, which fact appears to be generally consistent with observations of the prevailing winds. But it is far more important to observe that such currents would produce the establishment of a system of aerial motion, and this would be similar to that previously discussed for Arctic currents, acting cumulatively upon the aerial resistances, and inducing a form of motion in them that could be reversed only by a frictional mode of motion; therefore such a supply current, although oscillatory, or even partially reversed at times, within the elasticity of the system, would be more or less constant even after the forces that caused its projection ceased to act. Therefore the first effects of expansions in returning spring would, under this projection, open out for themselves a less frictional channel in some other direction rather than that of the established currents.

n. Taking the whole matter of European and Asiatic projection from all the causes given, it follows that the internal contraction and expansion of the aerial fluids in this great continental area engender an internal whirl system which is continuous with the aerial and vapour systems proposed, following approximately parallel lines within the vapour system shown in Fig. 168, which may be very marked at certain seasons of the year, although no doubt subject to a certain amount of frictional deflection.

o. In the whole of this subject I have taken *lines* of projection. The lines I suggest are only representative directions of forces in currents whose area includes the whole motive system, such lines as indicating directions being themselves under the influences of deflective forces from the configuration of the earth, and differences of local friction in parts of the system of projection.

p. Other systems of indraught of aerial currents upon the globe at periods of contraction, are generally more definite and simple from contiguity of oceanic aerial currents. Thus the indraught of North America would be derived directly from the great equatorial whirl, entering about the mouth of the Mississippi. That of South America at about the mouth of the Amazon. That of Africa by the Gulf of Aden, and of Australia by the Gulf of Carpentaria.

CHAPTER XIII.

DIRECTION OF CURRENTS DERIVED FROM VERTICAL MOVEMENTS OF WATER PRODUCED BY THERMAL CAUSES. DEFLECTIONS UNDER RESISTANCE BY WHICH VERTICAL CURRENTS OVERFLOW AND UNDERFLOW. COMPOSITIONS WITH REVOLUTION VELOCITIES OF THE GLOBE. OBLIQUE WHIRL SYSTEMS. TIDAL INFLUENCES.

147. Vertical Oceanic Circulation.

a. The motive principles of vertical circulation have already been partly discussed as effects of thermal forces, articles 112, 113, and 116. I now propose to take certain conditions of local effects which may be rendered somewhat evident in the actual motions of oceanic currents.

b. To the motive forces of the air alone in winds Sir John Herschel attributed the entire motion of the ocean, not only for waves, but for currents. Dr. Carpenter by his researches in deep-sea soundings has shown that the oceanic surface currents which flow fairly in the direction of the wind, where this is constant, may be of very inconsiderable depth; underflowing currents taking most frequently an entirely opposite direction to surface drifts. Upon the accurate observations made of these facts, I think that Dr. Carpenter has fully assured us that oceanic circulation exists entirely independent of aerial movements. The principle of this important physical law appears to have been made out, as before mentioned, by Professor Lenz of St. Petersburg, by demonstrations obtained in the voyage of the *Kotzebue* 1825 to 1828, from which he first showed by temperature soundings that there must be a constant inflow of heavy polar water to the tropics, and an equal overflow of the lighter heated surface waters therefrom. This important theory he propounded in 1845 but it met the usual fate of nearly all great theories, that it clashed too much with preconceived ideas educationally attained to be accepted, or even examined; until the entirely new researches of Dr. Carpenter in the voyage of the *Challenger* in 1868 proved, on new grounds, the entire theory of Lenz. It is now generally clear to careful observers that polar undercurrents must flow to the equator, as polar water, by the evidence of temperature, is found by soundings at much less depth at the equator than in intermediate latitudes. It is further evident that such inflow in some form is necessary to restore the equilibrium of projection towards polar areas, which is observable in the directions of large oceanic currents, such as in that of the Gulf Stream. These undercurrents being also active in supplying the loss from excessive evaporation over the tropical oceanic surface and therefore so far in *excess* of the overflow.

c. That the lower inflow towards the equator is evidently of polar water, has been further assured negatively, by sounding in seas cut off from polar currents by the oceanic entrance of such seas being of less depth; in which case, there is no low deep-sea temperature in undercurrents. This we find demonstrated by the equable high temperatures of the Mediterranean and Red Seas at great depths. Further underflowing polar currents have been traced by Dr. Carpenter from near the polar region without intermission to the tropical. On the other hand, the overflowing currents, in the N. Atlantic Ocean particularly, have been clearly shown to extend far up into the Arctic regions, as I have before discussed. Therefore, I consider that the principles of the active aqueous vertical superposition of forces, so far as the functions of these have power to direct the flow of the great equatorial currents northwards and southwards, have been quite made out by the researches of Lenz, Carpenter, and others, as previously stated; quite independently of the certainty of the principle, by the laws of gravitation evident in the movements of temperature or density systems of circulation in liquids. To the principles of vertical circulation given by the above-mentioned philosophers I shall be able to add but little, except by endeavouring to show that aqueous displacements act through whirl systems of motion, although these may be actually under certain conditional restraints.

a. If for demonstration of the equatorial vertical circulation we conceive the oceanic system to be one in which cold dense water covers the ocean bed everywhere to a certain depth, which is proved to be the fact experimentally by soundings, and we further conceive that there is some source of radiant heat above the water surface that has power to penetrate the water and to expand it in some proportion to depth, for a certain depth, which the heat force can penetrate. Then if the heat-force were distributed equally over the entire oceanic surface, the result would be only, that after a certain time the whole of the water would remain in vertical density equilibrium, the coldest or densest stratum being at the bottom, and less dense strata consecutively above up to the surface, where the heat is assumed to be most intense.

e. If we now imagine such a system as that given above to be established over oceanic areas, and that we change the conditions by placing the source of heat in such a position that it can act upon one *end* of the aqueous system only; then in this case it is quite clear that the expansile force that rendered the water less dense would be active at this part only, and that, supposing the heat upon the whole the same as that at first proposed for distribution over the surface, the heated part would be most expanded by the local concentration of the heat-forces.

f. If we now imagine that another portion of the same liquid system at some distance from the first is open to space but shaded from the given source of heat, then by the law of interchanges of heat established by Prevost, this shaded area would become that of the greatest radiation of its own heat; so that it would quickly lose its initial heat and become as cold and dense at the surface as at a depth taken for the constant temperature of the lower water. Further, the dense surface water, cooled by radiation, sinking to gravitation equilibrium of its density in relation to the water immediately beneath it, would replace this less dense water until the whole system over the source of surface self-radiation became one of equally cold dense mass. Now if such heat expansion and cold contraction remained immobile, we must have after a time, a division vertically in the fluid system in which a light and a dense fluid would rest vertically, against each other. This it is quite clear, by laws well understood in hydrodynamics, would be impossible, but what would occur is that we should have through the action of gravitation forces present, the parts of the fluid constantly moving to a state of density equilibrium, as we observe actually in the system pursued in the heating of buildings by water.

g. It is clear that the above conditions are those that actually exist in the oceanic system between equatorial and polar regions;

in which the density gravitation equilibrium, in equation with the average heat-force, is distributed by the differences of density; depending upon the forces of local radiation of the sun upon the earth, and of the earth into space, as previously discussed. Upon these principles we find that gravitation acts as a constant force quite independently of other forces; this force being active to restore the conditions of density equilibrium in the general fluid mass; consequently the aqueous system becomes motive, the colder denser water *underflowing* constantly to gravitation equilibrium, and the warmer less dense water *overflowing* to re-establish surface equilibrium. In such motion, however, the direction which currents necessarily take to avoid the resistances of frictional surface will considerably modify this simple condition, taken *per se*, as I will presently endeavour to show.

h. The system of overflowing and underflowing currents as applied to oceanic areas is graphically represented by an experiment of Dr. Carpenter, as shown in the diagram below. In which a trough with glass sides is filled with water at the temperature of the air. A heated body, formed of a hollow vessel in which steam is injected to represent the action of the sun's rays, is placed over one end of the trough, and a block of ice is placed at the other. Very shortly after, currents are induced, which are made apparent in the experiment by visible particles in the water of its own specific gravity, taking the directions shown by the arrows in the diagram below, Fig. 170.

Fig. 170.—Ex.—Dr. Carpenter's, showing Circulation of Water by Thermal Forces.

i. In this experiment we have the principles of a rotary heat engine, the motive power being engendered by the rising and falling force from difference of specific gravitation in parts of the water.

j. One particular in which this experiment does not entirely meet the conditions of the circumstances of the oceanic waters, is in that we have no solid boundary in the ocean representable by the end of the trough. But in this matter we can conceive that the polar ice, which is of considerable depth, will represent sufficient resistance for the cold end of the system, and the oppositely directed underflow from the north and south circumpolar regions will terminate this projection near the equator or where these polar forces meet, and oppose each other, and thereby represent the heated end of the trough. Some further conditions of which I will consider, on principles of whirl-motion, hereafter.

148. Influence of the solid surface of the land upon open oceanic systems. Midcurrents.

a. By the conditions discussed in the fourth chapter, the resting surface upon which a fluid moves, will be sufficiently frictional to the fluid to ensure motions of rolling contact of its parts. Now, assuming the actions of thermal forces in water to produce displacements in overcurrents in one direction, and undercurrents in compensation to move the water in the opposite direction. The undercurrents assumed to be of the greater density by temperature must then move upon or near the resting surface of land in a very close system, and such a motion would be very frictional. But if we take these motions to be active in a liquid system of great depth, and assume an undercurrent necessary either as a compensating current or otherwise to thermal effects, it is in no way necessary that this should extend to the *entire depth of the system*, as it is quite evident that motion in an intermediate depth, as being more distant from the resting surface, would be less frictional. Therefore it becomes extremely probable that an undercurrent in deep water, or other fluid, will have a constant tendency to take a middle or intermediate course where the resistances from the upper current flowing in an opposite direction, and the friction of land surface are brought in equilibrium about the axis of projection. Upon these premises it is very possible that active undercurrents in the ocean do not extend to the land surface in deep water, but that they flow in intermediate space, leaving the land surface in almost perfect repose.

b. Under the above conditions the rolling contact of an oceanic undercurrent, or as it would be really, as now proposed, a *midcurrent*, would deflect waters upwards and downwards, therefore move on biwhirl principles. The evidence of such form of projection may be found in 77 and 81 props., except that the influence of density is not taken in the latter proposition. This is, however, taken in 88 prop. *b*, but here we lose some of the conditions of freedom of whirl projection, as the resultant of continuity of like projectile action.

c. As this matter is important it may be well to refer again to experiment to show the possibility of such a form of motion as may produce a midcurrent in a very deep fluid, in the projection of upper and lower whirls, although the experiment I shall propose will not exactly answer the conditions of a varying density system.

d. If we take a tea-chest as in Prof. Tait's experiment, 66 prop. *e*, and instead of covering the open top with a light canvas we fix the canvas round the top so as to form what is known technically as *bellows body*, we may then project a current of ammonia chloride as before, either intermittently or constantly for the motive extent of the bellows action. If we further, instead of making a circular hole in the front of the chest as before described, make an open slit entirely across it, we may then readily project a flat horizontal current. In this case the current will flow upwards and downwards by projection upon the resistant air in equilibrium, and resemble in its projection a midcurrent, making rolling contact in upper and lower whirls.

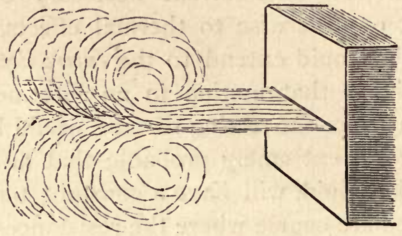


Fig. 171.—Ex.—Motive Principles of Wind-current Projections.

e. If we now refer to actual conditions and take the temperature soundings of the Atlantic Ocean as given by Dr. Carpenter, we find that there are temperatures downwards from 40° to 38° Fahr., which we may assume to represent the temperatures of most active polar density projections, we find that these temperature depths spread out as they near the equator; in fact that a cone of impression to these polar currents, appears to be formed near the equator in a stratum of about 39° Fahr. The temperatures 38° to 40° occupying less than 200 fathoms in latitudes 38° north and south but about 600 fathoms at the equator. By the above I imagine that the axis of undercurrent motion is in about 39° Fahr. or perhaps more nearly 38°·6. The temperature depths above and below this appear to spread out generally in opposite directions extending north and south, particularly from the tropics to the equator.

f. I have taken the following diagram from one given in Dr. Carpenter's paper before referred to, but here drawn to a smaller horizontal scale. Midcurrent whirls being indicated at about the position I assume that they may occur. The depth of the whirls will, however, vary with the longitude from other causes, the motive system being by composition with revolution velocities in oblique plane, of which the diagram only theoretically represents a vertical section. Secondary biwhirls possibly form at the surface exterior to lat. 20° north and south, which are indicated in the engraving.

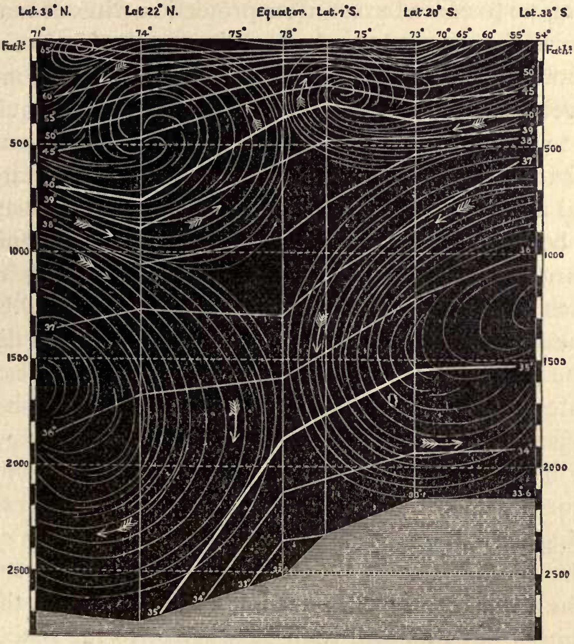


Fig. 172.—Diagram—Oceanic Section through the Atlantic showing Temperature Depths from Dr. Carpenter. The fine curved lines showing Vertical Elements of the Whirl System I propose.

149. Overflowing Forces caused by deflection from conic resistance. Vertical whirl force in oceanic currents.

a. If we duplicate the whirl system shown in Dr. Carpenter's experiment on each side of the equator, as before suggested, we shall have in the denser lower water a certain volume approaching by undercurrents towards the regions of heat of the thermal equator of the season, and have expanded and lighter overflowing and outflowing currents above on each side of the same region. The projectile properties of such flowing forces of matter, carrying momentum proportional to their masses into their velocities, we cannot imagine to come to rest after projection without maintaining an impulse as the momentum, deducting only the friction of contact on contiguous parts. Now I have assumed that fluids moving by their own velocities of greatest accommodation, as in a liquid system returning to gravitation equilibrium, suffer very small loss by friction; so that in this case the projectile momentum is largely conserved, and although the actual motion of translation may be conceived to be very slow, the masses affected in an oceanic system are very large, and the forces are effectively duplicated by opposition to each other, so that the entire momentum in collision is as great, or nearly so, as the united forces from which it is derived.

b. By the principles of whirl motions that I have endeavoured to demonstrate in many propositions, fluid projections are shown to be never perfectly resisted within any fluid system, the forces being conserved by deflections from the resistances, to move into the least frictional paths. Therefore in this case, if we have two density underflowing currents of cold polar water directed upon each other towards the equatorial area moving with any force whatever, small or large, the united forces will not directly oppose each other as perfect resistances, but they will both be *deflected* to the spaces offering the least resistance.

c. In the case given above it is not difficult to discover the area of least resistance, as the opposing currents cannot be deflected far towards the solid earth on which the water rests, nor laterally without encountering equal resistances from other approaching parts. They must therefore be deflected *upwards;* so that we may imagine a certain area at the oceanic lower surface to remain static by compression from the opposing flowing forces on each side of the equator, and this static mass to form a cone of resistance to the flowing forces that will themselves, by the momentum they carry, deflect the opposing direct currents upwards as whirls, over the cone of impression the one flowing force forms to the other; as in all other cases of fluid projection of which I have given many instances.

d. It is quite clear that the overflowing surface expansile systems would thus be forces that would enter into motive composition with whirls projected from the lower inflowing system, and complete the whirl system in underflowing and overflowing currents, in which both parts would be active, subject to certain conditions of friction upon the resting surface, discussed in the last article.

e. By the above principles we have movements of liquids upon the globe that maintain certain positions, which we may denominate forces of *cold*, engendering northern and southern underflowing currents constantly flowing towards the thermal equator; and we have equal expansile currents, which we may denominate *heat* forces, overflowing from the equator; in both instances directed to restore the density equilibrium disturbed by the forces of *radiation* from the sun upon one part of the globe, and from the earth in another. The momentum of underflowing forces being conserved by deflection at the meeting line which occurs at the region of greatest intensity of the expansile heat-forces. The general principles of the above may be represented by the following diagram, where A represents the thermal equator for the time being; C the conic area of resistance, formed by static water resting on the bottom of the ocean, or upon any deep motive plane that may represent the extent of whirl circulation for the stratum taken.

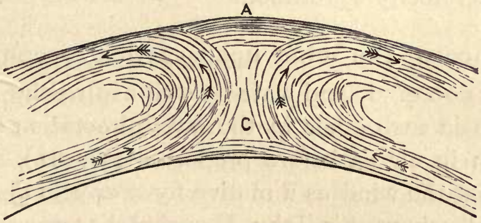


Fig. 173.—Diagram of Equatorial Oceanic Whirls.

f. It will be seen in this system that a secondary cone of resistance to the rotary incipient whirl systems right and left is formed by the upturned momentum of the deflected undercurrent force under A. This remains in the equatorial system as a band of still water, and the overflowing forces commence at some distance from this, where the incipient whirls may nearly reach the surface.

g. It may generally be conceived that the above conditions relate only to the elevation for overflow of parts of the aqueous system which may not by the density of parts quite reach the surface to form the upper current, or to the greatest depth to form a lower current. But if the overflow approach *near* the surface it will act as a *carrier* to the strata above.

h. Further, the projectile forces by the impulses derived from thermal interchanges, as before discussed for another case, would not be very great by any imaginable single effort of thermal force; but by the results of continuity of efforts of separate units of motive force added cumulatively together, with the almost frictionless modes of motion that fluids follow at velocities capable of the greatest accommodation, the system becomes finally in a certain condition of steady motive equilibrium.

i. A gravitation motive system acting by local thermal densities, whose active forces may be separately considered as small, may, under the greatest accommodation that is possible in fluid matter, be compared to the continuity of motion of a massive wheel centered upon a perfect axis which maintains a large mass of gravitating matter in motion by small constantly intermittent impulses in the direction of its motion, although it would at first require great instant force, or a long continuity of smaller impulses, to produce the same velocity of rotation.

150. Influence of the wind upon vertical oceanic surface.

a. The adhesion of air and water is well assured by the solubility of these fluids in each other in the natural aeration of water and of evaporation in air as before proposed (119 art.), so that there is no doubt that the wind as a motive force directs the oceanic surface by its adhesion. Sir John Herschel has considered this of sufficient activity to cause ocean currents, as before stated. There is no doubt, I think, that wind is sufficient to *direct* such currents as purely surface forces. The currents themselves, on the other hand, are formed by the causes discussed in the two previous articles quite independently of any action of the wind, although actual displacements of the surface would be less without the aid of this force.

b. Within a large portion of the tropical area I have previously considered the trade-winds to *oppose* the direction of the overflow produced by the thermal effects of surface expansions and evaporation. But if this overflowing heated water is active as a *force*, the influence of the wind represents only another *force*, which enters into composition with it, and deflects the fluid affected to the area of least resistance under the composition of all forces present. In this manner the expansive overflow acting as regards thermal effects only as a force directing the surface water directly from the equator to the tropic with small loss of revolution momentum, and the trade-winds moving directly towards the tropic, with a strong minus latitude momentum, gives by composition a westernly drift, the opposite polar and equatorial momentum of the two forces being apparently suppressed at the surface. Upon the above principles the united momentum of overflowing oceanic and wind forces conspire to produce one area of elevation or directive force upon the western sides of the oceanic areas near the thermal equator. This action is clearly auxiliary to the forces considered, 115 art., for revolution momentum of matter newly arriving at the equatorial zone, and modifies by composition the conditions of vertical circulation considerably.

151. Influence of Latitudinal Revolution velocity on vertical systems of displacement in equatorial oceanic systems producing oblique planes of motion.

a. I have discussed the principles of the action of revolution velocity upon oceanic surface systems for horizontal currents, 115 art. It remains now to consider some conditions which particularly relate to the direction of undercurrents whose force lines take the place of compensating systems, by their moving in the opposite direction to overcurrents, their directions being outward from the polar regions, as the thermal surface currents are inward.

b. The heavy cold water as it is projected as an undercurrent from polar regions towards the equator, in moving to gravitation equilibrium in equation with its density, carries a constant minus revolution velocity to the latitude of the earth it reaches, as before stated, therefore, it constantly drifts towards the *western borders* of the oceanic basins; whereas the overflowing surface waters moving with plus revolution velocity were shown, 128 art. *i, j,* to drift towards the *eastern borders*, this we may take as a general principle. Therefore these drifting undercurrents have motive direction towards the very points of the oceanic system where the direction of the trade-winds, and with them the surface waters are mostly drifted, that is to the same western sides of the oceanic basins.

And as the impulse of these currents meet opposing resistances, they must, to conserve their momentum, suffer deflection, which is necessarily into the path of least resistance, as before stated. In this case, taking the Northern Atlantic system, the polar minus revolution momentum carries the cold dense polar water projected southward as an undercurrent to westward along the coast of Greenland, and thus projects its impulse upon the Greenland coast and onwards across the mouth of Davis' Strait; but here the current meets the frictional resistance of the outflowing waters issuing from about the Florida coast by causes already discussed (128 art.), so that the opposite impulses deflect such meeting currents directly in an easterly direction across the Atlantic, the westerly direction for deflection being blocked by land; at the same time the deflected forces so far as they can absolutely meet, engender such a compression by the resistance, that the polar currents are thrown off the point of impact in every direction that does not perfectly oppose the escape of conserved elastic force of the currents they meet from the Florida coast.

c. I have already discussed the conditions under which overcurrents flow over certain oceanic districts, in which they maintain a certain amount of radiation of heat into the circumpolar atmosphere without such loss as would convert them into descending cold currents, until they have attained high latitudes, 113 art. *f.* In this manner the currents upon the western sides of oceanic basins near polar areas are by the polar projections, just discussed, much colder than the opposite eastern sides; these colder areas, therefore, act upon the thermomotive oceanic systems, *exactly as though they occupied the polar area, that is, the undercurrents from such cold western areas direct by their gravitation forces, irrespectively of revolution velocity, an underflow to the opposite warmer eastern areas.* Further, by such conditions, the opposing momentum just discussed, in principle (150 art. *b*), brings currents directed as regards revolution velocity to equilibrium; so that they acquire the *latitude velocity of their own positions*, and therefore take an easterly, westerly, or other direction by composition, or by any exterior impulse impressed upon them, as, for instance, the direction of the constant density-gravitation impulse in the case now discussed, which directs them from west to east in undercurrents, and east to west in overcurrents.

d. Confining our attention to the Northern Atlantic only, this being best known, here we find the eastern side of the basin including the western shores of Spain, France, the British Isles, and the coast of Norway are bathed by warm currents by causes discussed, and that the directly opposite shores near the coast of Newfoundland, Labrador, and Greenland are bathed with cold currents; and for the most part loaded constantly with ice. Now, as regards these opposite positions, by thermal gravitation effects simply, the underflowing currents are directed from the one position to the other in proportion to the differences of temperature density of the water according to well established laws for fluid forces known as a principle of hydrodynamics, and made use of practically in heating buildings by hot water. The effect of the surface extension of cold undercurrent water towards western borders of the basins, by the above causes, permits dense cold water to flow in continuous volume towards the temperate areas, and the lighter thermal equatorial currents to flow into the polar oceans by the eastern sides by continuous circulation. The direction of underflowing dense currents and overflowing lighter warmer currents under actual conditions, as in the regions of the Gulf Stream, are therefore compelled to cross each other in superposition at a certain point in equation with the directive forces derived from thermal causes into revolution velocities simply.

e. As overflowing currents, moving by direct thermal effects, take directions which are most free from land in the North Atlantic, or any other area, and such directions are also mostly under the influence of the motive force of the wind established by other causes, already considered, these overcurrents generally tend to establish the actual direction of surface forces, as before proposed, to the influence of which the undercurrents may be considered to act as complementary in order to complete a circulatory system.

f. Taking the above conditions for the whole of the Northern Atlantic area, beginning at the cold or greatest density projection, we find a cold undercurrent from the polar regions constantly flowing southward past the icebound coast of Greenland, Davis' Straits, and the coast of Labrador, and the same current then underflowing onwards by its density in an easterly direction in the same latitude. It therefore underflows the area of the Gulf Stream, carrying its density momentum across the Atlantic nearly to the coast of Africa, where it is possibly projected upwards, and laterally, by the deflection of its direct momentum against the resistance of the coast, here gradually, being warmed by the sun's rays it enters the surface in what may now be considered as a compensating system to the northern polar projection; and appears to be further deflected in rising as a surface current by the trade-winds in composition with its own direct impulse. In this general course now proceeding westerly near the equator it appears to continue until it is again deflected by the eastern coast of America. We may assume that near the equator, having its temperature raised by its less dense heated waters, it now overflows its previous direction in the Gulf Stream or general north-eastern drift, where its direct impulse is further carried onwards into high polar latitudes to become cooled down by radiation into space, so as again to enter its previous course drifting along in the Greenland current until it falls once more in the course of projection as an undercurrent off the coast of Newfoundland.

It is not necessary in the above sketch to suppose, although this may be the general drift of the aqueous forces discussed, that it is absolutely the same portion of water that passes through the entire course depicted, this is quite improbable, although possible for the course of any single particle. It is clear that any part of the system may be evaporated at the tropics, or left frozen near the poles, or be deflected from the course pointed out; but in this case other motive water takes up the forces lost by transference of impulse.

g. The following diagram will show by the arrows the principles here discussed, the underflowing and overflowing forces forming the figure 8 for a certain limit of circulation.

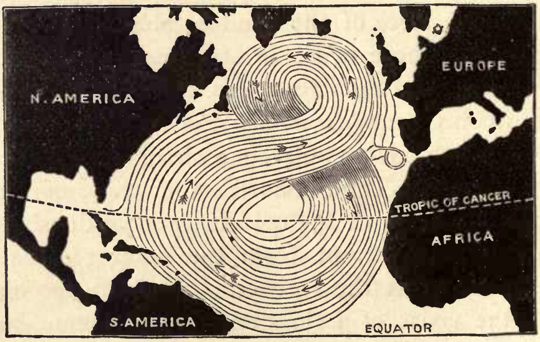


Fig. 174.—Diagram—Undercurrent, North Atlantic System.

h. It may be observed that this system in no way interferes with that proposed, 115 art., for horizontal circulation, which as a superimposed system, may be complete. It really forms a part of the same system of motive forces that could not be entered into at the time, or until further principles were discussed. These undercurrent directions of density systems may be taken in composition with surface whirl systems; such undercurrent systems very generally attaining directive forces, through which the same systematic whirls are engendered in the lower stratum, as were previously discussed for the upper ones; so that the figure eight only represents the direction of a certain axis of *biwhirl* deflection, forming in both areas of the two loops of the figure a complete whirl *in another plane* of motion; by the force of which a part of the water, in this case of the N. Atlantic system of the northern whirl, will be thrown into the more southern system, and a part after deflection on a cone of resistance which occurs at about the coast of Portugal, for some portion of the current, back into its own whirl system. This could not be shown in the engraving without complication. In this diagram I have taken a certain plane in two whirls only, but the same system no doubt in my mind extends to currents upward into the extreme northern regions by similar deflections which, taken separately, would form the like figure for certain current lines.

i. Again the undercurrent here proposed, leaving the Newfoundland coast projected in its axis in an easterly course as a density current, would at its southern borders be deflected southward to form not only the circumferential parts of an undercurrent whirl extending near to the equator, but would also fill by whirl deflection the entire centre of the system, so that this *infusion*, if I may so term it, added to the minus revolution velocity it experiences in moving southwards, again deflects a part of the force westward, causing the entire polar current to be deflected over nearly every part of the Atlantic towards the equator with some part of its force, the axis of direction only being in the lines proposed, in the last figure.

j. Further, the undercurrent motive system, being active upon similar principles to the upper system, it must follow lines similar to those discussed in 128 art. for the Gulf Stream. Thus there is in the undercurrents probably a considerable conic bifurcation off the Newfoundland coast, another off the African coast, sending a deflection to the Straits of Gibraltar, and another further south, sending a deflection from Cape Verde into the South Atlantic. This last possibly acting as a compensating current to the western superficial deflection, of which a part ultimately forms the Gulf Stream.

k. In the oceanic system we have two elements of motion which tend to project currents in a westerly direction:—That of the minus velocity impulse active upon all waters arriving by any cause from circumpolar areas towards the equator, which are either deflected upwards by opposite equatorial resistance, or in polar undercurrents impressing a certain motive energy, or carrying force upon waters above; and we have the impulse of underflowing aerial currents active upon the oceanic surface in the same direction. The entire western drift by cumulative impulses from these causes being strengthened as it reaches western boundaries, so that we find that currents that keep widely apart and flow gently by horizontal deflection (128 art. *i*), on the eastern tropical oceanic areas are generally condensed by convergence of impulse upon the western areas where currents are most active. This occurs in the Atlantic, Pacific, and Indian Oceans.

l. The whole of the above conditions are naturally subject to local resistances from the positions of land and oceanic depths. So that by density forces, upon principles discussed, whirl and biwhirl projections in oceanic systems generally take certain elements of *obliquity*. Further, an equatorial projection falling obliquely against a coast may project its waters by one whirl of a biwhirl to a depth considerably below its density position, whereas the opposite of this may be evident in superficial currents. And in the like manner polar projections may crowd their waters against a coast up to the surface for one whirl of a biwhirl, and project the opposite whirl in equilibrium deep into the interior water. This is probably the case with north-easterly currents of the southern ocean in forming whirls in the South Pacific, South Atlantic, and Indian Oceans, where such currents fall upon the immense deltas of America and Africa. The deflected whirl currents on their western sides crowd to the surface against the coast, whereas the eastern deflections, under less resistance, form cold undercurrents on the eastern sides of these continental deltas; where they are overflowed by oppositely directed warm currents. Such overflowing and underflowing currents producing uniformly stormy seas; upon conditions evident in the production of wave-motions to be discussed in the next section.

m. As currents flow with greatest facility in the deepest waters by principles discussed, 50 prop., and with least friction about an axis of inertia, 124 art. *c*, there will at all times be a tendency in currents to maintain the same areas of projection, they will also tend to wear out the deep channels, the section of which will constantly approach a semicircle, except that the tangential force of whirl currents will always throw the axis of the current outward, so that the section from the centre will be actually more nearly pear-shaped, as shown in the diagram below.

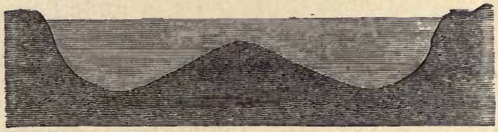


Fig. 175.—Diagram—Theoretical Section of an Oceanic Basin showing ultimate effects of Whirl Action.

152. Influence of the Sun and Moon in Tidal Phenomena in constantly restoring Gravitation Equilibrium.

If we imagine a fluid projection engendered by thermal expansion, condensation, evaporation, rainfall, and revolution impulse to proceed from polar to equatorial regions to establish gravitation equilibrium between a couple of forces, we can at once see that such motions, within the velocities of perfect fluid accommodation of the least frictional form, must be very slow. But we have in the intermittent attractions of the sun and moon constant separate efforts of local superior motive force, which at each effort raises a certain mass of water above the point of mean static equilibrium; and although this force is diurnal and intermittent, it is superior in immediate effect to thermal and evaporative forces, therefore it moves the liquid mass at every effort beyond the plane of equal mean gravitation equilibrium, leaving it when the attractive force is withdrawn to fall to the natural gravitation equilibrium of the system. By this means, of oscillation beyond the point of mean equilibrium, every force out of equilibrium in an extensive fluid mass is *accelerated in its direction towards the equilibrium which is constantly being only partially restored*, so that we may add tidal forces, which act cumulatively, to other modes of motion which I have offered as being active to accelerate the motive impulses previously considered in restoring equilibrium, and thereby rendering such *permanently* active in their motive directions by composition with all forces present in actual fluid systems.

CHAPTER XIV.

WHIRL AND BIWHIRL SYSTEMS OF VERTICAL CIRCULATION IN THE ATMOSPHERE—COMPOSITION WITH HORIZONTAL SYSTEMS—OBLIQUE SYSTEMS—CYCLONES—HURRICANES—ANTI-CYCLONES—CLIMATIC INFLUENCES.

153. General principles of vertical aerial circulation.

a. It appears to me somewhat curious that Sir John Herschel, who has negatived Lenz's theory of under and overflow of oceanic currents produced by thermal causes, should have himself proposed or supported a similar theory for the under and overflow of aerial currents as he has done in art. 52 of his "Physical Geography" in the eighth edition of the *Encyclopedia Britannica.* In this he offers what appears to me, generally, a sufficient theory for the direction of the dominant winds of the earth, that flow towards the equator in the trades, and for polar or westerly winds also, although not so definitely or logically.

b. Upon the same principles as those given by Herschel for equatorial currents, I shall for the most part discuss the vertical directions of aerial currents, generally, adding only the influences of whirl motions which I consider to be active in vertical atmospheric circulation, in the same manner as I have endeavoured to demonstrate this form of motion in other cases of fluid projection under every form of resistance.

c. In following this matter, we may take it for granted in the first place, as before stated, that as a general principle the sun's direct rays cause expansion of the air or increase of elastic force in proportion to the amount of heat he is able to communicate, which will be within certain limits upon the earth's surface, approximately *per area,* inversely as the sine of the sun's obliquity. At the equator therefore there will be a great expansion of aerial fluids; at the poles very little (112 art. *e*).

d. As before stated, the influences of equatorial expansions cause the air near the equator to have less specific density, and the constant excess of radiation of heat near the poles produces greater specific density, the air being held to the surface of the earth by the attractive force of gravitation, acting with greatest intensity on the colder, more condensed, and nearer parts of the air, which parts are therefore urged to flow at all points to an equal surface of gravitation density. The warmer, lighter, or more expanded air, in this manner, receives less gravitative impulse, and takes its position above the heavier fluid in the same manner as air takes its position by gravitation above water. This it does in proportion to its motile force, or power of accommodation for one part of the system to displace the other, that this arrangement may occur.

e. Now it is apparent that the above-mentioned systems of forces must be active in the manner described, but as terrestrial currents do not entirely follow such directions as here implied, of simple density superposition, we may conclude that as we have taken fluid force to be persistent, that there are modifying causes which deflect aerial currents into the directions we find them by observation.

f. To follow this matter into the principles by which aerial matter is projected upwards in certain regions of the earth, we find that the sun's heat is equally active over nearly the whole of the broad tropical band, where the sun's meridian altitude is never less than 66½ degrees. Therefore in these tropical regions the most material expansions of the air take place, thereby forming upward currents of great breadth which can have only a limited projection against gravitation before being thrown out of equilibrium to the general gravitative system of the globe, and hence they must overflow in order to continue the vertical thermal projection caused by heat expansions, as before proposed. In this case there must also be an equal inflow of air to the overflow, for the atmospheric pressure to remain nearly constant as we actually observe it does, and this must occur, as before stated, by undercurrents, which extend to the equatorial band of upward draught. As auxiliary to the direction of this inflow, we have also, by the aerial friction upon the oceanic surface, a region of induced oceanic surface movements, which I have shown are influenced to flow in the same direction as the aerial, and which, by reaction, consequently establish a consistency of direction for aerial inflow in the least frictional course. The aqueous and aerial forces by cumulative efforts being throughout the system auxiliary to each other's motion in the directions of flowing forces, so that in accordance with the above we have in similar positions upon the globe a like direction of active aqueous and aerial forces over the least frictional surfaces, observable, for inflow particularly, in the regions of the trade-winds, which by the causes given, may be sufficiently accounted for.

154. Establishment of the vertical aerial whirl system of the tropics.

a. If we now take the trade-winds flowing from their origin in circumpolar areas towards equatorial ones, as equal systems throughout the entire equatorial regions, and consider the equatorial direction of forces, without for the present taking into account the natural deflection brought about by the difference of circumferential velocity of the parts of the earth or the kind of surface over which these winds pass, we have then a similar case to that discussed for flowing waters (149 art.); that is, two fluid forces flowing from northern and southern polar areas in direct opposition to each other, meeting near the seasonal heat zone; and we may at this point, I think, again look for similar results to those discussed for the oceanic aqueous system.

b. If we assume for a moment that such volumes of flowing air from north and south could absolutely meet at the heat zone, the first effort of such meeting would be to produce an immediate elastic condensation, which, on the supposition that the underflowing force is from north and south, would as far as possible deflect the current eastwards and westwards, upwards and downwards, in fact in every direction normal to the lines of direct flowing forces. Now, suppose for the instant that the inflowing air forms a complete system, moving at every point from polar to equatorial regions, then the eastward and westward deflections could not occur since the system would be in equilibrium of pressure to the extent of both these directions throughout the entire equatorial regions. The elastic compression must therefore in this case, be deflected upwards and downwards only. But in the downward direction we should have the solid resistance of the earth or ocean which would only act to conserve the pressure by the elasticity of the system, hence omitting all considerations of circumferential velocity, the whole momentum of the system would be deflected by the entire resistance in one path only, that is, *upwards.*

c. I have supposed in the above case of two opposing systems of flowing air to the equator, that absolute contact is made between them, but if we analyse the possible conditions we see at once that this is impossible, or only possible, for an infinitely thin stream of opposing flowing fluid. In the first place, for the meeting in one area of two volumes of air there must be space for the air contained in the two currents, which press against each other, and, as air is an elastic fluid, the meeting currents would form a *volume of condensed air*, which assuming the forces equal would immediately produce by reaction a broad band wherein the air brought to equilibrium would rest static. Further, after this, any inflowing equal current moving upon this static band would increase its breadth and therefore its resistance; so that the band would finally represent a broad static mass of condensed air, which would be sufficiently extensive from its inertia alone to form a resistance to any future approaching flowing aerial fluid. We may further conceive that the influence of this central resistance would be perceptible in the approaching air so far as the elastic reaction of the condensation could act.

d. We find in consideration of further conditions that the trade-winds do not consist of a narrow band of matter, but of a constant stream of large volume; consequently, although we may imagine a small portion of the trade-winds to reach the point of resistance just described, the larger part could in no way reach this point. Now, by the laws of flowing force fully discussed, a fluid will uniformly flow to the points of least resistance, and as I have shown that the opposite flowing currents are resisted at all points but one, we must conclude that the *whole of the flowing force* so far as the equatorial or thermal directions of its momentum is conserved, will be directed upwards, as before discussed.

e. Taking at this point of our theory two flowing fluids brought to a plane of resistance and deflected upwards, we may follow the components of force-direction necessarily engendered thereby. The first impulse of the resisted elastic fluid being directed upwards, from both sides of the heat zone, there would be elevated a volume of air and vapour, but we cannot imagine that this would be raised to an *unlimited* height; for after the elevation of a certain volume of the flowing mass, the weight of this mass alone would afterwards become a pressure upon the rising current, so that as the inter-tropical atmosphere was constantly elevated it would form constantly a resistance to further elevation, and would finally have no vertical momentum superior to gravitation. Therefore as it mounted upwards it would be brought at a certain altitude to a state of rest.

f. We may further conceive from the atmosphere being elastic matter, that as it rises it is constantly expanding by the minus surrounding pressures and its power of elastic extensibility (4 prop. *c*), so that by this expansion it overcomes the resistance of the surrounding lateral matter, and at a certain elevated position, when it is brought to a state of vertical equilibrium, it rests upon the aerial matter below as a pressure of static mass, in a similar manner to that which the oppositely directed horizontal forces, just mentioned, bring the inflowing currents from north and south to equilibrium near the equator at the surface of the globe.

g. It will be seen by the causes offered above, that we have finally near the equator a mass of quiescent aerial matter whose inertia simply resists the flowing force of currents by compression upon its elastic mass. In this we have not a mere plane of resistance at the meeting point, but an extensive *volume* which becomes in a certain sense in equilibrium of inertia to the inflowing forces of the entire meeting currents; so that the equatorial directed undercurrents are first deflected at the point where they encounter elastic resistance, from opposition sufficient to deflect the momentum they carry from a horizontal direction, and then under this deflection which is carried to verticality, they encounter an equal and similar resistance from elevated matter above, which induces the vertical impulse to overflow its original plane, and direction of projection.

h. I have just pointed out that air and vapour could not be projected indefinitely upwards, since after a certain mass was so deflected, its gravitative force would equal its projectile force. Therefore we can imagine that when a certain mass of air is elevated to a certain height, it occupies a position in which it forms a static resistance to the rising currents, so that it now acts upon these currents as a *conoid of impression*. Further, as the elevated fluid, under the above conditions, rests supported by the force of the equal currents deflected upwards, it then becomes the superior deflecting force acting as a circumscribing wedge-shaped conoid of impression, having its point directed downwards, over the entire surrounding thermal zone. Upon this ring-like conoid of resistance, inflowing aerial fluids after deflection at the tropical surface attain a certain altitude, and are again deflected by conic resistance, so that they overflow by a *circuit* from the point of original projection, and induce thereby a circumscribing biwhirl system.

i. Taking the whole of the conditions above given, and assuming any possible inert volume of air laterally contiguous to the directly rising aerial fluids situated outwards from the position of the more active upward thermal projections, such fluids will be affected by adhesion to the rising motive currents acting tangentially upon them as before considered for liquids, 149 art. Therefore if we assume a static volume of air to exist laterally above the undercurrent of flowing force exterior to the heat zone, such a volume will be moved tangentially by the constancy of the undercurrent, it will also be moved in relation to its centre of inertia in the same direction, tangentially, by the constant friction of the upturned deflection of the undercurrent, and in like manner by the constant overflow from causes given, supposing these forces so far to circumscribe the centre of inertia of the lateral mass.

j. Under the above circumstances active in inducing whirl systems by tangential force, of motive parts of the system, whose tangential moments may complete the whirl for an entire circuit by their separate impulses, with such deflections as the many correlative conditions bring about, I will now endeavour to show diagrammatically, by representing the entire phenomena under certain limited conditions.

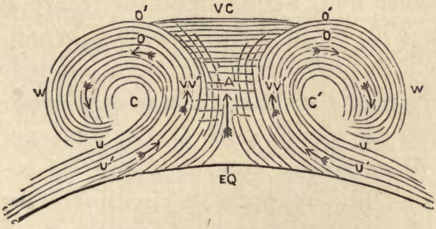


Fig. 176.—Diagram of Aerial Forces active about the Thermal Equator.

E Q. Position of the thermal equator towards which inflowing currents move from circumpolar areas.

U, U′ Undercurrents of flowing force having the greatest density and mass velocity at U′U′.

V, V′ Deflected undercurrents moving upon the area of greatest static elasticity over E Q by which they take vertical directions.

O, O′ Overcurrents, deflected by the aerial matter raised by vertical force towards V C upward to form a superior cone of resistance; the upward direction being constant until the matter raised against gravitation attraction equals the force of projection. The greatest velocity and elastic force remaining in continuity of projection at O, O′, active upon the whirl systems induced by the now inward, upward, and overflowing exterior forces.

C, C′ centres of inertia to the lateral whirls.

k. In the above scheme I have intentionally left incomplete the whirl at W. It will be seen that the motive effects offered would only perfectly account for three sides of such a whirl, but we may conclude that if the circumferential resistance upon the fourth side does not exceed the general resistance of the motive system, that the tangential momentum of the other three sides in moving a unit cohesive system upon its centre of inertia will carry the circumferential momentum over the fourth side; there is, however, on the descending side of the whirl a special form of resistance necessary to be considered, which is found in that we have in the descending side of the whirl at W, the tangential forces of the motive parts constantly encountering a higher resistance by the greater density of the air, as the motive circumference of the whirl at W descends nearer to the earth; so that the whirl to complete its spiral system must throw its lighter circumferential parts constantly against the lower denser parts. In this case the superior elastic forces and density of the lower fluid acting upon the rotary system throw off by tangential force a part of the descending current represented by the side of the whirl at W, leaving another part only to continue in the system by cohesion; so that we have a bifurcation of the overflowing projection in which, were the system purely aerial, the resistance to a downflowing current would be so great that the whirl could not possibly complete its circumference, but the underflowing and overflowing system would under these conditions be a deflected one of the class shown, 83 prop. *b.* By the influence of vapour condensation, to be presently considered, the whirl will be found to be completed in certain components of its directive impulses downwards; whereas other components may be represented by purely aerial conditions which produce an overflowing system not entering the whirl, but causing it to extend the overflowing currents possibly into polar regions.

155. Influences of vapour force and condensation upon the equatorial aerial biwhirl system.

a. In reconsidering the action of the expansive force heat, in the terrestrial aqueous vapour system; I have already observed, 117 art., that vapour possesses special functions of vertical lifting force derived from the expansion of water to nearly 1000 times its volume at an average tropical heat and pressure. This vapour may therefore be considered, upon its production, to consist of new matter, which replaces an equal volume of air, as before stated, since we do not find that it is chemically absorbed in any way by the air; for if this were the case, the air would be of greater specific density by the intrusion of vapour, whereas it is really of less.

b. It is very probable, that over oceans the evaporating force varies directly as the verticality of the sun's rays falling upon the surface of water exposed to its direct heating effects, and inversely as the humidity of the air above it. In equatorial regions we may entirely neglect all functions of humidity of the air, as the diathermacy of the atmosphere appears almost entirely to depend upon the deficiency of humidity; so that possibly there will be little difference in entire elastic force derived from the sun's rays, whether the atmosphere be moist, and the sun's heat be retained to expand the moisture or cloud to elastic vapour; or dry, so that it evaporates the surface water directly; as in either case it engenders an elastic aerial system over one area that acts possibly as an equal force of aerial extension in both.

c. The small declination of the sun's rays possible over an equatorial band extending from 10 to 20 degrees, will leave this zone approximately equal throughout, in its vapour-elevating force; the deflection of uprising currents would therefore occur generally before they reach this area of greatest intensity of heat-force by the *volume* of inflow of the lower surface currents. I think from this cause it is extremely probable that the vapour raised within the limits of an equatorial band of about 20 degrees is not deflected considerably to overflow, but that the vapour rises to a point of saturation, and forms a resistance to inflowing currents, being for the most part condensed to water over the *same area* when the vapour pressure exceeds the elastic thermal force of the average lifting power from penetration of the sun's heat through it, to the oceanic surface of evaporation beneath. This leaves the position of the inflowing currents, *exterior* to the equatorial band proposed of 20 degrees at the surface of the globe. The direct lateral impulses, however, would carry the inflowing currents much nearer together at a more elevated position, where the earth's superficial resistance would be less felt, so that they may possibly approach at the elevation of mid-atmospheric altitude of projection to within possibly about 10 degrees of each other. Therefore the evaporation taking place up to and within the cone of impression V C, Fig. 176, would be returned to the earth as rain in the vicinity of its elevation.

156. Whirl effects of condensation of overflowing vapour.

a. As the heat derived from the sun is a constant diurnal force, and aqueous vapour retains radiant heat in an exceptional degree from being very slightly diathermous; it is most probable that the saturated atmosphere in rising in the manner described, will fully maintain the elastic force in its vapour system as a perfect gas during expansion under less compression when rising directly in the sun's rays, so that no condensation to *cloud* may occur from the expansion during daytime in tropical areas. Now, in the area over which I assume there is an overflow, that is, not the tropical area of central or conic resistance, marked V C, Fig. 176, the vapour system in the air will represent an equal elastic force to that assumed for the purely aerial system, and by this it will be expanded to overflow, in the same manner as conceived for purely aerial currents. But in the continuity of this vapour projection into cooler areas in overflowing it will not maintain its elasticity as a perfect gas, but will condense and form cloud, producing the same diminution of elastic force by its condensation directly to water, as the elastic force it previously attained on evaporation, that is, to about ·001 of its volume. The entire amount of vapour force for the rising current and its after condensation will nevertheless partly depend upon the amount of surface evaporation of the area traversed, that is, it will be greatest over oceans, least over land.

b. If we conceive a flowing force in air saturated with vapour moving by backward pressures, and supported by the elasticity of the aerial vapour system, such flowing force would be retarded by any forward condensation from loss of elasticity, in such part of the system as consisted of vapour. But if we theoretically separate the elastic forces of air and vapour, we find that the air diminishes in volume in approximately equal ratio for equal loss of heat; whereas on the other hand, the condensation of vapour is nearly instantaneous for such volume of the vapour as may be converted into cloud or water; therefore the loss of elastic force in the vapour system, supposing this to proceed in regular proportion to distance from the thermal equator, would be in a much higher ratio than that of the air, and although the condensation of the vapour would release a certain amount of heat-force which would be communicated to the air to increase its elasticity, yet as the angle of declination of the sun from the thermal *equator* became greater, this force would be constantly lost in a much higher ratio by radiation into space, as before stated.

c. Further, we can imagine that at a certain angle of declination to the thermal equator, the aerial and vaporous system would lose elastic or heat-force and form visible cloud. That at this area the sun's heat being shaded from all lower strata of vaporous air in the same system, that rapid condensation would occur beneath the cloud, the direct influence of the sun's ray being taken from the entire intervening space between cloud and earth; so that, if we imagine some latitude temperature whereupon a cloud-covered area could rest there would be by the causes discussed, here constant condensation into which the sun's rays could not penetrate to preserve the elastic force in the lower stratum of vapour. In such an assumed area of condensation from the immediate minus pressure that the condensation would produce, less resistance would be offered to the downflow of the overflowing vapour, which would be drawn in to make good the loss of elastic force by condensation from the nearest direction. Further, by this means, every overflowing influx of aqueous vapour to this area would be retained as cloud, so that the area of condensation would be upon the above principle somewhat definite in locality in a certain latitudinal position, where we should have such an amount of vapour saturation in the air as could no longer be maintained in the vaporous or cloud system, but must fall as rain.

d. The conditions previously discussed for the continuity of an aerial system from equatorial to polar regions, would be active for purely aerial forces only in dry air, and the vapour forces would fall far short of the polar area, for condensation of the largest amount of vapour in moist air; as such vapour forces would not be able to maintain their elastic gaseous systems for the greater part of the thermal force they embody, very far from the tropical area where the sun would rapidly lose its sustaining force upon them. Therefore the elastic force of vapour projection would be nearly lost under the excessive condensation of vapour that would occur, and the vacuum produced by the condensation would not only cut off the projectile force of the overflowing currents, but would retard the aerial overflowing projections beyond, and induce them largely to follow the gravitation forces of the condensed cloud system that would be dissolving locally into rain. Under the above conditions there would be, as proposed, at a certain distance from the equator an area of *condensation*, which, if all forces were equal upon the surface of the earth, would surround the tropics as a band on both sides.

e. We may nevertheless imagine that if there were any specific cause for condensation in any part of such a band, as for instance by a land slope of greater declination to the sun's rays, this would be an area of greater proportional terrestrial radiation, and the condensation would concentrate particularly near such position. Further, towards this area, whether wide or local, the overflowing vapour forces, by minus elastic resistance, would be directed by the surrounding general gaseous expansion, and the vapour condensation being nearly instantaneous, aerial forces from polar directions as well as from equatorial would be drawn to this area; so that there would be a meeting at this place of aerial overflowing forces at the altitude of the cloud system, formed by condensation.

f. At such a position of meeting of aerial forces by the minus elastic forces of vapour condensations, from the condition of cloud when formed containing visible particles of water, it must occur that the entire condensation of the invisible vapour will form a mass *heavier* than the aerial system in which it rests by the entire amount of the visible matter contained in the cloud. For it is quite certain that the purely aerial system does not lose elastic force by the presence of cloud, neither can its condition be assumed to be in any way altered; therefore all cloud systems, by this superior weight, will be descending, and in their descent they will also carry some part of the adherent air with them as before stated.

g. At what declination from the thermal equator such an aqueous aerial system as that proposed above would generally occur might possibly be discovered from known data of elasticity and condensations of vaporous admixtures of air and water at certain temperatures, but this I am unable to follow. I presume from geographical observations it occurs at about thirty degrees of latitude from the thermal equator for the time; being influenced also by the local radiational force of the area upon which the system is superimposed which makes it one of greater or less latitude.

h. One matter in the above is important, namely, that by the superior weight of vapour condensed to water or cloud over gaseous vapour, the condensation produces a *descending area* or causes descending currents. The condensations offering at the same time space for the presence of such by contraction. Further, the rain system produced by condensation drags a certain volume of air by adhesion downwards with it. We may thus imagine that the whirl system, shown Fig. 176, page 439, would, by the addition of a descending force, on its polar side be completed in an elongated ovoid form, terminating at about 30 degrees of latitude; for we have now by this descending force the four sides of our whirl supported by active tangential action. Further, we have in such a whirl by the continuity of flowing forces a system, which once induced, would support the continuity of the weaker, that is, the descending side of the whirl, that would bring about continuity of whirl projection, even when the descending force was very weak, or possibly even at times slightly negative. In this manner the whirls proposed, 154 art. shown at W, Fig. 176, are rendered so far complete that a large element of the whirl motion would be continuous in areas lateral to the equator extending 30 degrees N. or S., more or less according to local and seasonal conditions over every area where evaporation was sufficiently active to nearly saturate the air.

i. We must, however, in the above conception observe that the fourth side of our whirl moves constantly to denser air as it descends, as before shown, by which although the air and vapour may be assumed to constantly change density from resistance to resistance downward as the flowing force is impressed, nevertheless the resistance encountered by the lower air is a constantly retarding force, and it is therefore only by condensation of the vapour to water that it gains sufficient gravitative impulse to enable it to be pulled through the complete whirl system.

j. Under the above conditions it is most probable that the resistant descending side W, Fig. 176, of the whirl generally forms a conoid of impression and divides against the resistance of the denser air, so that a biwhirl is again formed upon the resistance of the earth's surface, one whirl of which only enters by underflow into the primitive equatorial system, the other being deflected outwards from it toward the circumpolar area. This would possibly cause the terrestrial vertical aerial whirl system to take the form represented in the following diagram for vaporous aerial systems, particularly over oceanic areas.

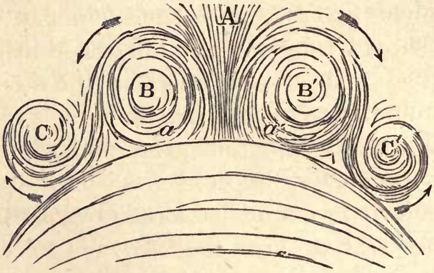


Fig. 177.—Diagram of Vertical Biwhirl Systems.

A represents a cone of resistance formed by continuity of projection of thermal elastic forces.

B and B′ biwhirl engendered by conic resistance at A, the whirls returning by reflection into their own system as shown at *a*, *a′*.

C and C′ secondary whirls formed by tangential forces thrown off from the exterior of the whirls B, B′ by a cone of resistance upon the earth where these whirls intersect, their downward directions being largely maintained by vapour condensation. The complete whirls C and C′ are purely diagrammatical, and the exterior upturned arrows are superfluous at the position shown, as these whirls would be ellipsoidal systems, and much displaced by rotational velocities not now taken into consideration.

k. If by the above propositions we take one arm of the lower biwhirl C, as being deflected northward in this hemisphere, and composed of warm visibly vaporous, therefore more dense air, it will be readily conceived that as it drifts by its projectile direction as a density undercurrent further northward, it will by loss of heat constantly become more dense, that is, more visibly cloudy; this will particularly occur over and near the ocean surface where such surface by its warmth from initial overflow (149 art.) supports a stratum of vapour-saturated air. The path of such a current by its constant radiation in space above will be one of constant increase of fog or of rainfall, that by cumulative action of constant condensations will upon the whole engender or pull forward a general flow of air in the same direction; by contraction of bulk in proportion to the extent of the elastic vapour force still retained. This will partly account for the direction of aerial surface currents northward by contraction upon the cold surface of the globe.

l. By the conditions of descending visible cloud currents, we have not the necessity of the descending region being a more rainy one than contiguous parallels of latitude, in fact there are some reasons that it should be less so, unless the downflowing current meet a cool surface. This is seen in that the downward pressure upon the air would increase its elastic force in such a manner that it would be able to maintain proportionally more vapour, so that the vapour would not be condensed until this pressure was somewhat released by lateral deflection and expansion.

m. It is also necessary to limit the extreme projection of a vapour system under condensation, as considered above, to oceanic areas where continuous evaporation is possible to support it. Thus it would be materially interfered with by such extensive tracts of desert as occur in parts of Asia and Africa where the dry land area is for a considerable distance continuous. Indeed in this case we must for the most part omit the consideration of the condensation aqueous system altogether, as the air is so much under saturated that it maintains its vapour as a purely aerial system whose overflow and underflow may extend from polar to equatorial regions upon principles discussed, art. 40.

n. We have evidence in several instances of the purely aerial system being persistent by continuity of overflow. Thus, over the great mass of northern land forming Asia, and by continuity in a north-easterly direction of the larger portion of Africa, which immense district has but the small discontinuity from the presence of inland seas which are not extensive. Here it is said that the north-easterly winds which take the direction of the oceanic *trades* flow continuously during the time that the sun is north of the equator. There is no doubt that this inflow is mainly due to the expansion of air only by the sun's heat being now greatest upon northern areas, but the fact that the same winds do not uniformly traverse equally oceanic areas is due to the presence of vapour, which must be manifestly increased in air traversing an entire area of constant evaporation. The aerial system is therefore one of greater amplitude in proportion as its mass is more dry, so that we have not over land areas the tropical whirl complete in about the 30th parallel of latitude, but the overflow continues as a purely density system until its temperature is reduced by radiation in the polar regions, upon principles discussed, 140 art.

157. The meeting-place of oppositely directed aerial fluids near the equator is not an area of great atmospheric pressure, whereas contiguous areas are so.

a. It will be seen by previous discussion that although the locality of meeting near the equator for inflowing aerial currents is an area of resistance, and therefore of more compressed air at the earth's surface, that as soon as the inflowing air is deflected by the resistance to approximate verticality, that the direction now taken by the inflowing air would project it upwards; therefore, although the surface stratum upon the globe would suffer lateral compression, the deflection would at the same time tend to remove a part of the vertical pressure above the area of deflection by the upward direction given to inflowing lateral currents. Thus the locality of opposite resistance would *not be a parallel of greater aerial pressure upon the globe*, but by the direction of deflected forces might possibly *be one of much less*, although of the largest amount of superimposed aerial fluid. As this is important it may be useful to explain the principle here given by diagram.

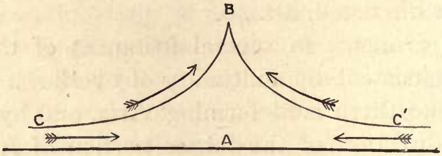


Fig. 178.—Diagram of Meeting Currents.

b. In the figure above let two currents of fluid forces C and C′ be directed, as shown by the lower arrows, towards A, assumed to represent the equator, and let them form by equality of resistance a static band about this position; then by continuity of the same direction of fluid forces C and C′, the currents projected will be deflected from the static plane to a position B; and the gravitation forces upon A for the space C B C′ will be relieved by the upward direction of forces from C to B and C′ to B. The pressure of gravitation in the system falling mostly by reaction upon the resistant points C and C′. Pressures at the equatorial area A being proportionally less. This is evidently the actual condition, and I assume the principal cause of minus atmospheric pressure observable near the equator.

158. Effects of the earth's rotundity upon aerial fluids inflowing towards the equator.

a. In flowing fluids we have heretofore considered the flowing force as following the outline of the earth's surface; it is quite evident by natural laws that material bodies can only flow in this direction by deflection under the force of gravitation, as we are fully assured that it is the property of all bodies impelled with any force to proceed in direct lines. Therefore there is a constant tendency in a horizontal projectile fluid to move tangentially to its last horizontal position on the earth, which will influence it to move in an upward direction. By the same causes currents, in proportion to the mass-velocity of flowing matter they contain, will have a tendency to tangential projection, except as they are withheld to the earth's surface by gravitation.

b. Now, an aerial fluid in flowing towards the equator will, as before shown, be constantly under the influence of the expansive force of the sun's rays, therefore as its projection continues gravitation forces become less *per volume*, and supposing the globe covered with an equally dense aerial fluid at every point, the expanding fluid will flow with less resistance in direct tangential line to its previous position; every possible deflection being equal to a certain amount of resistance. The influence of the above conditions nevertheless form only a small component of directive forces in inflowing currents, from the constancy and equality of gravitation.

159. Effects of radial velocity upon vertical aerial and vapour systems.

a. The latitude velocity carrying displaced aerial matter in currents has been already discussed in 137 art. for the horizontal motion of these currents. In the above considerations of vertical currents I have neglected the influence of the constantly active force derivable from the rotation of the globe, for simplicity of demonstration, the effects of which, added in all cases to the directions of the overflowing and underflowing currents given, should complete the directions of the projection of aerial matter upon the globe. The discussion of the active principles of latitude-velocities on the vertical movement of the aerial system will now need our consideration, for a few particulars only, that the discussion of the motive directions of vertical currents have now rendered necessary in relation to the surface of the globe over which these are drifted. It has already been shown that by thermal forces aerial fluids have a rising force over tropical areas; there is also a further condition, that as aerial fluids rise from this region, the radial distance from the centre of the earth will also increase; thus by direct upward projection from thermal effects, the circumferential velocity becomes relatively less, and in this we have an additional cause of westernly drifts increasing with elevation. It, therefore, becomes clear by composition of these force-directions, that there will be a strong tendency for aerial fluids to take an *oblique* upward direction at the tropics, *westward from the surface of the globe.*

b. Now by these causes, if the earth were an equally frictional sphere of revolution, and equatorial whirls were engendered by causes discussed, 116 art., and by condensations, 117 art., such whirls being completed, then we should have surrounding the globe two complete spiral whirls, whose motive planes would be oblique, rising to the westward towards the equator, and falling to the eastward towards the polar area. The equatorial zone by conditions, 156 art., would then from upward projection be a band of low pressure; and exterior to this the globe would be surrounded by two bands of high pressure. The equatorial whirls being assumed by causes given to press obliquely and slightly inclined to the equatorial plane, they may attain to high latitudes with only small deviations from a nearly circular cycloidal orbit, and therefore be engendered with small internal friction.

c. There is a further condition necessary to be considered in that the equatorial region being an area of resistance and repose from the equal opposition of forces of the northern and southern inflow of aerial undercurrents, by which these regions possessing a broad base on the earth's surface, will resist motion of aerial matter until it becomes in, or approaches to, velocity-equilibrium with the revolution of the earth beneath it. Therefore aerial fluids that impinge upon the more static equatorial zone are resisted by it in parts in proportion to its power of inertia, adhesive in matter upon the tropical land and sea whereon it rests. Hence the inflowing currents, after a certain westerly drift, come to relative rest by friction of contact upon this resistance, and overflowing currents that are deflected from it, as they overflow, take so much of the revolution momentum of the equatorial band, or plus velocity to the latitude into which they now overflow, that they, relatively to the earth's latitude-velocity in moving towards polar areas, possess in overflowing, a strong easterly drift.

d. By the whole of the conditions given above, it will be seen that the inflowing currents, as they first rise in space, have a strong minus momentum to the greater circumference and latitude-velocity of the earth, so that they rise with a westerly drift. That this drift is finally and gradually lost by friction on a static equatorial band; whereas thermal impulses being continuous, these, in overflowing outwards from equatorial latitudes after a certain distance from the equator, have their direction reversed relative to the latitude-velocity of their new position so as to take an easterly course.

160. Influence of land resistances upon vertical atmospheric currents under displacement.

a. As the conditions of land resistance vary according to locality, I will discuss this matter for one area only; and that upon general principles, omitting many details that evidently represent active forces in the system.

b. By the disposition of land and water I have pointed out the probability of there being established, two great horizontal whirl systems in the northern hemisphere; the movements of which are particularly shown, 140 art. In this case the direction of mountainous land on the north-eastern coast of South America was shown to materially influence the deflection of equatorial horizontal currents in the North Atlantic area. Now, we can imagine that the same influence would also deflect the westerly vertically directed currents, as previously discussed; that is to say, that a given resistance, as a mountain, situated in a direct flowing current, would not only deflect the current round its sides, but by its inclination, upwards as well. Therefore, such mountainous land opposed to the westerly directed rising currents would give these currents greater inclination to rise, so that, especially near the eastern coasts of continents that resist the equatorial currents proceeding from oceanic areas, the whirl plane would be thrown upwards obliquely by such land resistance.

c. There are some further conditions which establish the locality of an oblique plane, as, for instance, in the North Atlantic, by the effects I have pointed out in 144 art. of the natural frictional resistances that must occur at the meeting of two whirl systems under deflection of land resistance. In this case, at about the region of the Caribbean Sea, I have shown that there will be a reflex whirl from the Pacific system which will fill, as it were, the interspace between the Pacific and Atlantic whirl systems. This reflex whirl I have marked *a* in the diagram, Fig. 168. If such a complementary whirl exist it forms a further resistance to the direct projection of the westerly drift of the rising and overflowing equatorial current; which would cause greater deflection and produce further elevation than by the causes previously discussed of land resistance only. Therefore it is possible that the whole plane of the N. Atlantic whirl system, by these causes, is thrown oblique, having its highest point somewhere over the Gulf of Mexico, and its lowest about the Azores; where the momentum of the downward aerial drift may be perceived as plus barometrical pressure, as before stated.

d. The above assumption of an oblique whirl-plane, does not assure us that there will not be also surface whirls, as previously discussed, but only that in this direction there will be a down-flowing of vaporous aerial forces which, from other causes given, are directed from the equatorial areas. There is, no doubt, also a superimposed more purely aerial system, the principles of which were discussed, 141 art. *b*, that upon the whole, is not very greatly affected by the influence of vapour forces which, in condensation, compel

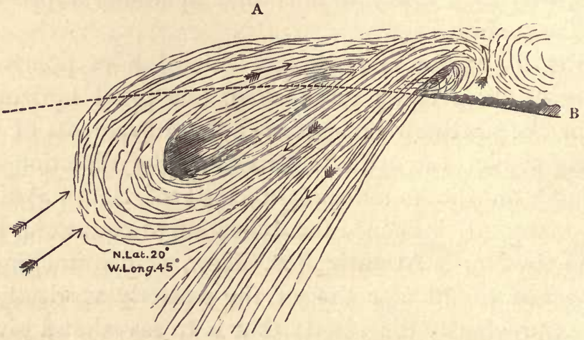


Fig. 179.—Diagram—Proposed Oblique Whirl Plane of the N. Atlantic.

certain strata of the air only to take a smaller and differently directed area of projection.

e. The purely aerial forces are completed in the plane of gravitation equilibrium suffering possibly deflection to whirl-current lines above the areas of vapour condensation, maintaining at the same time, by bifurcation, their direct projections over dry land areas in the direct whirl lines proposed, 140 art., aerial forces being deflected to the earth at a much greater distance from the equator than the vaporous.

f. The principles of this oblique plane of aerial motion under the influence of vapour forces, for the North Atlantic area, may be represented roughly by the diagram Fig. 179 where the dotted line represents the equatorial surface. The directions of forces shown about the arrow under A are represented impinging upon the area of resistance of American coast towards B, shown by shading to the right, here meeting also the reflex whirl above B.

g. The oblique overflow is directed downwards, the whirl impinging upon the surface of the globe in about lat. 20° N., long. 45° W. The superaerial system (Fig. 168) not being here shown.

161. Terrestrial surface resistance to winds.

a. By the general adhesion of air to all other bodies, its movements over the earth cause a deflection of flowing forces downwards to the surface in whirls, as previously discussed for other cases of side resistance. The whirls thus formed are general over every kind of terrestrial surface, being of small amplitude over still water; of greater amplitude over rough water and level land, and still greater over irregular hilly land and in front of obstructions upon it, as of mountains, trees, or rocks. This frictional deflection would, therefore, produce whirls of rolling contact from overflowing currents, the dimensions of which would vary from fractions of an inch to several hundred or thousand feet, according to the velocity of the current, and obstructions capable of engendering conic resistance, the general conditions of which will be the same as those discussed, 98 prop., for cohesive and adhesive systems of fluid matter, or for near surface, by the conditions of rolling contact demonstrated, 46 prop. *b*, and otherwise.

b. The immediate influence of resistance to aerial currents at the earth's surface may possibly be best observed by the directive impulse that the air gives to the drops of falling water in a shower. The drops themselves being drawn down directly by gravitation, the motive impulse of the wind enters into composition with this attraction. In this manner, supposing the air in currents moved with uniform velocity at a certain height and at the earth's surface; then, as the air increases in density, and consequently in momentum at equal velocity, the falling drop near the earth's surface would be accelerated by gravitation about in equal proportion to the increase of density or momentum of the air, so that its fall would be oblique in a nearly direct line. But if the earth's surface or obstructions thereon offer resistance to the current so as to form whirls, the paths of the drops become by this cause parabolical. The air nevertheless by its momentum could not deflect the drop-path to its own curvature, but only as the impulse of its momentum upon the surface of the drop.

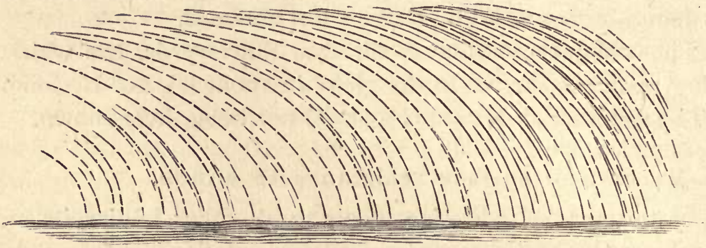


Fig. 180.—Ex.—Drifting Shower of Rain.

c. This parabolic form of motion I have frequently observed in snow, which is frequently laid quietly on the ground when there is considerable wind force above. In this case every snow-flake gyrates as it falls, so that the motion can only be considered as an equation of all the separate gyrating motions.

d. The same path of descent may be observed directly in rainfall. This I witnessed strikingly while standing for shelter in Cannon Street, London, the rainfall in this particular case was a heavy shower in a strong wind, and the path of descent could be well observed in the shade of the side of St. Paul's Cathedral. The average angular direction to the horizon that the rain drops were drifting, as seen in the shade on the side of the dome, was as nearly as I could measure 30 degrees, this angle increased constantly until drops were finally deposited at an average angle of 60 degrees, the distance traversed in falling being about 200 feet. The dome itself also considerably influenced the drifting storm on the side most direct to the wind. The rain and wind drifted also up the dome, moving evidently by rolling contact; this could be clearly made out by observation.

e. It is possibly to this cause we owe the greater amount of rainfall, that has been observed to take place, at a position a certain height from the ground than at a few feet higher. This Sir John Herschel mentions in his "Meteorology"[1]—that Dr. Heberden found in twelve months from July 7, 1766 to July 7, 1767 the amount of rainfall at the top of Westminster Abbey to be only 12·099 inches, while at the top of a house close by, much inferior in altitude, it was 18·139, and on the ground 22·608 inches. Thus also Mr. Phillips found the fall of rain at York for twelve months in the year 1833–4, at the height of 213 feet from the ground, to be 14·963 inches, at 44 feet 19·853 inches, and on the ground 25·706. Similar phenomena being observed at Paris and elsewhere. There is no information as to position of lower catchwaters whether exposed to north or south. Sir John Herschel says, "The effect cannot be due to *obliquity* of fall at higher than at lower level since the same quantity of rain must fall on the same horizontal surface after changing its obliquity as before." I am unable to see the force of this argument generally, but understand it to be perfectly true for an infinite area of rainfall; but rain falls for the most part locally in very irregular quantities and varying intensity, and by the general principles I have demonstrated, will be deposited most where there is space for incurvature of its path to deflect it downwards. This may be very clearly observable frequently with snow, which we may take for argument sake to be visible rain, although of less density in proportion to surface exposed. Snow may be observed in windy weather upon hilly lands drifting along so nearly horizontally that the land surface is left quite bare, whereas in shady places, or where there is greater space to direct whirls downwards, it deposits continuously. I anticipate also, that the greater deposit in the extreme cases given by Herschel, there would in all instances be found some obstruction to the direct action of the wind to cause local whirls. Further, the same observation of the horizontal direction of snow just given may be witnessed in the case of rain if we are placed in a lofty exposed situation, as may be seen by the horizontal position that we are compelled to carry the handle of an umbrella to protect ourselves.

f. With a perfectly direct rain, falling by gravitation without wind,

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[1] 8th Edition, *Encyclopedia Britannica*, article "Meteorology," § 109.

I anticipate there will be little difference in the amount that falls on a horizontal surface, unless there is much greater difference of altitude than in the instances given by Herschel. On mountainous or hilly surfaces, which possess northern radiational fronts by the possible extent of downward deflected whirls, an overflowing saturated current will deposit its moisture much more readily here than on a level plain, as the extent of resistance would destroy the motive energy of direct horizontal projection, and the cool surface would as constantly condense overflowing vapour.

g. We have frequent evidence of vertical surface whirls in hot summer weather, when this is accompanied by winds which blow at the earth's surface from some northerly point. In such cases, clouds if visible above, move from a southerly point, the aerial whirls, of which they form a part, being deflected to the earth in a more northern latitude. The northerly wind we experience is the reflex side of the whirl or counter current. In such cases of very hot northerly wind, storms commonly follow to the northward.

h. Similar conditions also occur in mountainous countries from dry polar winds, which by whirl deflections overflow in such a manner that a cold *southerly* wind is experienced upon the earth's surface in low positions for a certain time. When the wind is sufficiently constant and regular whirls of smaller amplitude are produced, then we have the normal conditions of southerly warm winds and northerly cold ones in this hemisphere; the wind nevertheless in such cases follows a parabolic path downwards as regards its vertical components of direction, but the radius of curvature is smaller.

162. Local vertical aerial whirl systems.

a. The diurnal force of the sun's heat upon small areas of land surface would by expansion of air and vapour produce slightly lateral, but principally upward projection of the air. If this effect be produced over an island, the sun's rays cause more rapid expansion over land than over the surrounding water, and at first, an outward land breeze from the shore, but where the island is large this difference of resistance is not material, and the upward current only or principally is engendered by the expansion. In any case the upward current produces by its tangential force active upon lateral aerial mass an overflowing whirl, which must return to complete its curvature over the contiguous oceanic area as a sea breeze;

taking possibly, if the land is unclouded, an approximately equal time for equal space projection in the upward side of the whirl that it takes in the downward side over the oceanic area.

b. I anticipate that there is a general local whirl system over every land surface exposed to the sun's direct rays, and that the projectile force is greatest over portions of land which present the least declination to the sun. The deposition of dew, so satisfactorily accounted for by Dr. Wells, I think might possibly, in certain cases, as regards its amount, be due to the reflex action of vertical whirls bringing air of minus temperature charged with vapour by return whirl currents, at sunset; particularly from the sea. We may further observe that as it is the property of water by convection currents, active under differences of temperature through surface radiation, to bring about a constant change of its parts, so that the warmer or lighter parts are constantly brought near the surface; it thus follows that water by radiation will be constantly giving out heat to colder air, should this exist above. Therefore in winter the positions over the sea and near the coasts are warmer than inland. By the same conditions, as just discussed for rising currents over land, it follows that at certain times air resting upon warmer open water will, by the heat it acquires from the water, form rising currents, which engender by their tangential action whirl projection upon the more static air overland, and thereby produce overland winds and rain or snow; unless such whirls are otherwise deflected by forces present, as by the influence of prevailing winds along the coast.

163. Cyclones.

a. I assume all atmospheric motions to be *cyclonic* by principles of rolling contact and conic resistance, fully discussed. Cyclones being engendered by every plus or minus pressure in parts of the atmosphere from any cause. The most general source of the larger cyclonic motions of the atmosphere is most probably derived from local condensation of vapour into cloud or rain. As we find vapour condensations from contact of cool currents or from surcharged electricity of one sign are *sudden* from the point of saturated air to cloud; in like manner, from the same cause, cyclonic propulsions are generally *impulsive*, and resemble exactly in principle whirl projections of the class described, 67 prop. *e.*

b. In tropical latitudes, the vapour and atmospheric projections acting obliquely in an upward direction to the westward, as before stated, cause by a sudden condensation of vapour to cloud, a greater upward projection of the more easterly vapour area towards a more westerly condensing one, and the concussion of meeting engenders electrical discharges with immediate downfall of rain.

c. In extropical or temperate latitudes the normal conditions of overflowing vapour forces will be to direct currents downward to the surface of the globe (154 art.); these projections being thrown upon a plane of resistance take *biwhirl forms.* In this case the equatorial side of the biwhirl is carried towards areas of *greater revolution velocity;* and from its inclined direction being *eastwardly,* the equatorial side of the biwhirl may be *suppressed,* from loss of excess of easterly momentum over the normal. On the other hand the exterior, or polar side of the biwhirl now represented by one whirl only, being deflected by whirl projection to latitudes of less surface revolution velocity, thereby *gains easterly momentum* so that its projectile force is increased. From this cause generally winds in cyclonic motion rotate from right to left in the northern hemisphere, and from left to right in the southern.

d. In cyclonic action there is a general tendency of the cyclone to move towards the polar area by its deflected currents impinging upon cooler air and suffering thereby greater condensation upon the cool side which produces at once a minus pressure in this direction. By this cause also the entire cyclone will generally continue projectile, since in proportion as it moves towards the polar regions it constantly gains plus easterly momentum, which in composition gives the cyclone a north-easterly projection in the northern hemisphere, and south-easterly in the southern hemisphere exterior to the tropics.

e. For the non-periodic or general irregularity in the occurrence of cyclones, we may conceive that if there were no obliquity of axis of the earth the whirl forces would be due purely to the diurnal action, and would from many causes be less intense. This is particularly seen, in that, the changes of season by displacement of the seasonal thermal equator, displace the local systems laterally with diurnal disturbances according to the local frictions and resistances on the land and sea surfaces; the local friction whirls being either drawn forward or pushed backward towards the poles with varying effects according to local resistances. If the obliquity of the earth's axis to the sun were constant the same exact *local* whirl systems would be formed, which would remain fixed and constant, the vapour these whirls carry being uniformly condensed at a definite area of deflection, where indraught contact would be established with cooler circulating currents.

164. Hurricanes.

a. Hurricanes occur where there is the greatest extent of equatorial oceanic projection and the greatest amount of resistance from western oceanic boundary. These conditions are met in the North Atlantic, Indian, and North Pacific Oceans. Similar conditions are also met with in the Southern Pacific as regards oceanic projection, but the resistance of land being imperfect to the south of the Eastern Archipelago, from New Guinea to Australia; from the influence of Torres Strait, the southern equatorial projection thus loses a part of its impulse and adds this part to the Indian oceanic system, so that we have here no considerable hurricanes. In the Southern Atlantic the equatorial currents are deflected by open oceanic areas towards the northern system (126 art.), and thereby lose a part of the southern whirl deflection, such as would naturally be produced by a normal resistance to the impulse of the westerly equatorial current. Therefore a part only of the southern equatorial aerial projection is thrown overland in moderate cyclonic action to greater distance into the valley of the Amazon, so that this can scarcely be pointed to as a region of hurricanes.

b. For more exact conditions it will be convenient to take one area of hurricane projection in consideration. The area of which we possibly know most is over the North Atlantic, where we have the record of so many hurricanes that we may fairly depict within wide limits the average hurricane path.

c. If we accept the general principles I propose of one great ellipsoidal whirl system being active partly over the North Atlantic area, so that the ellipsoidal aerial circulation has a focus approximately in about lat. 33° N. and long. 50° W., and that a *complementary* cool whirl meets this, thrown off the Pacific system about the Caribbean Sea; the meeting-place becomes at the time of condensation of the western equatorial projection upon the cool whirl, one of minus pressure. This condensation by contraction of mass increases the impulse of the westerly equatorial aerial drift, and as by general cohesion the whirl system continues its impulses of induced velocity under deflection into the ellipsoidal path, it carries its impulses onward into the Atlantic Ocean; throwing off certain elements of its force tangentially at the region of greatest curvature, that is, into the interior of North America.

d. If we now consider the conditions of overflowing forces only across the North Atlantic area in about 30° to 40° N. lat. we have here, by my theory, the greatest activity of overflowing vapour forces, which are active in completing the overflowing equatorial whirl (154 art.). Therefore along this parallel, assuming sudden condensation of vapour into cloud or rain to occur, we have at the times of such condensations a local minus pressure, the effect of which will be again, as by causes proposed in the last paragraph, a means by which the impulses of the ellipsoidal whirl will be greatly accelerated in that part of its path which is towards the western deflection of the ellipsoidal system, or about the position of the West Indian Islands. If the condensations over the Caribbean Sea project the equatorial whirl with greater force at certain times (163 art. *b*), and the condensations of the overflowing whirl are active at the same periods, we have then, by the conditions by differences of local pressures, forces capable of producing a *hurricane.*

e. By such causes as the above the general ellipsoidal currents will be more or less accelerated around their focal points, and as we must conceive every flowing atmospheric current to make rolling contact upon contiguous quiescent or differently directed parts; the whirls or cyclones formed by this means will have dimensions and velocities proportional to their impulses and the local freedom from surface resistances in all cases.

165. Anticyclones.

a. In the case of any aerial projection meeting an area of resistance, a cone of impression will be formed about this area, which will be an *anticyclone,* as it is termed. This may be represented more exactly as a *pressure* possessing generally no cyclonic character about it whatever (70 prop.).

b. In cases of continuity of aerial projection a current will be motive in space irrespective of the conditions given, 154 art., as we may imagine the cone of impression to be constantly wasted away by the continuity of projection upon it (73 prop.).

c. It is quite possible also for a whirl to move in opposition to the direction of projection, which will occur when the cone of impression (anticyclone) is derived from a narrow motive current which, having force to deflect the whirls constantly backward upon the projection, will in this case resemble the projection of a solid, as of a ship in water.

d. The cone of impression or anticyclonic area will occur generally over land where the earth's friction offers the greatest resistance to the direct force of the flowing current. Where currents are constant the anticyclonic area might be frequently clearly mapped for position upon the earth just in the same manner as whirl systems can be mapped for oceanic currents; always remembering that a cone of impression in a fluid is formed at a distance in front of the resistance. In cases where air and water are flowing in one uniform course upon the oceanic surface, as before pointed out, there would be less resistance than in the case of the air moving upon static land surface; further, the resistance would be greater, as before stated, in high lands and mountainous countries, which by their elevation would support the cones of impression to wind impulses more rigidly than upon level plains or water surfaces. We also find that regions of large radiational surface, where the air is cold and therefore condensed, form resistant areas to lighter airs, which have their densities further increased by the impingement of flowing force upon them, although the flowing force would be mainly deflected by whirl motions into areas of less resistance.

e. By the above conditions certain regions of the earth constantly receive the impulses of aerial projection, particularly where such are derived from the carrying force or minus friction of oceanic currents. These are regions of high barometric pressure under normal conditions or permanent anticyclonic areas (cones of impression).

f. In the anticyclonic area, as it is termed, the cone of impression engendered by any form of resistance is a condensation of the air in the condition that it happens to be at the time, such condensations by increasing the elastic force render any vapours present invisible, and produce fine weather over the district affected. It will be seen that the above propositions reverse the generally accepted doctrine—that the cyclonic area is one of ascent, and the anticyclonic one of descent. Upon the principle of whirl motions they would be the opposite of this, and I assume this to be the case so far as horizontal motions of a cyclonic character are concerned. It is nevertheless quite clear that a descending current would produce a pressure if it impinge on an area of resistance, as any part of the earth's surface, or if it act in such a manner that by its direct impulse it forms a cone of resistance. Such special conditions I have discussed already for vertical motions (154 art.).

166. Cyclonic motion in the air productive of rain.

a. It is generally assumed that there are rising currents in the centres of cyclones; but if we may take evidence from the whirl systems of visible gravitating matter, as of liquids in whirlpools, it will be exactly the reverse of this; the tangential exterior forces of the circumscribing current, by rotation alone, removes the lower central fluid by the tangential action, and causes thereby minus pressure in the whirl centre. It appears to me probable, however, that there is in all cyclones a vertical downward inflow of the more rarefied air from above which partly equalizes that removed by tangential force near the terrestrial surface.

b. Assuming the cyclonic centre, as here proposed, to be one of minus pressure from the tangential velocities induced, by a contiguous current producing exterior plus pressure in whirl motions; then the effect of such tangential force, if it act as proposed, that is, in the manner visible in aqueous systems in whirlpools, it will exhaust the air from such centres by tangential action. Now the result of this upon an aerial or vaporous mass of equal elasticity, would be to decrease the central elasticity; therefore, for any aqueous vapour that might be present there would be an immediate tendency to condensation. This would occur by the difference of vapour tension under reduced pressure, according to Dalton's law. The condensations may also continue from the descent of overflowing cloudy cool air within the cyclonic area, so that the rainfall within a cyclone may be much greater than that produced by the condensation of the air at first within the original cyclonic whirl.

167. Cyclonic action may produce water-spouts in certain cases.

If a cyclonic area occur where there are clouds above, the clouds being heavier matter, will be drawn down into it, by removal of elastic resistance. If the cyclone be in rapid rotation near its centre, which it may be, by continuity of inward deflections of tangential forces (whirl motions), this will remove the central resistance except at quite near the surface of the earth or sea; and in this case the rainfall may be very rapid within such whirl. Further, the intensity of tangential forces will remove the plus elasticity engendered by the latent heat of condensation of vapour which will again make such condensation much more rapid, and under certain conditions, by direction of vapour to the area of condensation, might make the rainfall enormous near the axis of revolution. This would be a probable cause of a rapid downflow in a small visible cyclonic area, but whether answering to a water-spout I do not know, although it appears to coincide with certain descriptions that I have read, but have never actually seen. There is also a further condition in small intense cyclones, which answers to the general descriptions of water-spouts, that is, that the centre of the cyclonic area is by the action of tangential forces an area of less atmospheric pressure, and the cyclonic circumferential area one of plus pressure. There would therefore be over any surface of water on which such a cyclone rests, an inscribed circular area where the water would be elevated in the centre by the minus pressure, and this may even, near the axis of greatest intensity, elevate the central waters into a conical form until it meets the axis of the cloud system above, which by the tangential forces acting on the surface waters, causes it to form a part of the system. This would be exactly the reverse, but upon the same principles of tangential rotary force as we find active in the formation of a *whirlpool* which produces a perfect pointing of the superficial air downwards in the rotating water where the whirl forces are active. It is most probable in the phenomena of water-spouts that the whirl has a downward direction through condensation as well as a horizontal whirling motion which causes a circumferential pressure over the area on which it rests greater than the difference of atmospheric weight as it would be caused by a purely horizontal motion. It is possible certain forms of water-spout are produced by electrical conditions which I cannot now discuss. Dust whirls appear to be produced by the same conditions as just discussed for the elevation of water in water-spouts.

168. Causes of local changes of season—climate.

a. Continuous observation for years has proved that we may fairly plot upon a globe mean local annual summer and winter temperatures and pressures, which will remain approximately constant in certain positions. The same will apply to rainfall areas, so that local changes of season which appear to us to take considerable ranges of differences, are really active within very definite limits of change for most districts. It is very clear that the uniformity of such local conditions as we observe, depends upon the nearly equal intensity of the sun's heat under an equal obliquity of the earth's axis, the equality of revolution velocity of the globe, and the permanency of positions of the areas of land, water, and ice; so that we can imagine, if these forces and conditions were quite constant, we should have exactly recurring circumstances at the same annual periods. There must be, therefore, such changes from average annual conditions as to produce the partial differences we observe, and these changes most probably follow definite laws, which we may utilize for prognostication when we are so successful as to discover them.

b. From observations of many philosophers there appear to be differences of the sun's annual heating influences ranging in periods of about eleven years, depending, as is reasonably inferred, possibly upon the amount of spots upon the sun's disc. A periodic difference of emission of heat from the sun may be otherwise possibly due in some way, as suggested by Sir John Herschel,[1] to the influence of the large planet Jupiter, whose revolution is in 11·9 years. In our ignorance of the causes of gravitation it may probably occur that gravitation force between the sun and planet excites the heating influences of the sun, or the reverse of this. For our present purpose, if there be the differences assumed, in the annual heating power of the sun's rays, there will be differences of vapour impulsion into circumpolar areas, differences of impulse in oceanic currents, and in the amount and position of ice that remains in polar regions, and from these effects combined, differences in the direction and force of aerial currents which are active in producing local climatic variations.

c. Besides the differences of short period, we have the certain influences of the astronomical action of gravitating matter; that produces cosmical effects which we may conceive to be, in relation to short periods, of constant regularity. Perhaps the most important of these effects, in producing changes:—are variation due to the eccentricity of the earth's orbit; precession of the equinoxes, and differences of obliquity of the ecliptic.

d. The present *ellipticity of orbit* throws the earth three millions of miles nearer the sun in our winter than in summer. This ellipticity varies within fixed limits, but is at present growing slowly less, so that the earth's orbit will become nearly circular in about 23,900 years. The greatest difference, according to Lagrange, may

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[1] "Meteorology," art. 7.

be fourteen millions of miles; the least, half a million. The influence of such changes of ellipticity is found in that the sun's rays penetrate the atmosphere with greater force at one period of the year than at another. Perhaps the most important effect produced by variation of ellipticity is seen in that as we approach nearer the sun in winter, that this nearness aids in maintaining northern oceanic areas open to the drift of currents at this season. These differences must also be important, as pointed out by Mr. Croll, in their influence upon the amount and extent of snow that may be deposited or may remain in polar regions. This deposition being even possibly so great at certain periods as to lower the oceanic surface, and affect by difference of gravity in polar regions the general distribution of fluids upon the surface of the globe.

e. By the precession of the equinoxes and a general change in the direction of the major axis of the earth's orbit, our present relative winter position of being nearer the sun than the southern hemisphere will in about 10,400 years be changed to the opposite condition, and the southern hemisphere will take our equivalent position. Under this change, by the same causes as discussed above, § *c*, the northern will become the colder hemisphere in winter, and the directions of currents will be naturally changed by the advance of arctic ice.

f. The *obliquity of the ecliptic* which produces our seasons is at present about 23° 27′. This obliquity diminishes on an average not quite half a second a year. Such change in a short period would evidently produce only small results, but as the force of projection of thermal atmospheric currents into polar areas so materially depends upon the amount of obliquity, such small changes cannot be without influence.

g. Supplementary to the above conditions, we have *nutations,* which render annual conditions irregular. The whole of the above changes tending at the present time to make the northern hemisphere colder in winter; although these differences may be conceived to vary the amount of thermal force in either hemisphere slightly only in short periods. Nevertheless, with regard to minor local climatic changes, the course of projection of an equatorial current may be materially influenced by very small differences in original direction and by the positions of the local resistances which it afterwards encounters by which it may be deflected. The directions and deflections of equatorial currents will again rule the position of friction-whirls necessarily formed contiguous to them, by which climatic conditions are much influenced. Changes of direction being most particularly active in the local complication of whirl systems where the general equilibrium is most sensitive; as, for instance, in certain districts of the United States of America and of China (*a a′*, Fig. 168). These changes again extend their disturbing influences deep into the current systems normally established as in those projected to the British Isles.

169. Established aerial currents produced by astronomical causes have a constant tendency to modify the configuration of the surface of the globe to the conditions present.

a. The general causes for conformation of motive oceanic forces to astronomical conditions were pointed out, in 125 to 131 articles; the oceanic conditions being also assumed to most materially influence the aerial. There must also necessarily be a certain influence upon the position of terrestrial matter due to the action of winds only.

b. By the action of the sun's direct rays, and by frost and rain, the most solid rocks of the earth become disentegrated, and by the action of the winds the disintegrated parts in dry weather are transplanted. Therefore, where the wind is constant in one direction, such loose particles are drifted by successive movements to localities forward of the wind's influence. This may be very commonly observed in short periods in the elevation of sand dunes, as at Yarmouth on the eastern coast of England. But it is more particularly seen as a geological feature in the general lower altitudes of districts traversed by drifts under the influence of constant or frequent winds, and in the proportionally higher altitudes of land forward of the drifts which generally finally produce deflection of the wind into whirls. This we find as already pointed out in the great ellipsoidal systems (140 art.) and in districts where systems of indraught pass through lower lands (146 art.), in these lands taking a certain conformity consistent with the actual conditions; so far as these may rule in composition with volcanic forces present and past, which I have not considered.

SECTION III.

AQUEOUS SURFACE WAVES.

CHAPTER XV.

FORCES ACTIVE UPON LIQUID SURFACES WHICH ENGENDER WAVES. PROTUBERANT, COMPRESSILE OR LOOPED SYSTEMS. CONCHOIDAL FORMS. VELOCITY OF PROPAGATION. TANGENTIAL ACTION. MOTION FROM ROLLING CONTACT OF MOVING AIR. WHIRL IMPRESSIONS. DIVISION IN SEPARATE UNITS BY OVERFLOW. DIVISION BY CORRUGATION.

170. Conditions under which waves are formed.

a. A liquid surface, by the perfect mobility of its parts moving sensitively to the constant equal action of gravitation upon it, is, when at rest, in a state of perfect equilibrium. In this state any part of the liquid surface may be represented as a body resting upon a perfectly level plane of repose upon which it is free to move, when impressed by any directive force whatever, capable of overcoming its mass inertia, and the resistance of cohesion to other parts of the liquid system.

b. Now, as we find a liquid surface in the state of equilibrium of repose, is of less superficial area than the same surface would be if under any possible state of disturbance, we may conclude that *any exterior action that disturbs this surface will produce greater surface curvature upon the whole or in parts, and that this curvature will necessarily form either protuberances or hollows, or a combination of both.* Such protuberances and hollows as may be formed, meet conveniently the general denomination of *waves.* Therefore a wave *is caused by any possible motion that disturbs a liquid surface* from a state of rest. Taken in this manner, we may conclude that a wave is the *resultant* of one form of motion or another which produces surface disturbance. In open water, as in the ocean, it is generally a resultant of all component forces that may be active upon the liquid surface. Waves will therefore have as many *modes of generation, as there are means of disturbing the liquid surface.* This fact becomes important in the investigation of the subject before us, as it is by no means necessary that free fluid matter should be affected by one system or direction of forces only to produce waves, but rather that waves are the necessary consequences of *any* possible disturbing influence. Neither is it necessary that the wave should be of one form or dimensions. One constant condition, however, is evident, namely, that gravitation enters into composition and acts uniformly through all other disturbing influences, and in this tends to induce a uniform system of motion, which may disturb the gravitation plane as little as possible.

c. The possibility of propagation of forces in fluids to a distance, is evident both in the motion of a solitary wave referred to, 34 prop. *d,* and in the motion of sound through air and water. A general mode of propagation of forces to a distance in fluids, is probably by impulses from molecule to molecule. The principle of which is made evident experimentally in the propagation of an impulse through a long straight series of hard balls of equal size suspended in contact with each other and resting in equilibrium. In which experiment, if we strike one of the balls at the end of the series by a like ball, one ball from the other end of the series will be projected with nearly equal force, the whole series of balls otherwise remaining apparently motionless. If two balls strike the end of the series at one time, two balls will be projected from the other end in like manner. If three balls strike, then, three will be projected; so that by this we infer that an impulse may be transferred through matter in equilibrium, retaining both its force and character by sequential impulses upon the intervening parts.

d. The density of water, evident by its resistance to deflection under compression, gives inference that any amount of impulse possible to be applied to an open free surface, would not decrease its bulk to a measurable extent; therefore forces impressed that disturb the general equilibrium will be propagated for a certain distance until the disturbed parts can find positions in gravitation equilibrium, without excess of impression by weight upon contiguous parts in the same gravitation plane. We may also assume that any force impressed normally to the surface will be resisted instantly by the inertia of the impressed parts.

e. A free liquid being a perfectly mobile system of matter, and the air resting above it being also in a free state and of great elastic flexibility, possessing also a certain function of adhesion to the liquid, it is presumable that any force impressed by adhesive moving air above sufficient to overcome the resistance of the system of the liquid at its surface, would move a portion of this liquid forward upon or within that in front, and as the liquid is assumed incompressible, this motion would necessarily produce a protuberance upwards into the air. The air always offering a plane of less resistance than in any other possible direction for the displacement of the liquid.

f. Now, taking the ordinary conditions that surround us, we observe that a surface of water may exist as a level plane when it is undisturbed; or it may have protuberances or waves engendered upon it, as, for instance, the ocean may be calm or rough. It is also quite clear to us that certain exterior forces, as the wind when active upon the surface of the water *do produce the roughness*, or the motions we term waves. We also conclude that waves are not produced upon the ocean unless such exterior forces are active, excepting ripples derived from currents. It therefore becomes reasonable upon the above premises, which is all important for our investigations, to assume:—*That any force whatever that is capable of producing the wave will be certainly capable of maintaining its existence*, so that the principles of the genesis of the wave is the demonstration of the conditions of its continuity; both phenomena being clearly derived from the same set of motive causes. Further, we may assume that if the wave-producing force, whether wind or other agency, should cease to act, that there would be still constantly present the active force of gravitation, which would cause the surface to return to equilibrium in such space of time that the potential of the elevated wave could dissipate its force, by composition of its initial momentum derived from the disturbing force, with that of gravitation, and of resistance by molecular friction, ever present.

g. In the above conception of the subject we meet with two important conditions that may be conveniently and separately considered:—Firstly, the elevation and maintenance of waves by the impression of exterior forces; and secondly, the dissipation of waves, or restoration to equilibrium by the evident action of gravitation; and as we may assume that all wave motions are caused by exterior forces, active in the first instance upon a surface plane of repose, we may further observe that the elevation of the wave from any force whatever, applied upon or within the surface of the liquid, must primarily possess a mode of action *contra* to the force of gravitation. The active conditions being generally, that if the volume of water remain the same, a portion of the water *below* the plane of gravitation equilibrium in one position is elevated *above* this plane in another. If the entire surface of a liquid were equally impressed by a force, this surface would equally resist in all its parts, and no waves would be produced. It is therefore necessary, in every system of wave motion, that there should be *local impression of force.*

h. The most natural force-system that we may consider is that of air or wind acting upon water. The wind, by the irregularities of land and oceanic surface, as also by the necessities of its rolling contact in motions upon these irregularities, becomes itself very intermittent in the impression of its direct momentum upon any object upon which it impinges. This we not only feel in the intermittent gusts of wind upon our bodies, but we may observe it in every mobile body under its influence, whether it be wind passing over a field of corn, the fluttering of a leaf, or the oscillation of a blade of grass. When the wind blows powerfully our ears are also sensitive to the note produced, and we know that this is caused by the intermittent intensity of motion, as our ear is not sensitive to uniform pressures or impressions as sound. This evident intermission or division into separate impulses in the action of the wind upon water, becomes important in experimental investigation, inasmuch as it leaves us the option of observing motions produced by *separate* impulses, whereby such isolation of a part of a general system of motion is far more easy to be grasped by the mind than that of a complex system, such as a wavy surface of water moved by the uncertain constant action of the wind.

i. As the action of gravitation upon a liquid undisturbed by other forces, produces a perfect plane in equilibrium at its surface with the intensity of gravitation on its parts; the differences of radial intensity of gravitation being very small for small distances of elevation, we must conclude that a liquid must be almost infinitely mobile to establish a surface of such perfect equilibrium as we witness on a calm surface of water (15 art.). Where perfect equilibrium is established by the extreme mobility of the system, we find that any disturbance possible to be produced upon the surface will throw more or less liquid on certain local positions, and we can again imagine that this excess or deficiency, separately considered, will find no position of equilibrium until it is dissipated or diffused throughout the surface of the entire liquid system of which it forms a part. Therefore we may conclude that any increase or decrease of mass elevated or depressed locally would be as a free body of liquid urged constantly by gravitation over the surface until it finally found rest by distribution to equilibrium.

j. Under the above conditions we may imagine that any force of displacement in water, as for instance, that caused by the launch of a ship, would by the local elevation, that the pressure of the intrusion of the hull of the ship occasions, throw the whole aqueous system of the ocean out of equilibrium. Theoretically, this would be so, if the system of the ocean were in perfect equilibrium to gravitation so as not to be affected by intermittent or other opposing forces. But at the same time we must conclude that the pressure by increase of bulk of liquid that the ship would throw above gravitation equilibrium, would only represent a certain force that would be active upon the inertia of the mass under equilibrium of gravitation already established, that is, active upon *another force.* Therefore the pressure caused by increase of bulk of the ship would combine with the force of inertia of the system under equilibrium of gravitation by general laws of resolution of forces, that would cause a certain displacement of a *part of the liquid system*, which would afterwards move to equilibrium in equation with the motive values and directive influences of all forces present.

k. We find when we further analyse the above, that the mass of water elevated by a force, as for instance by the intrusion of the hull of a ship, would have by its horizontal projection lineal with the surface of the water a certain momentum, which to diffuse itself must move the projected water against the inertia of that resting quiescently near the surface of the area affected; therefore by the equality of inertia volume for volume of the projected and resistant water the projectile impulse would soon come practically to equilibrium with the resistance; but as we should still have by *intrusion of the ship*, a large portion of the elevated water representing a pressure by its potential force, left above the surface of equilibrium upon which gravity would remain active as at first, the further movement of the elevated water, caused by the displacement, must be practically by the force of *gravitation alone* upon it. The original horizontal momentum of the displaced water having been nearly exhausted by the resistance of the inertia of equal matter within a small radius, leaving only a small residuary fraction of projectile impulse.

l. Upon the above conditions we may also imagine that if upon an aqueous surface there was a depression from any cause, as by the withdrawal of the ship, assumed previously to be launched, which depression would instantly produce a hollow, that gravitation would immediately act to restore the equilibrium of pressure by the more elevated parts of the system moving towards the hollow. But in this case, from the action of the surrounding pressures from all directions, there would be complicated directive influences, the general equilibrium being much disturbed locally and the phenomena more complicated, as I will hereafter show.

m. In the important original researches of Mr. J. Scott Russell, in the British Association Reports, 1844, the reading of which induced me to take the illustration above of the launch of a ship, he shows, experimentally, the effects caused by increase or decrease of bulk upon a liquid in equilibrium, by two methods. One of these for the increase of bulk by the intrusion of a body of water or a solid; and another by the abstraction of a like mass. In these experiments Mr. Russell shows that under certain conditions a single wave may be propagated in both cases which will move continuously over the liquid surface, with a velocity in equation with a function of gravitation upon it; which is found to be under the conditions taken, equivalent to that of a free body falling through a height equal to half the depth of the water upon which the wave is propagated. The wave when formed moves forward until gravitation equilibrium is restored. I need not now follow the particulars of these important experiments, as I shall return to them further on. There are, however, certain theoretical principles in these motions which I hope to make clear by propositions and experimental demonstrations. 1. That the increase of bulk locally upon a liquid in equilibrium from any cause will for the instant produce a protuberance upon the surface, the liquid being structurally too solid to deflect in order to admit the extra bulk, and that this protuberance will act as a *compression* or hydrostatic pressure upon the surface of the liquid system until it is distributed by a motive wave or waves to gravitation equilibrium. 2. That decrease of local bulk from any cause will produce a *hollow* which will in like manner be surrounded by hydrostatic pressures, so that the pressures will constantly urge the liquid to fall into the hollow by *Infractions* with a certain motive velocity that will not come to rest until the composition due to the constant forces of gravitation, general momentum, and molecular friction causes it to do so. This principle of motion was first pointed out by Mr. Flaubergues in a paper in the *Journal des Sçavans*, 1789, to which I will refer hereafter for the discussion of the principles of central disturbances.

n. It will be convenient now to follow the conditions of the production and maintenance of waves by protuberances only, and to discuss the mechanical principles by which forces of *compression* propagate themselves upon a liquid in equilibrium; some experimental demonstrations of which may be found in the important papers of Mr. J. Scott Russell. I will leave the conditions of abstraction of volume and of minus compressions in gravitation systems, active upon the surface of water, for consideration in the next chapter.

Deep waves formed by horizontal compression. Looped systems.

171. PROPOSITION: *That the effects of a compression upon the open surface of a liquid will be to move the compressed parts consecutively within the surface plane towards points of equilibrium with a velocity of motion proportional to the mobility of the liquid, and its freedom from the influence of the resistance of solid surface near or beneath it.*

a. The principles of the action of consecutive compressions in a liquid having a free surface may be possibly best considered by taking some of the conditions which may be made visible under certain conditions in solids. Thus, if we take for an instance, the application of a pressure to an elastic body, such as india-rubber, we may assume in this case that the body yields at first at the point of compression, but this point being thrown thereby out of equilibrium with all other parts, that it will in proportion to its elastic flexibility, have the forces of compression that it at first received distributed in all directions in its mass, and if the india-rubber be not fixed and the impressed force sufficient, the elasticity by reaction will move the whole mass forward from the area of compression to a more free area, and thus regain static equilibrium throughout its parts. So that we find the forces impressed in a material elastic system, that suffer restraint under compression, will react by movements towards areas of less restraint, seeking equilibrium as quickly as the inertia and resistance of the parts of the system permit.

b. The mode in which a compression is communicated to areas of less resistance may be taken in another case for a physically incompressible body, that is one practically incompressible in relation to the force used. To illustrate this we may place a long straight metal rod or wire of small diameter upon the level ground and apply a compression to one end of it. We then find we cannot push it bodily along by a longitudinal compression at the one end, as the resistance of the surface of the ground will offer too great friction for us to do so, and that the general inertia of the heavy material of the rod will cause it to bend into a loop rather than continue the impressed force in a direct line; that is, the impressed force will be found ready to wholly escape to any point of less resistance open to receive it.

c. As the above experiment would be somewhat difficult to follow to observe the exact conditions that I wish to define, we may simplify the matter by placing the metal rod or wire, before mentioned, firmly in a groove in a piece of wood open at the top as shown full size in the section below.

Fig. 181.—Ex.—Section of Wood with Inlaid Wire.

d. The length of the wood may, for this experiment, be eight or ten feet, and the rod or wire laid in the groove be of length sufficient to project, say a quarter of an inch from each end beyond the wood. In this case it will be seen that the wire is supported by the wood in three directions, namely, the two sides and bottom, and that each of these sides will offer resistance so that it will have one side only, the top, free. If we now apply a compression to one free end of the wire it will bulge out as observed at C, in Fig. 182 below, showing that the wire by the compression upon the resistance of the groove is thrown out of central equilibrium, and that the force of compression moves from the resistance to points of less restraint, which are in this case situated at the open side of the groove. In this direction, therefore, it exerts its force to equilibrate the compression that it receives moving by its flexibility into free space, in every possible direction open to receive it. It therefore *accepts*, if I may so express myself, the resistance of the groove consecutively from point to point. If we were to suspend a free body at the opposite end of the wire to that upon which a compression so resisted was made, it would not in this case move the body during the compression; thereby showing that the distant end receives no part of the force impressed. A mechanical means well adapted to demonstrate a compression, is by the apparatus technically known as a *snail*, shown at A, this being turned in the direction of the arrow, causes a compression continually on the end of the wire.

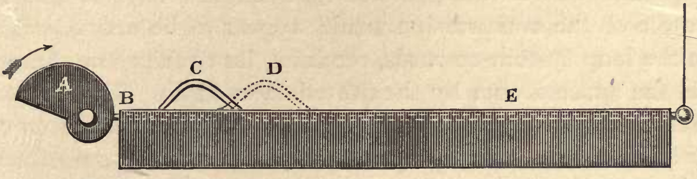

Fig. 182.—Ex.—Loop formed by Pressure at the end of a Wire in a Groove.

e. By the known properties of fluids we can imagine the resistance to compression as being at no point perfect unless the fluid is entirely surrounded by rigid matter. This is demonstrable from the equal mobility in all parts of the mass. Therefore the swelling of the wire about C, which remains in static equilibrium after we have given a certain space-compression at the end, could not remain in such a condition if the wire were a *soft* body or a liquid, as in this case, from the imperfect rigidity of such body, gravity would pull it down and flatten the loop out immediately after the compression had raised it. This would occur, for instance, if the soft body were of the flexibility we recognize in textile bodies, as in a rope or cord.

f. If we take the case of a soft body, as, for instance, a rope pressed as the wire was proposed to be in the last illustration, and assume the pressure at B to be a point of firm resistance; then, on the supposition that all parts of the rope are equally mobile, the pressure upon the loop C would be in all parts in equilibrium;

but as gravitation would now act upon all parts equally, the resistance would be greater at the point of rigid pressure B. Then the part of the system that would be *weakest*, and therefore *the first to give way*, would be that at the opposite point of contact to B, where the rope again meets the plane of repose. Therefore at this point the whole of the compression would appear to be active and thus move the loop system onwards, retaining its equilibrium afterwards within the space shown by the dotted lines at D. The resistance still remaining strongest towards B, it would move further onwards toward E with a velocity proportional to its mobility and freedom from resistance, until the loop would arrive at the free end F, where we could make the impression visible if desirable upon the suspended ball at that point. In the above we may notice that the raised loop is *always in equilibrium* throughout its system, and at both points of contact upon its plane, and that the compression which makes the loop progressive is simply derived from unequal local resistances which cause the loop to move from the most resistant part of the system to the least resistant; therefore we may conclude that any small directive impulse given to the loop from the point B will be sufficient to cause the loop or wave in equilibrium to move constantly forward by continuity of impulse, and this would also have been the case with the wire except that the immobility of its parts resisted the influence of gravitation; whereas a softer or more mobile body would have moved the loop forward under this influence. Hence we might conceive it a law that the propagation of a free compression is motive in direct proportion to the mobility or fluidity of the matter compressed.

g. The principles of this mode of propagation of free compressions

Fig. 183.—Ex.—Loop or Wave formed in Water by a Pressure at the end of a Parallel Channel.

may be better shown by taking a long rope and placing it in a straight position upon the ground. Then giving this rope a direct compression and impulse by a vertical wave of the hand to raise the near part against gravitation, to form a loop similar to the one produced in the wire, and afterwards by holding the hand rigid, this loop being in a soft body, the compression or wave formed will go continuously forward, and each part of the rope will rise in succession at a greater distance.

h. This motion of compression free in one direction may also be exemplified directly in liquids where greater mobility renders the motive system more perfect. Thus, if instead of the rope fixed in a groove we have a long trough, a part near one end of which is illustrated, Fig. 183, this being 20 or 30 feet long, by 10 inches wide and deep. The trough being placed quite level, half filled with water; having one end arranged by means of a movable board so that it can be pressed upon the column of water in the trough for a given distance as shown above, from A to B. Then by moving the loose board by the plunger C, this distance, the first effect of the pressure upon the water will be that the water will resist by the inertia of the mass in front and by the adhesion to three surfaces of the trough; it will therefore swell up, as the wire in the groove of the previous experiment, near the point of compression, and thus a loop or wave will be formed, which as a compression will afterwards go forward constantly towards the free end of the trough moving from the point of greater resistance, or compression, consecutively to each freer point in its immediate vicinity, with velocity of transference of impulse proportional to the fluidity of the water and inversely as the resistances of the surfaces of the trough in contact. The force of gravitation being equal upon all parts of the system, the wave retains only a small element of the directive force, this being active to constantly overthrow the equilibrium of the elevated mass in the direction of its original projection.

i. To save misapprehension, it should be understood that the impressed force of compression discussed above, must be applied to soft or fluid bodies only as *compression* in the manner indicated, or the equivalent of this; as it is also quite possible to send an impulse or wave along a pliant body, that shall not be a wave of *compression*, as here defined. This may be done by throwing a wave forward with great projectile force in a long pliant body of relatively small gravitating mass, wherein the *projectile impulse* alone may produce a wave in the pliant body, the wave being formed by the release of the consecutive parts. In this case the impulse or wave is *tensile*, to the parts of the pliant material where it consecutively acts, and such a wave may be produced in a chain with loose links laid horizontally along the ground. A wave may also be produced as a pendular motion of parts of a vertical or suspended system. These systems of wave-motions are merely mentioned to save misapprehension, they are forms of motion that have possibly no equivalence in liquid waves, or at least so, in the cases that I am now considering.

Motive equilibrium of a Protuberant Wave.

172. PROPOSITION: *That a protuberance or wave formed by compression of a liquid will have an initial energy of propagation if its gravitation figure is out of central equilibrium; the motive direction taken by such wave, being towards the side of the figure where the greatest mass has the greatest protuberance and is supported upon the smallest base.*

a. For liquid waves formed by compression, the influence of the compression may be entirely neglected after the wave is once formed, as its gravitation figure may then insure the continuity of its propagation upon a free surface. This is shown in the diagram below, which may be taken to represent either the outline of a loop in a shaken rope, or the contour of a surface wave upon water.

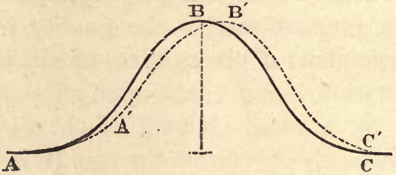

Fig. 184.—Diagram—Equilibrium of a Loop or Wave.

b. Let A B C represent a conchoidal wave equal or symmetrical on each side of B. This in a free liquid would subside on the spot under B by the equal pressure upon the surface at A and C. Let any impulse upon the mass A B C throw the wave represented out of equilibrium into the form of the dotted lines A′B′C′, by pressing over the most free part of the system, so as to cause it to bulge out as shown by the dotted line B′ to C′ upon the opposite side of the figure to that receiving the impulse. Now if all parts of the system are equally mobile a wave of this form in a channel of uniform depth will move onward by gravitation only, for the following reasons:—The particles in the line A to B′ being straightened out take a *more rigid form for resistance* by the solidity of the liquid; whereas gravitation will be more active at C′ than at A, and the *curvature of support* B′ to C′ being greater will be less rigid. Therefore for the stability of the loop or wave, the liquid at C will move forward by excess of compression through gravitation above it and greater curvature in front, and A will remain more rigid by less curvature, and fall by gravitation upon its own surface. As water is practically an incompressible fluid and therefore gravitation acts equally in all parts of it, the wave must go forward constantly to maintain the loop system, so constructed out of static equilibrium, and if such a form of mobile gravitating mass could be rendered frictionless upon its plane of motion, it would be possible, by the principle of continuity of momentum, as there is no part of the system antagonistic to the direction of the wave, that it would go forward eternally. But the molecular friction of any possible moving fluid matter will cause the wave or loop to gradually subside by constant loss of impulse upon the surfaces of resistance, present in any possible containing vessel, or medium.

c. The wave of the form here given being out of central equilibrium for its entire figure upon the water surface, will produce a slight depression upon the front of the wave, below the average surface; but as this hollow will be only a weight deflection of the rigid system of resistance of the liquid mass, it will not be great, or affect materially the general principle of equilibrium of projection, here proposed.

173. PROPOSITION: *The compression that forms a loop or wave within a horizontal mobile system in equilibrium acts as a motion of displacement of the compression in the mobile system of energy, in comparison with other loops or waves, in the lineal ratio of the circumference of the loop or wave to its chord.*

a. It will be observed in 170 art., in the case of a compressed wire in the groove, as also of a loop in a free rope, that for the wire to lie down quiescently in the groove or the rope upon the ground, after it is once compressed to form a loop, the point of compression being always supposed to remain rigid—that *space* for the loop in each case must be found *in front of it,* that is, on the side away from the impulse that causes the compression. So that for this lying down we require the possibility of a motion of displacement in the system equal to the lineal excess in the curvature of the loop over its direct chord. Therefore, upon this principle, under the constant normal action of gravitation, free mobile matter will possess energy to be permanently displaced by a compressible wave or loop in the lineal ratio of its circumference to its chord.

b. We may conceive that the same principles as the above may be practically applied to the compression of a liquid, for we may conceive a compressile wave as being formed by an infinite series of loops, one fitting within the other. In this case such loops would descend in altitude to the plane of repose, yet theoretically as each loop of the system possesses greater potential force in its centre by the extra thickness of matter necessary for its curvature in excess of any parallel loop system, *its force of displacement would be equal to that of a uniform system.*

c. The best experimental demonstrations of the principles here adduced, that I can offer, will be found in the beautiful experimental researches of Mr. J. Scott Russell, in the form of liquid motion, he has termed a wave of translation; described in the British Association Reports for 1844, p. 319. In this, the phenomenon of which I here only suggest the principles, is ably discussed in many experimental particulars, to these I must refer the reader. I may, however, mention that the looped compressile wave is so frictionless that the altitude of the wave is maintained with little decrement to great distances, and the velocity remains constant for uniform depth under the decreasing altitude from friction of the mobile system (see 34 prop. *d.*).

A Compressible Wave of conchoidal outline.

174. PROPOSITION: *That the contour of a surface under deflection will be conchoidal if produced by a compression lineal to a continuous homogeneous horizontal stratum of matter upon which there is active a transverse force, as that of gravitation.*

a. If we press a uniform straight free flexible system of homogeneous matter, of moderate length, at its ends, we cause it to bend, and the bend forms an arc of a circle. But if the flexible system is of great length, and is transversely held down equally in all its parts by a normal force, and the lineal pressure is in any central part only, a bend will be produced in the pressed central part of the system, which will be of conchoidal form.

b. This conchoidal form may be produced approximately in a simple manner by pressing a smooth parallel slip of writing paper near its two ends towards its central part; the paper being placed on a flat surface and the ends kept down with the hands.

Fig. 185.—Conchoid produced by a Slip of Paper.

c. Applying the above principle to pressure upon a liquid surface; supposing gravitation uniform on the plane A to C, at first level, and that the compression is made in the direction of the arrow at A which is resisted by the plane extending to and beyond C, the whole surface system having a certain flexible rigidity as proposed for aqueous surface by the presence of extensile forces, 16 prop., page 45; then, by the pressure, the loop B would rise upon the part of the system that was at the time from any cause least able to resist it. Assume this pressure upon a part of an infinite surface A to C, there would then be external flexure at B, and internal flexure at the points *d* and *e* in the figure, where the force first deflects the adhesive flexible system. Such compound flexure would produce a conchoidal outline.

d. Assuming a pressure as a continuous direct moving force upon a liquid plane, its motive elements of equilibrium may be taken in the following manner, as shown in the diagram, Fig. 186, below.

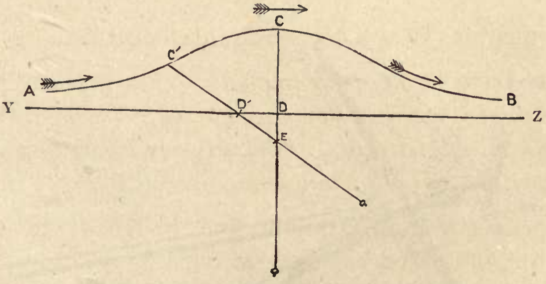

Fig. 186.—Diagram—Generation of a Conchoidal Wave.

Let the line Y Z represent a part of the section of a level liquid surface of infinite extent. Let a horizontal pressure beyond the figure to the left hand elevate the conchoid A C B above the plane of repose; and this pressure be such as will elevate the axis C D of the liquid conchoid for the distance C D, in the time that the pressure moves by cohesion the surface below Y Z upon an axis of inertia whose depth is equal to D E. Assume E the centre of rolling contact from this action. Then, supposing the system infinitely mobile, the potential represented by the gravitation forces in C D to the plane of repose Y D Z will be equal to the pressure at A which causes the elevation of the wave; and as all pressures by composition of horizontal impulse and gravitation will be directed towards E in the elevation of the wave, they may be represented at every place upon the plane of equilibrium Y Z by the space C D, in radial direction from the axis of rolling contact, E. Therefore, if the point E be taken as a fixed centre, and the part of the radius C D, as the edge of a free straight rod, be moved along the plane of repose represented by the line Y Z keeping the point D constantly on the line; the end of the rod C will describe a true conchoidal wave engendered by a horizontal pressure equal to the gravitation potential of C D to the plane of repose Y Z. This rod is shown in one position at C′ D′ E. It will be noticed that the rod slides upon the point E, therefore the radius D E may be assumed to represent in magnitude and direction the inertia-resistance to direct impulse, which constantly increases as the angle descends to the plane of surface equilibrium. The ratio of this increase is as the power of the horizontal force to deflect the equilibrium of surface into the free air above, contra to the cohesion and gravitation of the liquid system.

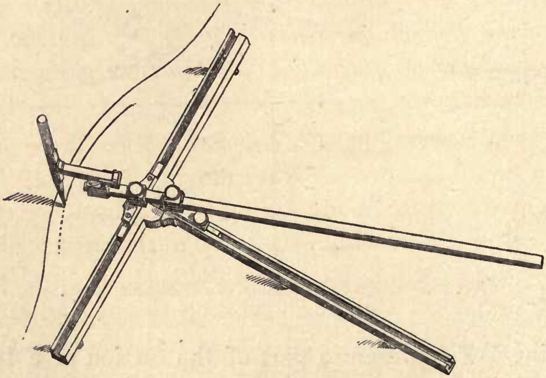

Fig. 187.—Instrument for describing a Conchoid.

e. I have constructed an instrument to describe the conchoidal wave; the idea of which I have taken from a description given of the conchoid of Nicomedes, described in *Nicholson's Five Orders of Architecture*, which is said to be the most simple means known of describing the contour of a Grecian column. This instrument is shown in perspective in the engraving, Fig. 187, describing an inner line. It was drawn for illustrating other matter.[1]

f. The conchoidal wave form may be described on a line with a straight slip of paper having the distance C to D, Fig. 186, marked near one end, keeping the straight edge of the paper E and the point D on the horizontal line Y, Z.

g. It may be seen that the proposition relates to a free wave only, so far as this is strictly compressile, the compression not being in excess of that capable of forming a *conchoidal system.* It will be seen further on that with greater compression the force of gravitation in higher waves acts otherwise upon the wave system, and permits certain parts of the compressed water to fall away from the most elevated part, as free matter until the conchoidal system alone remains active.

Velocity of propagation of protuberant waves in looped systems.

175. PROPOSITION: *That the velocity of a single free compressile wave moving in a horizontal uniform channel upon a surface of liquid at rest, will vary directly as the distance of its parts from the plane of support for the whole depth of the liquid; such parts being taken to be moving by rolling contact upon free axes or radii extending to the bottom of the channel; the velocities being in proportion to the force of gravitation acting tangentially to the radii the parts subtend to the axis of resistance.*

a. From the observations of Mr. J. Scott Russell we find that the velocity of a liquid compressile wave moving in a parallel horizontal channel is equal to a free body falling by gravitation half the depth of the liquid from the crown of the wave to the surface of resistance. By the principles of rolling contact of fluids (44 prop. *b*), the point of contact of a liquid on a plane in relation to any higher point, is as a radius of a free lever upon a fulcrum of resistance; so that tangential velocities are as the forces active on the free parts of the lever. The protuberance of a compressile wave as an infinitely mobile system in unstable equilibrium will be free to act by sequential

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[1] *Treatise on Drawing Instruments.*

force as shown, 171 prop. *b*, to the influence of gravitation. Upon these conditions the velocity of a free compressile wave of the whole depth of the channel, neglecting its conchoidal outline by taking a small section vertical to its crown only, may be resolved as follows:—

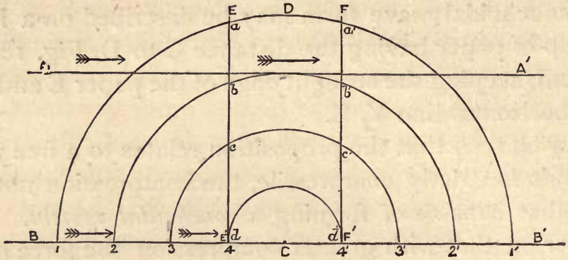


Fig. 188.—Diagram.—Equilibrium of a Liquid in a Parallel Channel moved by a Vertical Part.

Let A A′, Fig. 188, be the top of a liquid surface resting in perfect equilibrium on B B′, the bottom of a parallel channel.

Let C be any point of resistance vertically below the greatest protuberance of the compressile wave, whose summit is here shown at D.

Let the semicircular arcs 1 to 1′, 2 to 2′, 3 to 3′, and 4 to 4′ represent pressures in equilibrium, radial about the point C. The action of gravitation will then be vertically tangential to the termination of such arcs meeting at the points on the plane B B′, and with respect to the centre C, the compressile wave will be in static equilibrium.

b. If we now impress a force in the direction of the arrows as by movement of the vertical plane E E′, and suppose this by its tangential pressures to move the system to the point C, so as to cut off the gravitation arcs 1 to *a*, 2 to *b*, 3 to *c*, and that the tangential pressures *a* 1, *b* 2, *c* 3 are therefore cut off from the action of gravitation upon the system; then would the tangential pressures moving upon the fulcrum of resistance C be active to the influence of gravitation as radial velocities upon C, and the point D the crown of the compressile wave would be moved forward with a velocity equal to the action of gravitation on the tangential arc D to 1′, moving freely for this vertical depth.

c. Now let us suppose that the horizontal force impressed by the plane E E′ is equal to the impression of a force that would displace the segments from E E′ to F F′, this force being resisted by the plane

F F′. Then will the radial velocities of vertical parts moving upon the point C as a fulcrum be resisted with the greatest force at *a*′, as the excess of this radius over other vertical parts will the sooner meet the resistant plane F F′. At *b b*′ the radial velocity will be less than the above, but its radius of arc and therefore its freedom will be greater, as the resistance of the plane F F′ will be less immediately active. At *c c*′ the radial velocity will be again less, but the resistance of arc also less. At *d d*′ less again, this now forming a complete semicircle. Therefore forces impressed in a horizontal direction by the vertical, E E′ will move forward through the space included between the arcs, considered infinitely small of E E′ F F′ by equal effective horizontal displacements, as the restraint at different parts of this parallel space will be proportional at different depths to the radial velocities at the same depths. The entire velocity will therefore be in equation with the average horizontal velocities of the separate parts, and if this be from the action of gravitation acting normal to the system, as here assumed, it may be represented as the tangential action of free gravitation upon half the radial distance of the depth of the liquid, and irrespective of the surface curvature, if this is not great, at the point taken.

d. From experiments in respect to the absolute velocity of the compressile wave agreeing nearly with the above, as shown in the beautiful experiments of Mr. J. Scott Russell, I conclude that water must be a cohesive fluid with little viscosity, as it appears in these experiments, to move subject to its inertia alone.

e. To prevent misunderstanding I may note that if the liquid compressile wave be moved by a force impressed at the *surface only*, as by the wind, the restraint of resistance will be constantly as the radius of the depth affected, and equal to the surface resistances at F for every radius to which the liquid moves by its cohesion. The velocity of the compressile free surface wave in this case will be at its surface, but half that given above for the same depth of liquid moved. Further, the system of the compressile wave will not be in equilibrium to move as a looped system, but under restraint of the lower more inactive parts, there being no free radial velocities as in the case taken for movement of a vertical plane from E E′ to F F of the whole depth of the liquid, it will therefore soon come to a state of rest and subside, unless constantly maintained by an exterior force. There are also other conditions affecting the velocity which I will hereafter consider.

Superficial waves. Surface movement under cohesive restraint of deeper water.

176. PROPOSITION: *If a surface stratum of water be projected by a constant force over quiescent or oppositely moving water, the surface stratum will make intermittent rolling contact upon the water beneath in forming local protuberances or waves into the air.*

a. The above conditions may be those of the inflow of a river to an ocean; the opening of a sluice, the overflow of a weir upon quiescent extensive surface, or an undercurrent moving in opposition to a surface motion. In regions where oceanic undercurrents are oppositely directed to the surface currents we have the most stormy seas, as to the eastward of the Cape of Good Hope, and Cape Horn.

b. It will be convenient to discuss this proposition for a single unit projection upon a liquid surface plane in which the motive unit may be considered to move *tangentially* against the resistance by inertia and cohesion of the mass beneath; the whole system of the liquid being held in horizontal position by a *radial force*, namely, that of gravitation.

c. For the action of an equal tangential impulse upon a liquid I will take theoretically the condition of a small part, namely, that of one vertical series of molecules extending to great depth below the surface.

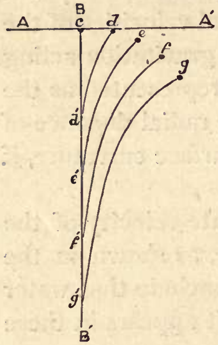


Fig. 189. — Diagram — Rotational Influences on Motive Surface of Water.

Let A A′ represent the surface of a liquid in perfect equilibrium, and let an adherent horizontal force parallel to A A′ move above this. Let a vertical series of molecules represented by B B′ rest upon a surface of support of any depth and be held erect, but in unstable equilibrium, by the force of gravitation. Let a particle of the liquid about *c* upon the surface plane A A′ be moved in A A′ tangentially to the line B B′ for a small distance to *d*. Now the movement of this particle for a small distance could not affect the underlying particle for a great distance in the mass of a mobile infinitely jointed system as that I have heretofore considered a liquid to be (53 prop. *b*). Therefore we will assume that by the general cohesion and pressure upon the liquid surface the movement of *c* to *d* deflects the vertical series of underlying molecules in B B′

first to a point *d'*; then a further effort of the tangential force moving a backward part again *d* to *e* would deflect *d'* to *e'*, and in like manner *e* to *f*, and *f* to *g*, and these movements would constantly disturb a greater depth of the liquid, say consecutively to the points *d'*, *e'*, *f'*, and *g'*, assuming the tangential force to affect a depth proportional to the horizontal displacement.

d. In the above diagram the vertical series of molecules is represented free, it would therefore in following the deflection of the single molecule meet with constantly more and more resistance as the deflection became greater, assuming it surrounded by like fluid. It would also meet with more resistance by the deeper displacement of the longer series of molecules. Further, the tangential force imagined, is directly tangential to the first static position of the molecular series only; so that as the deflection of the first molecule of the series becomes greater the tangential force upon the vertical series of particles becomes less by this displacement, so that at a certain point of deflection in this single series of molecules we can imagine the first particles moved to be brought vertical to the original plane of projection.

e. We may now consider the conditions of a single particle upon a continuous cohesive surface in which the particle is not free as in the last case, other conditions remaining the same. We will in this case suppose a tangential force constantly active on a *selected* surface molecule *only*, as before. In this the surface molecule being deflected, another molecule will come forward by the general surface cohesion and take its place, and another in like manner, so that each molecule will be deflected by the tangential force above it, and the next molecule that moves forward, equally so, and the next, and so on. Now as we assume that each of these molecules is moved to a greater deflection with respect to the forward surface, then as all surface parts of the liquid follow in like order, it is apparent that these *sets of deflection* from the surface will form motive directions

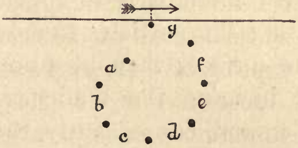


Fig. 190.—Diagram—Theoretical Continuity of Tangential Impulse.

which will follow each other as constant efforts of the same continuous tangential action, and the molecular displacement will circumscribe a definite point below the liquid surface; so that for the small area taken, the consecutive tangential actions upon the molecule may be theoretically represented by the diagram, Fig. 190.

Let the point to the left of *g* be a surface position taken, and *a b c d e f* the consecutive particles moved, then the angle of deflection from this position being set for a small unit of time by the deflection of the first movement of each particle; the deflected parts being constantly lengthened in lineal series would cause them to return by continuity of like deflection to the position from which each particle was first moved, and the whole would complete a *circle* under the position of the first displaced molecule. If we further assume a forward resistance upon the system this would induce a further constant cause for deflection, which would give a spiral direction to the series of molecules taken.

f. If we again refer this system to that illustrated, Fig. 189, we find that if the surface particle were deflected in a circle of a certain radius, the second particle below must be deflected to a less radius as the disturbing force would be less; and further, as the assumed units of tangential impulse act continuously upon the cohesive system, and that each deflection occurs in a certain time, the whole movement must be less and less the deeper the deflection enters the water. So that if we assume the tangential force to have a tendency to engender a large circle by directing the particles to move at a constant *angle* to the surface; then, at a not very great depth, we must imagine the tendency to deflection to be smaller, and, consequently, the resistance relatively greater, so that if the same *disposition to motion* be continuous to this depth, it must be limited to a smaller radius as it recedes from the tangential impulse, of which the diagram, Fig. 191, may in this manner theoretically represent the first positions of movement for two points, *a* to *g* representing the surface tangential action, and *a′* to *g′* the same action at a point disturbed in a lower position. If this were not the case a tangential action would move open water for an infinite depth, whereas it cannot move it for a greater depth than that which in the movement equilibrates the inertia-momentum of

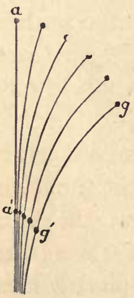


Fig. 191.—Diagram—Influence of Tangential Movements upon deeper parts of a Liquid.

the mass moved. Its power to impress force in the water beneath being derived from adhesion only.

g. The above conditions of surface motion presuppose the possibility of a surface plane entering the bulk of the water which is not evident by experiment; we have, therefore, to consider further conditions.

h. By the principles of rolling contact, a fluid rolls upon a surface of resistance, 45 prop. The same must occur by horizontal force movements of a liquid surface on the unmoved or resting liquid beneath, except that the assumed deeper surface, in this last case, is not a fixed or definite plane, but a hydrostatic plane which reacts upon any force impressed within by its own elasticity.

i. The action of a superficial horizontal motion upon a liquid surface may therefore be taken, by greater refinement, in the following manner for the displacement of a surface particle.

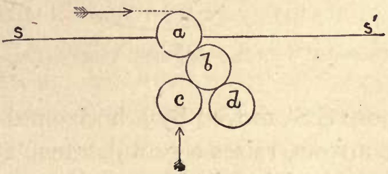


Fig. 192.—Diagram—Riding Action of a Surface Molecule under Tangential Displacement.

Let S S′, Fig. 192, be a plane of repose and *a, b, c,* three particles resting at first in the plane before the impression of the horizontal force shown by the arrow. Suppose the motion to extend at first to the depth of one particle only as before taken (Fig. 190). Let *d* be the first particle deflected, *b* the second, moving on *c,* and *a* the third. Then, as this series of deflections would move towards the resistance of the liquid beneath, and the aerial space above the liquid surface would be free from this resistance, the particle *a* would rise into the air by the resistance of the inertia of *b* and *c,* and be protruded above the surface with a part of the horizontal force with which it was at first impressed.

j. If we now summarize the conditions of active tangential forces upon a deeper position of repose, we find that by conditions here proposed, §§ *c, d, e,* and *h,* that the tangential action would be such as to produce a *whirl.* But that by the conditions proposed, § *f,* such a whirl would be perfectly free at the liquid surface plane, and under greater restraint, proportionally to the depth, at greater depths.

So that upon the whole we may assume by composition of such motions, under the separate conditions proposed, that the whirl formed will have excess of water in motion at the aerial surface by which it will be constantly induced to *lop or tumble* over in alternate parts in a manner which will somewhat resemble the motion of the human body in walking, wherein the foot at each step forms the axis of motion. The cramped form of whirl induced to secure rolling contact may be theoretically represented as in the engraving below, Fig. 193, except that the motion will be largely impressed by sequential impulses and not by entire displacement of the liquid parts.

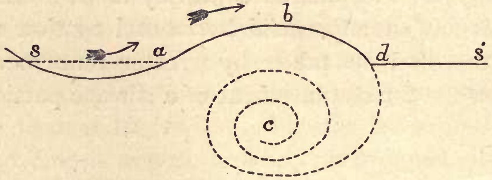


Fig. 193.—Diagram—Theoretical Whirl Deflection under Tangential Action of a Liquid Surface.

The surface of repose S S′, moved by a horizontal force in the direction shown by the arrows, raises a protuberance at *b*, and continues the direction of tangential velocities of the cohesive system by constant deflections onward to *c*, the assumed centre of inertia and resistance.

k. Under the above conditions the forward parts would be pressed up so that the deflection of the whirl represented at *d* would not cut the outline upon the liquid surface as shown in the engraving above, Fig. 193, but the pressed parts would fill in this forward angular space. Further, by principles of conic resistance fully discussed, the surface of the liquid would be a plane of greater resistance than the air, therefore, pressure of the free surface would divide the liquid that was moved forward upon this plane, and the whole form of motion might be theoretically represented as in the diagram on the next page.

Let S S′, Fig. 194, represent a surface of a liquid in equilibrium.

(*a*) A hollow produced by the action of a tangential force; conditions of which have not yet been considered.

(*b*) An elevation by resistance formed by continuity of tangential action on the elevated mass.

(*d*) A hollow formed by the tangential impulse of the whirl action, as a form of incipient biwhirl whose cone of impression is at *d* to *e*.

l. The amplitudes of tangential waves are proportional to the velocities of the overflowing force which induces the lopping action, a greater velocity engendering a broader wave, the tangential forces carrying direct impulse and suffering deflection inversely proportional to the constant force of cohesion of the liquid which induces the tangential deflection to form the protuberance.

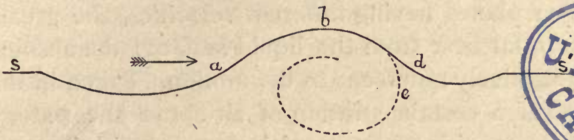


Fig. 194.—Diagram—Surface Equilibrium under Whirl Deflection.

m. If the tangential action be derived from a single impulse, as in water overflowing a weir, the first wave will be broad, being produced by direct tangential action upon the surface; another wave will be produced after bifurcation of the force at *c*, *d*, by the remaining tangential impulse of the surface deflection only *d*; therefore the second wave will be less, and a third again, by continuity of decrement, proportionally less, by like action. Waves upon the principles discussed may retain local positions upon the surface, or be carried forward by motions of general displacement of the system in motive composition.

Action of wind upon the surface of water.

177. PROPOSITION: *That the retardation of moving air by the adhesive resistance of a still surface of water and the greater freedom from resistance at any position above the surface, will cause each molecule or small part of the air to rotate towards the surface of resistance in such a manner that the result of its impact upon the water will not necessarily move the surface in the direction of impression.*

a. If there are present two unequal lateral resistances to the projection of a free particle or mass of matter, the greater resistance will retard the projection the most, and the particle or mass projected will rotate upon its centre of inertia over towards the greater resistance. This is shown in that the resistance of a moving fluid is greatest near a solid or other more resistant system of matter, and that the resistance will decrease in a certain constant ratio to every equally distant plane. So that from this cause a fluid passing such a plane will have parallels of velocities that will be unequal, 46 prop. *d.* If we assume the fluid to be air passing over water, any particle near the surface may be conceived to be placed between two planes having different velocities, the greatest being that which is farthest from the liquid surface; the absolute surface remaining relatively quiescent to the motion. Taken in this manner the impulse of a certain stratum of air above the water would be urging the upper side of the particle forward with its own velocity, whereas upon the under side of the particle where it was more particularly affected by the resistance of the static water near, would through resistance have less velocity. Therefore, the particle would be thrown in revolution upon its own axis of inertia as a resultant of the impulses of the two parallel forces of unequal velocity, as previously considered for rolling forces, 46 prop. *b.*

b. If we now consider this particle to be pressed forward and ejected by the continuous elastic force of compression behind, so that it forms one particle of a strong wind. It would then impinge upon the back of a wave that under the circumstances would be inclined to it, and upon this it would impress the resultant of all its motive forces. Thus, although by its entire initial velocity it might have power even to press forward or to penetrate the wave front, yet from the velocity of rotation that I have assumed it to possess, it would not eject or splash away the surface of contact in the manner of a direct percussion by reflection at the plane of incidence, but rather in the direction of its own rotation. This matter may be made more clear by the diagrams above, Figs. 195 and 195*a*.

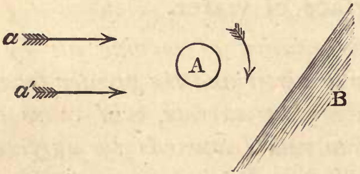


Fig. 195.—Diagram—Rotary Particle before Contact.

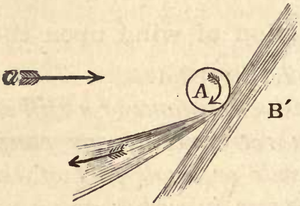


Fig. 195*a*.—Diagram—Rotary Particle after Contact.

Let A, Fig. 195, represent a particle of air about to strike the surface B.

(*a*) The motive stratum farthest from the surface of resistance in the water below, not shown in the engraving, therefore, that of highest velocity affecting the motion of the particle A.

(*a′*) Stratum of greater resistance nearer the resistant surface and therefore of less velocity than *a*, the particle A, Fig. 195, is therefore turned in the direction shown by the arrow in front of it. In Fig. 195*a* the same particle of air is shown striking the surface B′. Under such conditions it ejects the water in the direction of its revolution, as shown by the arrow below.

c. If in the above cases the forces are assumed to be direct, and no principles of whirl motions are taken to be induced in the general system, the velocity of revolution of the particle will be slow; but if the lineal pressure A or B deflect the particle, as well as cause it to rotate, the revolution to follow the whirl current will be quicker. The principle induced by the movement of a particle in rotation impinging upon a surface of repose, may be roughly shown by giving a hoop a motion of rotation, and at the same time projecting it to a certain distance upon the ground; in which case the rotation will not interfere with the general projection, but will after contact accelerate or reverse its projection by friction upon the surface of impact, so that the hoop when the rotation is in one direction will have its velocity increased upon contact with the ground, or in the other direction it may roll back to the point from which it was projected. In this case, nevertheless, the revolution in the hoop in no way neutralizes the projectile force, which makes its percussion on the matter opposing its direct projection exactly as though it possessed no initial revolution; except the action upon the surface of contact just stated.

d. By the principles of rolling contact in fluids, to which I am compelled so constantly to return in demonstration of every motion of one fluid acting upon another; we find that in the case of air moving upon water, by the adhesion of these two fluids, assuming as I propose that there is no slipping motion, the air will necessarily have the velocity of its surface of contact with the water reduced to the surface motion of the water whatever this velocity may be, and in whatever direction it may take. The effects of this adhesion upon motive air may extend by a decreasing series of velocities for a great or small distance, according to the velocity or pressure of the aerial force, whether it has power to break or bend the cohesive system of the air more or less near the surface of the water. If we consider the action of this adhesion as producing a reflex surface motion, from causes discussed, at the aerial aqueous surface, the retarded air will then return by deflection in opposition to the direction of original projection by the principles of rolling contact. We can also imagine that in this motion there will be the same forms of conic resistance extending forward to the more static water surface as those given in (88 prop. *e*, Fig 121, page 249), so that absolute reflex motions demonstrated for water will occur in this case in the air where the water, by its superior density, forms the resisting body.

e. I have endeavoured to show that this is true experimentally, but have not succeeded by any striking experiment. However, the following will aid our conception a little. If we cautiously and slowly eject smoke upon water, the smoke being heavier than the air, it will slowly advance upon the water. I have noticed in this case that the head of the projected smoke curves at the water surface, and has apparently only a slow motion of revolution of which the arrow shows the direction. As this is precisely the motion I have assumed for that of the least fluid friction, 48 prop. *g*, there is, I think, little doubt that the smoky air follows in this case the same law of rolling fluid contact as in similar cases previously considered. Under these conditions the general aerial impulse above the water will not have the power to carry this dense fluid in its *own general direction*, but it may, even by the principles of inversion of flow on contact, carry it in a *reverse direction* at the surface of contact. The air in this case being assumed to be at such a velocity that complete extensive whirls may be developed; but under any conditions upon principles of whirl projection, the wind cannot act upon the back of the wave in a manner to retard its downflow.

Fig. 196.—Ex.—Smoke Poured over Level Water.

178. PROPOSITION: *That the effect of the impression of an oblique impulse by air on water will be to extend the liquid surface to greater area with disturbance of the equilibrium of the static extensile state of the liquid surface; thereby producing ripples.*

a. The general principles of surface extensibility were discussed, 16 to 23 props., and of motive continuity of extensile surface strains 24 prop. The conditions under which aerial impact becomes oblique were discussed in the last proposition. The experimental evidence of the proposition may possibly be seen in every puff of wind that disturbs a still surface of water.

b. It is presumable that if an aerial current were of perfectly uniform velocity in all its parts, that its impression upon a surface of water, in perfect equilibrium to gravitation, would not ruffle the surface, but by the principles of rolling contact there would necessarily be separation of the air into motive units. This separation is also evident actually in the motion of air over any object in sensitive equilibrium, as a blade of grass, which does not bend simply to a curve under the wind action, but *dances* also in it. So that it occurs from the sensitive state of equilibrium of water, its tendency to surface extensibility (16 prop.) and from deflected impulses of impression, that actually its surface is always thrown into ripples by a wind active upon it, and these ripples are the first motive effects, which under continuity or increase of wind force, become waves.

179. PROPOSITION: *That in light winds over rippled water where motions of easy accommodation may be assumed to occur, the liquid surface will be impressed with hollows produced by lateral diffusional whirls of aerial projection.*

a. The principles of the above proposition were discussed for water, 105 prop., but the conditions now to be considered are changed by the liquid surface being one of partial restraint to the aerial motions from adhesion. The same principles applied to static surface were discussed in 109 *remarks*, page 319, wherein it was suggested that whirls would form in every indentation of surface (*d*) by the influence of *shading*. Upon the same principles when a surface of water has become ruffled by the wind by conditions of the previous proposition, every hollow upon such a surface is in relation to the horizontal elements of direction of the wind an area of repose from its action.

b. It will be further seen that even if the surface of water is not ruffled that any intermittent force that there may be in the wind from consecutive action (*b*, last prop.), would upon a resistant liquid surface of contact at once induce the surface to divide by the impulses, and thus form separate whirls which would be at first of small dimensions.

c. We may also conceive by the whirl system of motion in mobile air acting against the more static resistance of water, that the horizontal resistance of one side of the plane of motion would deflect the current naturally by a retrograde movement (89 prop. *c*) for the first immediate surface of contact. Further, by the formation of any possible whirl of contact in a free fluid, we have one of two conditions ensured; either that the whirl must continually increase in dimensions by the continuity of decreasing resistance within the area in which it is generated, or it must form a whirl over a certain area and leave the remainder of the surface free, either shaded from the current, or in a condition to form other like whirls. Now it would appear from my experiments that when a larger or deeper free space occurs in the path of the most static side or resistance, that an increasing whirl would continuously revolve until it filled the space, even if it were the bay of an immense ocean, but if there be restraint or inequality of resistance, whirl forms are at once produced at any point of greater repose in the system (109 Remarks).

d. In all motions previously discussed for air there is every evidence of unequal or intermittent action. Therefore in the air we may infer, from the musical note which comes distinctly upon the ear in the roar of the wind, that the intermittent action is in this and many other cases in equal time, following possibly a law of vibrational forces. This uniformity of impulse engenders equal uniformity of whirl action upon water surface which independently

Fig. 197.—Diagram—Influence of Lateral Diffusional Aerial Whirls in Producing Aqueous Waves.

permits the formation of waves, as the separate units of impulse are deflected by surface contact to form a system of whirls which occupy approximately equal interspaces upon the water surface. We may represent these conditions, as in the diagram, Fig. 197, consistent with the forms of rolling contact of air on the deflectable surface of water; either where hollows exist, or intermittent impulses are impressed from above.

e. In air passing solids, no planes of deflection could be formed in the solid, as in a liquid, and the adhesion of the air to the solid would possibly be less. But I have occasionally noticed the same principles of motion in drifting dust and snow, where a point of superior resistance occurs, so as to leave a more quiescent air space in front of this, equivalent to the hollow of a wave just considered. In this case the current of air moves continuously, and the deflection occurs at a certain point where a whirl is produced by the overcurrent nipping up a surface wave. This is shown by the direction of the arrows in the diagram below.

Fig. 198.—Ex.—Dust Wave formed by Wind.

f. The above described form of motion is very frequently caused by the interference of a wall or tree, but I have seen it often occur in simply undulating ground, or irregular solid surface, and observed it in the drifting of autumn leaves along a high road.

g. The whirl resistance under the above conditions still remains in the line of direct, or tangential impulse, of the whirl. It is uniformly greatest upon the back of the wave, where the greatest freedom from interference also occurs; so that the dust heap, leaves, and ripples we have considered maintain a general projectile force in the direction of the wind.

h. What we find important in this matter for the consideration of vertical whirl-force, in the free motions of the wind, so far as regards a liquid surface, is that wind may appear to produce a *retrograde motion*, and will do so naturally on the back of the hollow of the wave. Therefore if the surface of the ocean did not derive projectile force from the wind, the surface motion would be retrograde to its direction. That this is not really the case under the conditions, I will discuss in the next proposition.

i. I may also observe in the action of whirl motions generally in air, that a whirl is produced by a tangential impulse moving a free part of the air about a centre of inertia, therefore that its motive lines are by this cause uniformly *circular*, and take no other form, unless deflected from it by restraint (83 prop.). Therefore the intermittent action of separate units of air in the wind upon the relatively quiescent spaces of repose formed by the hollows of the waves, will cause the whirl motions induced to *constantly correct the form of water surface into semicircular hollows.* Wherein the waves differ from this is by the influences of other causes that I have already discussed for compressile forces, and will further consider.

180. PROPOSITION: *That the projectile force of the wind acting upon an extensive surface of deep water, if it exceed the velocity of easy accommodation for rolling contact, will drive forward the aqueous surface with a velocity of displacement which will be according to the angle of impact upon the surface and the momentum of the wind's mass, into the cohesion of the water at the horizontal plane; the hollow produced by the impact of the wind being equal in volume to the protuberance projected forward.*

a. Under the conditions taken in the three previous propositions the curvature in direction of impulse of air and water is an important element in the surface motion for the small waves considered. For larger waves we may assume that the spiral direction of impact of the air remains constantly within small radius. Therefore when the wind's force is greater, its general directive impulse will overpower the influence of any element of retrograde surface motion, occasioned by rolling contact. Further, if the velocity become greater in the air, the accommodations of rolling contact becomes more frictional, so that the resistance of the aqueous surface instead of completing an outward circular whirl in the air, is only able to complete such part of the whirl as to meet the water surface practically at an angle at which it will rather *project* than *deflect* it. For in this case of higher velocity the momentum of the aerial force may be conceived to overcome nearly all exterior resistance to flexure within its own cohesive system.

b. The principles of the above may be expressed diagrammatically as follows, taking three cases, of which the last only accords with the conditions of the proposition.

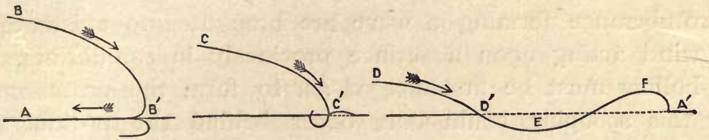


Fig. 199. Fig. 200. Fig. 201.

Diagrams of Aerial Impact on Water.

Let the space included between A and A′, extending through the three diagrams, be a liquid surface in equilibrium.

B B′, Fig. 199, will then represent the direction of motion of perfect accommodation by rolling contact of air; in this case, moving nearly frictionless upon the liquid.

C C′, Fig. 200, represents *deflection* of the surface of the water, so that the air in this case impinges perpendicularly to the surface, causing thereby greater instability of surface equilibrium, and some forward compression at C′.

D D′, Fig. 201, represents a higher velocity of the wind by the magnitude of the momentum to the small relative resistance by cohesion to the aqueous surface, the whirl taking a sharp parabolic form practically meets the surface at a less angle. In this case the water has a tendency to be projected from the surface in front of the point of impact, D′, and by deflection upon the mass resistance from E to F.

c. In the above diagrams, to complete the theoretical forms, I take the motion of single units of projection as before, and assume no interference. It is quite certain that this could not happen in a perfectly uniform moving wind if such a phenomenon exists. The principles here discussed show, that upon aqueous surface there will be very small resistance to gentle motions of the air, but great resistance to intense motions, and less surface stability under more intense wind forces.

d. Upon principles shown in diagram, Fig. 201, the impact of air in the direction D D′ by the action of which a hollow is scooped out, and a protuberance projected forward, we might conclude that the entire surface of a liquid impressed by such a local force, as that of a gust of wind on the ocean, would cause the water to be projected forward by the impulse. This is not evident by observation of the surface of the water, but that this action is actual, appears to be largely supported by observing the manner in which the wind drives forward oceanic waves; for we are certain that if a hollow and a protuberance forming a wave are brought into existence by the wind acting upon a surface previously in equilibrium, that the hollow must be first *scooped out* to form the protuberance, and this scooping could only occur behind the protuberance. Therefore the protuberance is formed from the liquid *pressed forward* out of the hollow. Upon this principle we may conceive the wave so formed to be urged forward over the whole surface, by continuity of like action, in pressing forward other parts of the surface, in so far as the separate units of the breadth of the wave are active, and although the deflection of contact resistance gives an immediate surface reflection to the direction of projection (179 prop.), still the mass of water will be urged forward by the general horizontal momentum of the entire initial force of the wind.

181. PROPOSITION: *That a steady wind over wavy oceanic surface becomes intermittent in its action upon waves by impressing alternate salient angles more than alternate shadings.*

a. The wind having once raised a wavy surface upon a wide extent of water, it is much less difficult to conceive the continuity of the impression of its force at a higher velocity producing a still more wavy surface. For in this case the wind impinges at first upon the rippled surface which it presses forward to form the wave, at the same time raising the wave to a more salient angle to its direct impulse, so that the wind does not only produce waves in moving horizontally, but produces these in altitude proportional to the wind's force and its direction of impact conjointly, in equation with the inertia and mobility of the water. Further, even if the action of the wind were possibly a steady motion, the inequality of its possible contact upon the back and front of the wave, after it was once produced, would, by the differences of resistance, render its action intermittent upon spaces forward; so that we have at all times, not only surface waves by contact of the wind, but we have them superimposed upon a system of larger motions induced by intermittent action from its passing previously over a wavy surface.

b. This principle of intermittent shading from the wind we experience in the shade of a hill bank or wall upon our own bodies when exposed to it. In this manner, as each wave is followed by an interval, before the wind-force can again be active to produce another impression, the motion induced in the backward parts of a ruffled system will be carried forward constantly by the now intermittent tangential action of the wind upon it.

c. To follow each separate wave according to these conditions, the following construction may be taken.

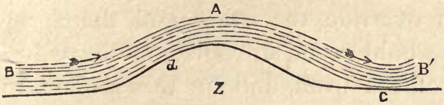


Fig. 202.—Diagram—Intermittent Action of Wind upon a Wavy Surface.

Let Z, Fig. 202, represent a single wave, and assume this wave to be formed by a tangential compression by the lower atmospheric stratum represented by B B′ moving in the direction indicated by the arrows, and let this impress the wave at the salient angle formed by its plane, *d.* Then will the assumed horizontal projection of the wind be deflected by continuity as a compression upwards at A, and its elastic momentum will not impinge again actively on the liquid surface until it reaches C, where, by its impulse, another wave will be formed as a secondary resultant of continuity of projection of the wind.

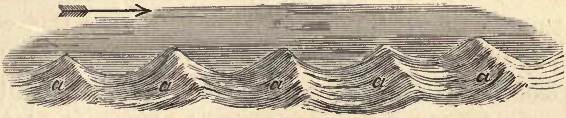


Fig. 203.—Diagram—Waves of Easy Accommodation.

d. The continuity of bite of the wind upon the surface of the rippled ocean, which produces larger waves, may not inappropriately be compared to the bite of the threads of a screw which produces a very perfect form of mechanical adhesion. The bite will be greatest upon the planes *a*, *a*, *a*, in the engraving above, Fig. 203.

Conditions of division of a liquid surface into waves by tangential compression.

182. PROPOSITION: *That a horizontal, or tangential, force impressed in adherent matter upon the surface of a liquid will raise separate units of the liquid into protuberances by the resistance of the inertia*

of the forward parts of the liquid. These protuberances or waves will conjointly rise to a gravitation potential of resistance in equilibrium with the momentum of the tangential force impressed. By continuity of impulses the tangential forces will act cumulatively, so that the whole system of separate units or waves will also be raised into larger protuberances in like manner.

a. In 179 prop. it is shown that every unit of a surface impressed by tangential forces is in unstable equilibrium, which induces one particle to override the other and thereby produce surface division. This would occur, of course, more easily under the intermittent action of the wind, and by this motion the conditions of wind action on water become of the same form, as before discussed, for horizontal surface displacement in 175 prop.

b. Certain conditions of surface division into many waves by the impression of tangential force at or near the surface may also be experimentally shown, for a simple case, by moving a small body along horizontally with moderate velocity at a small depth in the surface of still water. In front of the moving body a series of small undulations or surface waves will be formed by the compression (24 prop. *c*). The liquid surface, in this case, will appear to be *crumpled* up. This principle may possibly at all times be active in the formation of the larger natural division of an extensive surface into waves caused by the intermittent action of the wind, coherent upon and resisted by the forward parts of the water. It is also very probable that under tangential action from any cause the conditions of resistance, by the extensile strain that the aerial surface of water constantly possesses, as argued 24 prop., will be in a small degree an auxiliary cause of this *crumpling*. This appears to be so, as the surface of oil by itself, or of oil upon water is not affected in a similar manner, to open water by the action of the wind.

c. For the height to which any single wave of a system attains by the causes discussed above we may consider the conditions of the elevation of one wave. Thus if the whole surface of the water or a part be moved by a composition of the tangential and vertical forces acting upon the general inertia of the liquid system, the whole surface will resist by its inertia and by the general cohesion of the water. So that the surface becomes as a rigid elastic body under longitudinal compression; under this condition a compressile wave will be formed when the impressed force exceeds the elastic resistance. Now it will be quite clear that a tangential force sufficient to cause a compressile wave will by continuity of pressure, where the wave is raised to a more salient angle to the wind's impulse, cause its maintenance during the continuity of the impression. It is also clear that if this salient angle be once produced, that a greater force in the wind will exert a greater pressure and the surface will tend to expand upon the compressile wave. This increase will be proportional to the force of the wind since the wave increases in height as the horizontal forces press it forward. It increases also in area of base as the constant force of gravitation brings the figure of the protuberance to equilibrium as a conchoidal system, so that at a certain point of elevation and expansion of base, the tangential pressure by its momentum will come to equilibrium with the potential of the elevated wave, the inertia of its mass, and the force of cohesion of the liquid system combined; and it will rise no more. As one wave is formed so will others be, until the aqueous surface is covered by waves of dimensions proportional to the extent of pressures into the resistances.

183. PROPOSITION: *If by the contiguity of solid matter to a wave system the area over which the resistance is carried be confined, or the tangential force of the wind act with great intensity over small area, the liquid surface will be more compressed, rise higher, and flow less easily to a state of equilibrium, so that sudden forces impressed upon small areas and resistances to surface forces by solids, will produce relatively higher waves of smaller base.*

a. This is seen in a wave running inshore, where the resistance of the shore causes it to rise and break. Also by restraint of coastline as for instance the broad Atlantic wave becomes a shorter and higher wave in the Bay of Biscay.

b. The evidence of the above may also be clearly observed by looking over the daily weather reports issued from the British Meteorological Office, wherein we find the direction of wind upon a coast generally produces a rough sea, which is caused by the volume of the wave being *shortened* up and the surface pressed together by the resistance of the coast to the inflowing more free conchoidal wave engendered at a distance from the coast by the wind. It is possible that the entire momentum of the wind by its adhesion may be generally found in the potential of the water elevated by it above the surface of equilibrium, over the whole areas of resistance.

c. As long waves are produced by strong winds and ripples by light ones, there is no doubt that a certain ratio exists between the length of wave and the force of the wind, for a constant wind acting upon a definite surface agitated by no currents. But the difficulties of experimental investigation are immense, as wind forces act cumulatively to a certain extent on the volume of the wave, every portion of the surface of a large wave being subject to exactly the same impressions and re-divisions as a level surface if struck at the same angle.

d. For the general assurance of the fact of pressing up of surface water by the wind's force, observations of the most superficial kind teach us that the forward elevation of the oceanic surface is absolutely from this cause, and that no principle of mere oscillation of ridges and hollows will answer to experience. It is well known and easily observed that the wind blowing towards a coast or up a river carries the sea with it and elevates the water much above the average level. That setting from the coast or down the river it carries the waters outwards. We cannot in these cases infer that the transference of the water is in any way effected except by the force of the wind; neither can we imagine that the deeper parts of the water are greatly disturbed by the exterior tangential action of the wind; therefore the actual displacement must be by surface motion. The wind-force manifestation, as translatory motive power, in such cases where there are present near resistance from land surface, can be in no way comparable to the translatory motive power of the same amount of directive force in the wind when it acts upon a free area of the deep open ocean, where there is present no static resistance equal to that of an inclined surface of coast. Further, on the coast there is no possibility of bringing about any very frictionless motions of accommodation, where the wave breaks and loses in molecular friction nearly all powers of continuity of directive impulse; such as would remain perfectly active in a free ocean

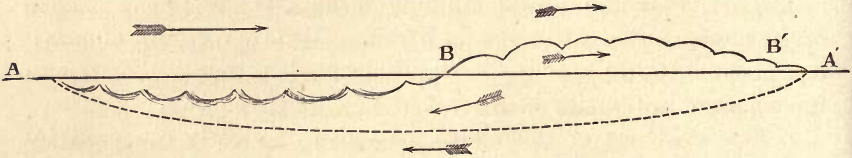


Fig. 204.—Diagram—Theoretical Storm Action upon Oceanic Surface.

where the equivalent wave would be urged forward as a looped system of compression.

e. Upon the principles here discussed the diagram, Fig. 204, may represent theoretically the action of a storm in the open ocean, the area of resistance being at A′ which may be either the resistance of quiescent or oppositely directed water, or of a coast.

Let A A′ represent by a line the plane of static gravitation equilibrium. The two arrows above, A B B′, represent the direction of the wind. The dotted line below the extent of the return current, showing possibly the depth of water affected by the surface movement within the amplitude to the wavy surface.

B B′ represents a great wave, which does not divide by its relative amount of elevation simply, but by the resistance to its impulse. In the open sea a wave divides when its elevation becomes greater than a certain function of the radius of the depth of water moving in the same direction in the wave system, probably when the depth is less than double the elevation of the wave above the plane of lowest motion of the system. The wave probably breaks more readily at the front of the storm B′, than at the back B, from conditions I will show in the next chapter.

f. If a storm be directed upon a coast, the coast resistance is felt for a long distance outwards from the land, so that the storm compression, at its greatest surface elevation, seldom or never reaches the coast. This is assumed by principles of conic resistance to liquid projections in flowing forces (88 prop.), there being always in the case of a storm impinging upon a coast an area of compressed surface, where the waves rise much above the figure of conchoidal equilibrium, the sum of the compressile force being largely lost in the separate potentials of the elevated crests (171 prop.).

g. Open oceanic compressile systems are increased in elevation by the impulse of the tide coinciding with that of the wind. The tides forming otherwise naturally compressile systems by the effects of tangential attraction of the sun and moon upon the oceanic mass.

CHAPTER XVI.

ACTION OF GRAVITATION UPON LIQUID PROTUBERANCES OR WAVES, WHEREIN EQUILIBRIUM IS RESTORED BY SURFACE INFRACTIONS. CONDITIONS OF HOLLOWS—OSCILLATORY MOTIONS—AND GENERAL COMBINATION OF FORCES IN LIQUID WAVE MOTIONS. CURRENT WAVES.

184. Minus Compression—Hollows.

In the commencement of the last chapter, 170 prop. *g*, a general statement was made that wave motions occur from two causes, namely, by *increase* or *decrease* of local compression upon a liquid surface in equilibrium. The former as regards increase of local compression, for effects produced by both vertical and horizontal impressions, where *protuberances* upon the liquid surface were shown to be formed, was developed in the last chapter. It will be my present purpose to endeavour to show the principles upon which local *minus* compressions, that form *hollows* in a liquid surface, dissipate impressed forces to equilibrium; wherein the action of gravitation becomes the all-important element. The hollows themselves, in these cases, being generally the results of tangential forces active upon the surface; as by the impulse of the wind, which engenders a compression upon the forward parts of the liquid system and therefore produces necessarily a hollow in the backward parts, 183 prop. The same principles will, however, be active if the liquid surface be impressed by any other mode of force by which the potential of a part of a liquid system becomes *less* in a certain locality near relatively protuberant parts; as the action of gravitation will in all cases act uniformly upon equal elevation.

Infraction of liquid surface.

185. PROPOSITION: *That if a motive impulse be impressed upon one part of a lineal homogeneous system in a direction to produce greater extension of a part of the system, than the general cohesive force can withhold by the traction of following contiguous parts, this impulse will cause a fracture which will release the motive part from its previous cohesion to the mass system. The fracture formed in this case will be normal to the direction of the force, and occur at the weakest part of the system behind the moving part. On a liquid surface under the like conditions of extension, a partial fracture only will occur.*

a. If we fracture in a long solid body of uniform section by a longitudinal strain, the fracture will occur at a point in the solid where the molecular force of cohesion is weakest, and this will be consequently the first part to give way under the strain. With a liquid, where every part is mobile, a like point of least resistance occurs only insipiently, as the general cohesion of the system may be assumed to cause the fractured parts instantly to reunite after the release of the strain. Therefore a liquid, by a kind of mobile equalization of strain, causes the parts disturbed by the strain to move consecutively from place to place until equilibrium is restored; so that a plane of fracture or *infraction*, as I term it, in a liquid, may be one that exists only for an infinitely short space of time, in any single local position. This form of fracture being necessarily at no period entire, as in the case of fracture of a lineal solid, where perfect detachment is possible.

b. This matter may be made clearer by reference to more particular instances of the fracture of solids under strains equivalent as far as possible to those I wish to demonstrate for liquids.

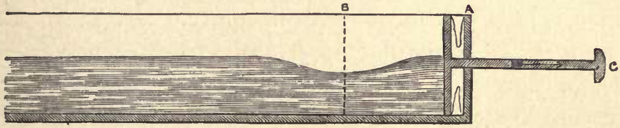
c. If we stretch a length of metal rod or wire, we can imagine that every molecule forming this rod will experience a *strain* against the cohesive force by which it is held to the next molecule in lineal series of parts, all molecules being united in a manner to form a mass system. If we continue to stretch the rod it finally breaks, and the matter in the two separate parts is now again thrown outwardly into the same material condition of static equilibrium as the whole was before commencing the stretching, and each of the two separate pieces forms thus a distinct although similar mass system.

d. If we examine further into the molecular or physical condition of the rod before the fracture, we find that the separation occurred only at the *weakest point* of the system. We may also conclude that if the rod had been weakened at any other point, at this point only, there would have occurred a *plane of fracture.* Therefore a concrete mass system gives way or separates by a tensile force that is sufficient to disunite it, in *one plane only*, which plane will be that of its weakest transverse area normal to the lineal direction of the tension; and although all or many other parts may be nearly as weak as this, and may be affected during the impression of the constant strain by tension in a degree almost sufficient to separate its contiguous parts, yet if the strain be within the range of the cohesion of the material body, no outward physical effect may be permanently or visibly produced and the system will generally remain, materially considered, as at first.

e. To take another instance of the fracture of solid matter which may be considered relative to our subject—we may observe a surface of high land broken away by the action of the sea, so that a cliff is formed upon the sea front. Gravitation is acting with great force over the whole extent of this land which is unsupported by other equally gravitating matter on the exposed front, and if it were not for the general cohesive or attractive forces existing within the separate parts of the mass of land, the cliffs formed would slide down as the sea broke them away, until a level surface would be produced by gravitation. But as it is, the cohesive force exists in every part over an extended area between particle and particle of the mass; therefore the sliding down when it takes place occurs only at some considerable distance inland where the entire gravitative force of the forward mass exerts sufficient strain to overcome the mass cohesion, and at this distant part a plane of fracture occurs and a large mass of earth slides down into the ocean. When such a phenomenon occurs the strain of gravitation, previously active in the mass contra to its cohesive force, ceases to act in the same manner, and the mass is then left in a state of comparative equilibrium; being separated at the same time from the general cohesive mass system of the land of which it originally formed a part.

f. If we apply such a system of fracture as the above, to a mobile symmetrically constructed homogeneous mass as that assumed for a liquid (3 prop.); then from the equal mobility of all parts, no distinct plane of fracture could be imagined under similar conditions, for if there were liquid-slips these would be unlike the land-slips, as they would not be able to withhold, as it were, the cohesive force until cumulative strains fractured a particular part; but all parts would be *equally weak* and would move down exactly at that point where gravitation was in equation with the cohesive force of the system. Therefore, under these conditions, the liquid would move down by gravitation if unsupported on the side of each hollow, by a constant *disintegration in infinitely thin mass fractures*, or as I will term it, by continuous *infractions.*

g. The system of motion here proposed for liquids slipping down into hollows, has been given as a part of the cause of wave motions by M. Flaubergues—to which I will return further on. I may here remark that although I use the term *infraction* and *slipping* in the above case, this is really an apparent effect only (prop. 43, *e*), the motion being one of rolling-contact; but as a regular hollow plane is nearly produced, it may be taken as a slipping motion until further refinements are offered.



[Fig. 205.—Ex.—Hollow Produced upon a Liquid Surface.

h. The infraction of liquids under the above conditions may be illustrated by taking the same long trough as described, 171 prop. *h;* this being partly filled with water and the false end being at the place it was left in the experiment, before described; that is, at the position B in the above illustration, Fig. 205. If we now suddenly withdraw the movable part from the position B, by the plunger C, to near the end of the trough, as shown in the engraving, the water will instantly form a surface hollow at about the position formerly occupied by the false end, as shown above under B. In this experiment, after the hollow in the surface of the water is formed, the water does not immediately slide down for a great distance or move gradually into this hollow by an inclined plane of motion, as might possibly be imagined upon principles of equality in fluid pressures under sliding motions, but it consecutively *breaks away* from the level surface as in the case of the land-slip just discussed, so that the hollow moves forward upon the liquid surface in equilibrium apparently by *con-*

secutive infinitely small slips until it reaches the limit of the extent of the water surface. These slips are for the instant planes of fracture, or as I term them—*planes of infraction.*

i. In the above case there are subsidiary phenomena; for instance, the water does not fall into the hollow exactly as the land-slip, a frictional mass; but by its fluid mobility, elasticity and mode of motive contact, it falls as an *active* force, which produces a rebound that elevates it to nearly the same height on the opposite side of the hollow space; so that the hollow goes forward upon the liquid surface as a true wave would. The reflected slips fall again consecutively and similarly to the first, as they reach the same altitude, so that one hollow is followed by another engendered upon exactly the same principles as the first, and this by another until a wavy surface is produced. To the more exact condition of this process I will return.

j. In the above there are also subsidiary effects which disturb the general principle offered. The falling mass of the infracted liquid adheres to the surface of the forward parts of the hollow upon which it falls, so that it never absolutely *breaks,* but the entire motion appears as a regular systematic transference of a minus compression, represented by the hollow, to areas of greater hydrostatic pressure to establish equilibrium. Further, each wave becomes henceforth a unit system, the conditions of which I will follow in separate propositions.

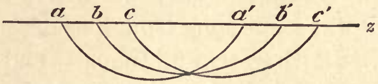


Fig. 206.—Diagram—Infraction of a Hollow upon a Liquid Surface.

k. The above diagram represents the mode in which a liquid breaks away from a surface upon the principles of infraction offered. Supposing a hollow formed at first from *a* to *a*′, and that the mass of liquid has its point of greatest pressure towards the right of the figure at *z* by surface elevation at this point, as by the mass of water in the trough in the experiment just given, Fig. 205. The liquid at *b*′ will first fall down the incline and rebound to *b*, removing the hollow as it were forward to *b b*′, it will then in like manner fall from *c*′ and rebound to *c* until the hollow passes to *c c*′, and by continuity of like action it will pass entirely over the surface in direct lines to the point of greatest pressure, towards *z*.

l. If there were no point of pressure *z*, to the left of the figure the plane of infraction would surround the hollow and the fracture would be equal on every side, but as this would produce residuary phenomena which would need our separate consideration, I therefore give the conditions of a hollow infracted on one side, in a parallel system, as the simplest case for primary consideration.

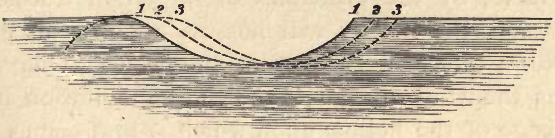


Fig. 207.—Diagram—Infraction Curve on a Liquid Surface.

m. Actually, by extensibility of surface (16 prop.) and the necessity for rolling contact of the motive parts of the system, the edges of the hollows are rounded off particularly on the following side, and in so far, do not resemble fractures. The surface adhesion of the separated water, and its momentum under deflection from a direct path, forms on the back of the wave an incipient compressile system (176 prop.) as shown by the continuity of curve 1 to 1, 2 to 2, 3 to 3, Fig. 207. The wave falling from right to left as in the previous illustration, Fig. 206.

186. PROPOSITION: *That large waves upon an extensive surface of water will grow larger by more intense wind force and will extend themselves over a much greater area than the region on which they were produced, by mobile compressions going forward over surfaces in relative equilibrium and infractions going backwards in like manner.*

a. The conditions of forward pressures producing large waves were discussed, 183 prop. The hollow also increases in the wave by infractions on the backward parts. The height to which waves rise in a storm is not from the effects of the average pressure of the wind over the area affected, but is in proportion to the intensity with which a local tangential compression in a unit of time elevates a certain mass against the general surface resistance (183 prop.). We may assume for the conditions of this proposition that an elevated wave mass exists, and that its protuberance is continually supported by the direction of the wind on its backward side, it will then approach in principle a free surface wave of the looped class discussed, 171 prop., but be moving upon a lower surface of restraint; wherein the lower strata of the liquid will have less velocity in proportion to the radius of depth affected by the force. In this case the loop or wave goes forward over a surface in equilibrium by a smaller constant wind force than that which was necessary to raise it, and thereby continues its motion forward beyond the locality where it received its original impulse. On the other hand, by the principles of infraction discussed in the last proposition, the hollow will possess a power of propagation in the *opposite* direction to that of the wind. This will be so, inasmuch as the wind will aid the force of gravitation in breaking down the edge of the hollow upon which it first comes in contact, at the same time that it supports the forward part of the hollow *against infraction.* Therefore the entire action of the wind upon a hollow in a wavy surface, once formed, will be to engender motion in two directions: the one driving forward the protuberant wave by direct compression, and the other breaking down the hollow against the direction of the wind. The above principles may be represented by the following diagram.

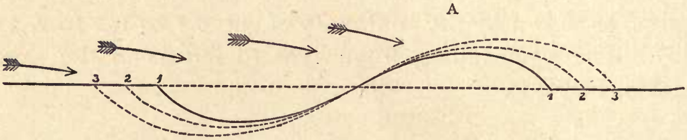


Fig. 208.—Diagram—Double Curvature of Infractile Wave.

b. Let Fig. 208 above represent a hollow taken out of a surface in equilibrium, as discussed in 180 prop., by the force of the wind moving in the direction of the arrows above. Then the parts of the surface 1, 2, 3 to the left of the figure will be consecutively broken down by infraction, in which gravitation will be aided by the direction of the wind. Let A be a loop of compression formed upon the liquid surface and be pressed forward by the wind's force in the direction shown by the arrows. Then the wave will advance constantly before the impressed force and move to consecutive parts as shown 1, 2, 3 to the right of the figure above, the hollow moving at the same time *backwards.*

c. By the conditions of rolling contact, and parabolic projection of the wind, 179 prop., a modification would be produced in the wave form in front of A, so that the forward protuberant wave would now nearly resemble Fig. 209 below which I have sketched from nature.

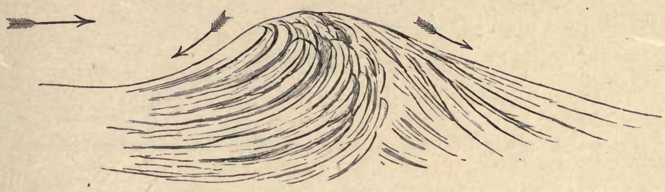


Fig. 209.—Ex.—Observed form of Wave.

d. By applying the conditions just proposed to the surface motions of a wavy sea—a wave being pressed forward by the wind at its back and infracted in front, there is found a constant tendency in the protuberance to override and fill up the forward hollow; at the same time the infraction is lowering the crest. In this manner also there is a constant tendency for a wave to extend its breadth. Further, as a wave at its complete formation, wherein its gravitation potential may represent the force of the wind (183 prop.), becomes immediately in a state of dissolution; so that the wave finally entirely subsides and forms the hollow of another wave, which is again pressed forward and infracted in like manner. These conditions are consistent with observation.

187. PROPOSITION: *A horizontal compression upon a liquid surface that acts too suddenly to enable it to produce a conchoidal wave, will produce at first a protuberance of central height in excess of the mass necessary to form a symmetrical conchoid; such protuberance, by its form being out of compressile equilibrium, will constantly fall away by infractions until the wave is reduced to a conchoid of compressile equilibrium.*

a. The conditions of a wave as a conchoid of compressile equilibrium are given in 174 prop. If a gust of wind impinge upon a conchoidal system, or a current in the ocean drive it against a coast, by the wave moving upon forward greater resistance than the open water, the surface system will be compressed, and its protuberance will exceed in central elevation a conchoid of equilibrium for a motive wave system (173 prop.), and in this case part of the protuberance will be immediately *infracted.*

b. Let the lower line A A′ A″, Fig. 210, represent the outline of a conchoidal wave in compressile equilibrium.

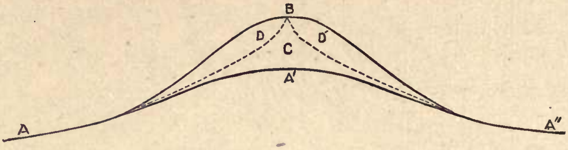


Fig. 210.—Diagram—Infraction of a Wave when above Conchoidal Equilibrium.

Let the elevation B be engendered by a horizontal compression on the surface of water greater than that possible in the time to form the true conchoidal wave of equilibrium A A′ A″. Now the entire resistance of the curved plane represented by A B A″ being weak, and the potential force by gravitation being great upon it; this plane will be *infracted* by the conditions of 185 prop., and the planes of infraction may then be represented by the lower dotted lines B to A and B to A″, the excess of mass represented by D and D′ being brought down to a lower position.

c. In this case the liquid in the inclosed space C would be at first supported at its base by the conchoid of equilibrium A A′ A″, but by continuity of infractions the part C would also be brought down and the conchoid only remain in motile equilibrium.

d. In the above diagram, if the entire wave A B A″ were impressed with a horizontal force so as to render it a motile wave, 172 prop.; its translatory force would resist locomotion by its mass, in such parts of the system as were not included in the conchoid of compressile equilibrium, and the whole mass in the space above the conchoid, here shown, would not support the back of the wave to form a rigid plane in equilibrium. This excess of matter would therefore be superfluous as a motive part, and would burden the wave as much as it would accelerate it by its compressile weight; so that the wave, whether high or low, would have equal translatory velocity, consistent with its general equilibrium of figure (172 prop.).

e. The immediate primary effect of the infraction upon this form of compressile wave, of the excess of matter necessary to produce the conchoid, would only point its crest, such waves being formed forward of a compression, whether this was caused by a backward pressure in the open ocean, or by drifting of the water against a coast; but the *elevation* and *subsidence* of all compressile waves must necessarily be of *conchoidal form only.*

Division of large waves by Infractions.

188. PROPOSITION: *That any protuberance or wave upon a liquid surface that is not by its conchoidal form, a motive compressile wave, will through the action of gravitation upon it, be dissipated to gravitative equilibrium by infractions; such infractions from the vertex of the wave acting as biwhirl projections upon its lower parts, which represent the cone of impression to the biwhirl. By the biwhirl action there will be a constant tendency to redivision of the wave surface system, to restore gravitative equilibrium.*

a. The surface of water under the conditions of this proposition is assumed to be raised in waves or protuberances by some exterior horizontal or tangential compression, and gravitation alone is now assumed to be active in restoring such surface to equilibrium. In this case the mass of elevated water in the protuberant wave represents a potential force pressing upon the mobile system of the water beneath. This force embodied in liquid matter would clearly resemble the pressure by which a biwhirl is engendered (76 prop.), from the local pressure of a liquid of limited area being active upon a free more quiescent part; and we may suppose that the local pressure would now move against the conic resistance of the relatively static liquid beneath it by deflections, in the same manner as that shown experimentally in the case given, 77 prop. *e.*

b. The liquid impressing force would, by the above conditions, divide against the conic resistance of the mass beneath, and flow down both sides of the wave surfaces with the projectile force derived from its gravitative impulse. In this case, if its direction through the altitude of the wave, form a large angle to the horizon, it may cause the flowing water even to penetrate the quiescent base of the wave, as in the case of biwhirls in previous illustrations. If we represent the potential force of such a wave in a unit of matter located near A, as in the diagram, Fig. 211, then the biwhirl projection through gravitation would be representable by the whirls right and left, shown by the dotted lines, for the single wave under A;

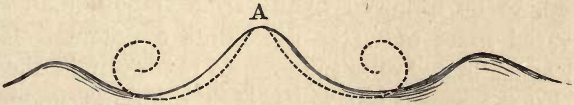


Fig. 211.—Diagram—Biwhirl Infraction of a Wave.

the area of conic resistance being supported by the incompressibility of the liquid beneath. The infraction of the surface would thus be constantly dissolving the wave, but by the directive influence of the biwhirls right and left, a new wave would be formed at its base.

c. If we apply the above principles to a wavy surface, the like potential forces would be resident in every protuberance, and oppose each other in the contiguous hollows. Suppose that we have two such waves, as represented in the diagram below, Fig. 212, then the action of the wave A will be the same as the action of the wave B, and the whirls derived from each of the biwhirl projections will meet in the interspace between the two as at B′, and in this position by their directive influences an intermediate wave will be formed, which will rise in equation with the forces derived from A and B, less the action of molecular friction in the process of formation.

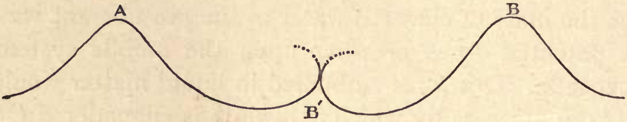


Fig. 212. Diagram—Biwhirl Formation of a Wave.

d. By the above it will be seen that, however high the ocean waves may be, that when the forces of compression cease by which such waves are elevated, the principle of infraction, active in biwhirl motions, will lower the waves by constant division from opposing whirls; so that a high sea falling to equilibrium, does so by the division of the highest waves, constantly, until a rippled surface is produced, to be followed finally by a smooth plane. Further, every elevated mass of water represents a potential force which acts at all times in a similar manner, so that even in the formation of waves by the tangential pressure of the wind, upon principles already offered, every compressile wave that is elevated above the conchoidal form of potential equilibrium is under the action of constant infractions forming transitory biwhirls, so that the crown of the wave is constantly induced to fall, and the interspace bottom to rise alternately. But if the compression continue, there will then be no interspace wave formed, or the hollow itself may be absorbed by a greater compression as before shown (183 prop.).

e. Experimental observation of the general action of gravitation upon the surface water of a wave may be made, I have found, at any time by a simple experiment as follows:—If we throw a few corks upon a rough or wavy sea the corks will be urged forward by the wave on its rising, but will retreat backwards on its falling, after the crown of the wave has passed the corks. This is more readily seen when watching two corks only, at a short distance apart in lineal range with the direction of impulse of the waves. These corks when on each side of the crest of the wave will move *from* each other, and when in the hollow move *towards* each other, by the same principles of surface infraction. These movements are represented by the corks moving in the direction of the arrows shown above the crest of a wave in the diagram below, Fig. 213.

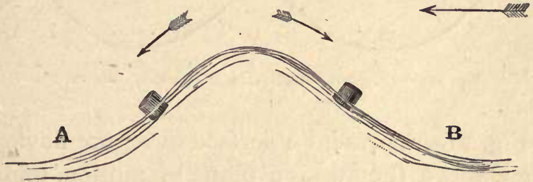


Fig. 213.—Ex.—Evidence of Infraction of Surface Water in High Waves.

Oscillatory Motions of Waves.

189. PROPOSITION: *If a hollow upon a liquid surface, surrounded by hydrostatic pressures, be more infracted by pressure on one side than on the other, the differences of such infractions or pressures will render the hollow locomotive in proportion to these differences, and the infracted water will produce a separate similar hollow or wave on the side where there is the least pressure. The velocity of waves formed in moving from the greatest pressure, or infraction, will be nearly equal to that of the swing of a free pendulum of a length from crest to crest of two such waves.*

a. By 185 prop., a hollow is propagated by infraction from a depression formed upon a surface of repose, and a series of waves are found to follow such hollow before the water can return to equilibrium. The surface plane in this case may be assumed to be an elevation of the liquid above the altitude of the hollow formed in the surface, the higher part therefore represents a hydrostatic pressure that facilitates the infraction of the water into the hollow on one side more than on the other. We can imagine the same conditions would also hold good if the water were higher on one side than on the other by any other mode of production than that given experimentally, 185 prop., as, for instance, if the water were locally elevated as a compression by the horizontal force of the wind. The action of a hydrostatic pressure thus placed, upon the infraction, will be found equal in its power of locomotion to a compressile force in moving to the most free area.

b. The infractions given above are also displacements of a part of the surface water, therefore the velocity of the wave movement must be equal to that of the fall by *infraction*, so that these infractions will regulate the distances apart of the waves, as pendular velocities of equal radius, as the following diagram may show.

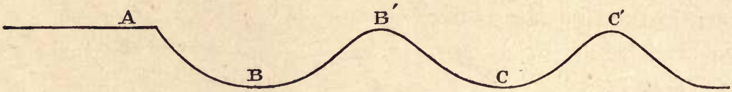


Fig. 214.—Diagram—Duplication of Infraction Waves.

c. Let A, Fig. 214, represent a surface of repose having a general potential force above the average plane of motion, under the conditions given, 185 prop. *g.* Let the lowest motive plane in the undulatory surface be represented by B and C, in the figure. Then any liquid falling from A will be directed by gravitation with pendular velocity through the arc or hollow B, by its falling momentum, and as this hollow will resemble the path of a suspended pendulum nearly, it will, like a free pendulum, by the impulse of the fall, project the momentum of the falling part from B to B′. This will occur by sequential impulse through the aqueous system; and the velocity of the impulse will be pendular, except in so far as this may be retarded by the molecular friction of the system.

d. The impulse of infraction from A arriving at B′ will be equally influenced when it arrives at this position of equilibrium to continue its movement to C, or to return to B, so that it will act about this point of equilibrium as a forward motion for one part of the liquid and a return or oscillatory motion for the other part, so that the water as it is infracted from B′ will move this surface of the hollow towards B and C by the impulse it receives through the pendular arc A B B′, and the potential of B′ will engender following waves in like manner.

e. In the conditions of the second elevation at the point C′, the elevation is shown above that given for the conchoidal wave of equilibrium, 187 prop., and therefore, by principles proposed, there would be infractions on the wave C and onwards, which would have equal oscillatory force to the infractions A to B′ per volume for the less volume that would flow from B′ to C. As these forces would oppose each other there would be some difficulty in assuring the pendular velocity of the infracted system A to B, and B′ to B simultaneously. In this case, we find that the wave is rendered locomotive, and of such period that the constancy of pendular oscillation from A to B′ impressing the side B B′ of the wave B B′ C overthrows its equilibrium and renders it projectile. The progressive velocity being ruled by the pendular infractions; at the same time we have a general reflex action of the surface water, as may be shown in the diagram below.

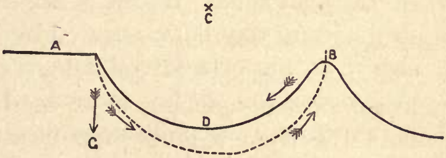


Fig. 215.—Diagram—Whirl Oscillation.

f. Let the *potential* of A, Fig. 215, by its direct gravitation impulse have a force downwards shown by the arrow G, this under resistance will produce the curve D by restraint of free gravitation and therefore resemble the action of a pendulum whose axis would be at C above the wave. This being the case, such potential force as is represented by the elevated part, having no restraint at C, as in the case of a pendulum, would have a certain gravitation momentum to enter the surface as a whirl, as shown by the dotted line to B, as before stated (188 prop. *b*). In this case it would complete so much of its impulse as may be contained in the curve, shown by the dotted line to the point B; now as B would at the same time be partially infracted at its surface towards D, the water would rise through the whirl and fall by infraction into the hollow; at the same time the entire projectile force of the whirl, derived from the potential A, would be locomotive in following the most infracted part. I think this motion may be very fairly assumed since the water to form the wave B is evidently removed from A, while the surface of the wave in this case is moving oppositely, B towards D. In such a form of motion, the parts of the hollow of the wave at D are moved in small circles by the differences of tangential velocities (46 prop. *b*).

g. Somewhat equivalent conditions of superficial oscillation to those offered in the present proposition § *c* are given by Newton in his celebrated 44*th prop.* 2*nd book of the Principia*, which he demonstrates by a bent tube partly filled with water in which the water oscillates with a pendular velocity equal to a free pendulum whose length is half that of the water in the tube. The tube experiment of Newton may be assumed to take the place of the hollow in the present proposition. Newton proposes, by inference of this experiment, that a protuberance or wave upon a liquid surface will *sink beneath* the surface of equilibrium, and rise again above it alternately upon the *same area* similarly to the oscillation in the bent tube. If this be so, it appears to me to be in no way demonstrated by his experiment which is that of a lateral movement of the kind I propose for *surface* motion. I think that as so much of wave philosophy has been built upon this proposition it may be as well to discuss the matter somewhat more in detail; for although I feel in this particular case the greatest admiration for the ingenuity of our great philosopher, this matter appears to me to be worked out without his usual care, and I cannot arrive at the same conclusion from the experiment that he does.

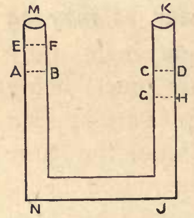


Fig. 216.—Newton's Tube Experiment.

h. With this bent tube Newton demonstrates that "water will ascend and descend in either arm of the tube in the same time that a pendulum of the same length will oscillate. That the force with which the motion of the water is accelerated and retarded alternately is the excess of the weight of the water in one leg above the weight in the other. The force with which a body moving in a cycloid is accelerated or retarded, being to its whole weight as its distance from the lowest place, to the length of the cycloid. The motive forces of the water and the pendulum describing equal spaces are as the weights to be moved. Therefore if the water and the pendulum are taken in a quiescent state, applied forces will move them in equal times, and will cause them to go and return together with a reciprocal motion." From this conclusion it is demonstrated that the reciprocatory motion of the water ascending and descending are all performed in equal times, whether the motion be more or less intense.[1] This principle of pendular motion carried out showed that the surface wave of the ocean would have a velocity in the sub-duplicate ratio to its breadth. Therefore that a pendulum whose length between the point of suspension and centre of oscillation is equal to the breadth of the wave, that is, of the wave from crest to crest, would perform a single oscillation in the same time that a wave would advance a space equal to its breadth. Newton notes that this would be true on the supposition that the parts of the water ascend and descend in direct lines. But really the action of ascent and descent is performed in a circle. Therefore the time proposed only nearly approaches the truth.

i. In this tube experiment there is no doubt the oscillation should be strictly pendular, the principles of the pendulum being fully involved in it; the resistance from friction and other effects being *equal* in each arm of the tube. If the same principles are applied to motions upon a hollow upon the surface of a liquid the same equality of conditions also occur; the action of gravitation being equal and the resistance equal for each oscillation on each side of the arc which forms the hollow.

j. But, if we apply the above to the direct vertical descent of the crest of a wave into resistant water beneath, the conditions are evidently different, as in this case, the raised mass which forms the wave is directly active as a hydrostatic pressure upon the body of water beneath, which is practically *incompressible* in the *vertical* direction of the pressure, whereas laterally it is nearly free to move on each side from the crest of the wave. Therefore by the nature of a hydrostatic pressure it would move the elevated water not directly *downwards*, but to the *free areas of the lateral parts*, so that the highest parts would flow into the hollow by deflection from the rigid resistance beneath; exactly as it flows in Newton's tube experiment under like deflection by the directive curvature of the tube. It appears to me we are bound to conclude that in this case by the conditions of the resistance present. For a free oscillation to move, as suggested, vertically into the surface of an incompressible liquid beneath, as Newton's theory demands, we should need a *vacant space* for it to move into, so that it might oscillate somewhat in the manner of a scale-beam in the air. Otherwise the small impulse of the falling water would move clearly only to the area

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[1] *Principia*, Book ii. Sect. viii. Props. 45, 46.

of smallest resistance, which would be into the free open lateral hollows, as I propose. Further, that it does move in this manner and not vertically is clear to observation.

k. I think it probable that the infraction of surface hollows may by local compressions and other causes, meet such interference as not to give exactly pendular velocities to the wave, but in so far as Newton's tube experiment may be applied to surface motions the velocity will be exact within the limits of frictional restraint. Nevertheless as we are assured by many observers that the wave spaces are as the pendular velocities, I propose that a *pendular infraction* of the hollow is the ruling cause thereof.

l. There is further one point to be observed that may appear a paradox, namely, that by *compression* upon a surface, waves are driven forward from the compression, whereas by this proposition waves move towards the infraction; that is, when the infraction is into space below the general plane of surface equilibrium; so that a rising tide which is equal to a *compression* sends waves *forward* to a shore, and a falling tide which is equal to an infraction sends waves also forward in the same direction towards the shore, and there is no difference in this case in the form or direction of the wave, whether for compression or infraction. This principle is fully demonstrated by 171 prop., and by this proposition; another instance of the same kind is also given by pricking a liquid surface with a needle (15 art. page 44), wherein the wave moves in one direction whether the needle enters the water producing a *compression* or is withdrawn leaving a hollow for *infraction.*

190. PROPOSITION: *That if a hollow be temporarily produced upon a liquid surface, and the parts of such hollow possess oscillatory motions, these motions will tend to produce exactly similar motions upon the contiguous superficial parts, by adhesional communication. So that an oscillatory motion induced in one part of a liquid surface will communicate like oscillation to other parts, so far as the intensity of the original force overcomes the inertia of the liquid surface moved.*

a. The above is proposed for oscillations in equilibrium of equal pressure upon each side of a liquid surface hollow; and may conveniently be considered apart from any locomotion that may be derived from compression or infraction. It will be necessary in this case to suppose the power of oscillation of a single hollow or wave, which produces other like oscillations, to be *maintained by some* *exterior force acting vertically upon the system.* In this case such means, for instance, will answer our purpose as that of moving the finger up and down within the surface of a liquid.

b. This principle of motion on a liquid surface may be shown as follows:—

Fig. 217.—Diagram—Equal Space Oscillation.

Let an oscillatory motion be engendered and constantly maintained in a parallel channel at A, in the above Fig. 217, by constantly impressing a force to compel this point to move upwards and downwards; and let this force occupy no area that would cause a displacement of the liquid, but be as an alternating hydrostatic pressure only, as for instance, if we can imagine such, by an alternate inequality of the action of gravitation on this small area. Then the downward impression of such force would engender a biwhirl under the motive point A by every downward impression, whose whirls would be projected right and left in the parallel channel until the projection was restrained by the liquid surface at the crest of the next point formed by biwhirl action at B and C on the surface produced by deflection of the whirl. Let this complete biwhirl be shown by the dotted lines extending from A to B and A to C, and suppose the part of the dotted line, shown in the aerial space above, to be retained by the cohesion of the water; then, such whirls moving upwards to complete an oscillation would adhere to the adjacent liquid on the surface from C towards E and B towards D upon which they would make rolling contact. Therefore other like opposite whirls would be induced to rise upon the surfaces C E and B D, as shown by the dotted lines and these whirls would induce others, so that the oscillatory system of a liquid, *engendered and maintained* by the means proposed, would resemble a series of equal surface *rockers*, one rocker moving the other by contact upon it.

c. We must only take from the cases given a purely theoretical idea of a whirl-motion, as it is quite clear water could not curl as proposed in air, but the principle of motion will be established if the above be consistent with the form observable, as this would be modified by gravitation acting upon it. Under the conditions assumed opposing whirls would fall as soon as their infraction figure permitted them to do so, and the whirls themselves, from the adhesion of the liquid, would project the liquid upwards to a ridge only. The modification may be shown by the diagram below, Fig. 218, where the completion of the whirl is shown theoretically by the dotted lines and the wave is actually represented beneath; the vertex of the wave being shown projected between the assumed whirls.

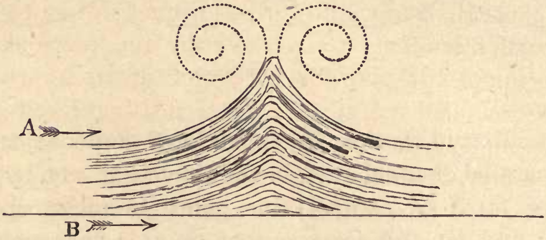

Fig. 218.—Diagram —Imperfect Biwhirl Action.

191. Current waves by underflow.

As a general principle undercurrents in liquids will produce only that small class of waves which may be termed ripples. On the same principle undercurrents would no doubt produce larger waves if the fluid systems were free from continuity of surface resistance and the forces equally intense in relation to density, as in certain conditions of aerial motions; but such intense motions do not occur frequently by natural causes in water where an opportunity is given for observation. On the other hand undercurrent winds from the free elasticity of the aerial system and its small density may, and probably do, produce waves of large dimensions upon the super-aerial surface.

192. PROPOSITION: *That an undercurrent in water, which is not a part of a density system, will engender whirls upon the upper surface. These whirls will form hollow waves that will have surface motions acting inversely to the direction of the current, and remaining at fixed positions on the surface if the resistance be fixed, or mobile if the resistance be mobile.*

a. The motive principles of whirls formed by undercurrents are the same as those previously given for other cases in many propositions in which the flowing forces make rolling contact upon liquid planes of greater resistance. Where a flowing undercurrent is resisted by any means, as by a less velocity or difference of direction in the surface water, the continuity of the carrying power of the undercurrent will be resisted, and the flowing water deflected from its direct course by the cohesion to the more resistant surface parts. The momentum of the flowing water by the continuity of its force will therefore deflect the surface upwards, forming whirls which give the projected parts a tendency to overflow the undercurrent.

b. The principles of the above become apparent when there is perfect resistance at the surface by a floating fixed object in a running stream. Thus, I have noticed at London Bridge and elsewhere, that a boat at anchor will quite turn the current ripples of the ebbing tide so that they will break against the flowing direction of the river and overflow in consecutive waves for many yards before the stream reaches the boat. The diagram below, Fig. 219, represents the direction of the motion somewhat exaggerated, each wave being shown curling upward from the boat.

Fig. 219.—Ex.—Fixed Surface Overflow.

c. Similar waves may be observed at the back of any paddle steam-boat; the paddle giving a downward direction to the water projects it backwards, whereas the surface water follows by traction. This is shown below, Fig. 220.

Fig. 220.—Ex.—Motive Surface Overflow.

d. In both cases the surface waves following or keeping at equal distances from the object of resistance are stationary when the resistance is fixed, or motile when it is motile. The same principle is shown by the resistance to static surface liquid in the experiment of Bossut given, 94 prop. *f.*

CHAPTER XVII.

PROPAGATION OF WAVES OVER LIQUID SURFACE FROM CENTRAL DISTURBANCE.

193. Equivalency of circular and parallel wave systems.

It is presumable that waves generated by central disturbances upon an extensive aqueous surface will be motive upon the same principles as we find them in parallel areas, the general conditions of which I have discussed in the two preceding chapters. However in the propagation of waves from central positions the resistances are materially different. In parallel areas the impulses are resisted by the lateral parts of the containing vessel, whereas in circular dispersions the resistances are circumferential, and vary proportionally to the squares of the radial distance from central disturbance, according to the common law of radial forces.

194. Conditions of the dispersion of small superficial circumferential waves.

a. For certain limited conditions of central dispersion of waves in liquids, for such cases as I am able to follow, which will be for small waves that affect the aerial surface only; I may not exactly assume that masses at greater depth will be affected precisely in the same manner; but I feel that there is every probability that all masses of liquids, which may be assumed to extend in equal gravitation strata, will be *similarly* affected. This we may find in that, although the aerial surface of a liquid may be in a state either of relative *distension*, as I have proposed 16 prop., or of *tension* as proposed by others, it will nevertheless be in all cases a surface in equilibrium by uniformity of horizontal pressures about any particle, and in this so far resemble the condition of a horizontal stratum of a liquid at any possible depth, to which a surface force which produces a wave could penetrate; so that we might expect similar effects in deep as in superficial movements.

b. In this matter the probability is that, assuming the surface to possess, as I have proposed, a certain special force of elastic extensibility, this will act so as to render it more sensitive under the influence of any distending force that throws it in one place out of level, than at a greater depth, where the equilibrium may be less sensitive, the disposition to extensibility reinforcing, as it were, a continuity of the compression. It is nevertheless clear that there is considerable elastic rigidity in an aqueous system at all depths, as evidenced in the persistency of compressile waves (172 prop.).

c. The fact of the facility of propagation upon a liquid surface of very small forces was shown, 15 art., in the pricking a liquid surface with a needle, by which a wave was shown to be propagated to the extent of the borders of the containing vessel. We may find that an equivalent motive propagation to this will be produced by forces that are greater. Thus, if instead of using a needle we intrude a larger object as that of a cone of a quarter of an inch diameter of base, this will in like manner disperse a motion from the centre, as we should conclude, but in this case the disturbed surface will be pressed more against the resistance of the elastic surface extensibility (16 prop.), and the surface will be crumpled up into not less than two protuberances and an intervening hollow. When the crumpled surface, or wavy form, is once set by the impulse, whether it produce one or more waves this form will extend to such an area of inertia that the resistance becomes in equation with the original impulse; leaving in all cases a plane of perfect repose after the wave or waves have passed over the surface. This repose is so perfect that I have found central dispersion of waves the best means of bringing a disturbed surface to rest for continuing other experiments.

195. Production of surface waves by falling drops.

a. To follow the effects of dispersion of small forces in liquids by visible waves, to which I limit my experiments, the best method that I was able to devise was by dropping water in separate drops upon a still surface. I found that by this means I could observe the effects of separate units of compression and infraction, in a manner quite within control. The waves produced in this manner extending in gentle undulation from the centre, I could also readily observe uniform effects by permitting the drops to fall at equal intervals, so that one wave followed closely after another. Indeed this effect was very approximately produced by carefully observing the motion of a single drop in the manner I will describe.

b. In my experiments with drops falling upon a smooth liquid surface, after having made a few trials by dropping single drops of water, first in a circular dish, and then in a large tub, I found the extension of the single drop-waves could not be followed for careful observation in a vessel of less diameter than 12 feet, which I was therefore compelled to construct. Having this vessel prepared I erected over the centre of it a simple apparatus constructed of a few pieces of wood screwed together, to which I fixed by means of a cord and pulley a glass dropping-tube at any required height, from direct contact to 3 feet from the surface of the water. Upon this apparatus I placed an adjustable shelf to support a tin pot, which was made to supply the dropping-tube by means of a small india-rubber syphon, as required.

c. The lower end of the dropping-tube was now fixed at a distance of five inches above the surface of the water over the 12-feet vessel, and the height of the tin pot was so adjusted as to permit one drop to fall at about every minute of time. The drops at the point of leaving the tube were about one-sixth of an inch diameter, as nearly as I could measure them.

d. I will now endeavour to describe as exactly as possible the action of the falling drop, to demonstrate the impulses which produce in this case surface waves.

e. A drop as it fell upon the surface of the water appeared to be flattened out by the percussion, and the surfaces of contact of the drop and plain were at this point possibly united. Apparently the drop united by a surface of about one-third of its circumference, as nearly as I could observe in the short interval of time. By the moment of projection of the drop after impact it appeared for the first very small fraction of time to produce a small hollow or shallow cup around it, which was surrounded by a ridge thrown above the level of the water by the percussion. The hollow appearing to be only equal to the amount of water splashed out to form the ridge, which was of irregular form; the outline being derived possibly from the crowding up of the liquid surface. The single drop in this case appeared not to have force to enter the water for any greater distance than the hollow was splashed out, except for a small element which will be hereafter considered. The evidence of the projection of the splash was readily seen by letting the drop fall near any small floating solids, as pieces of cork, which were instantly projected therefrom.

f. The first contact of the drop, as nearly as my powers of observation would permit, was as represented below, Fig. 221, which is enlarged about $1\frac{1}{2}$ diameters, the liquid surface being shown in section.

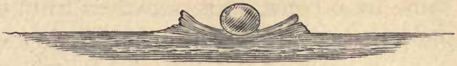

Fig. 221.—Ex.—Contact of a Drop upon a Surface of Water. (Enlarged $1\frac{1}{2}$ diameters.)

g. After the percussion of the drop upon the water surface there immediately followed a recoil, in which the drop of water rebounded, carrying with it a certain quantity of the surface water which adhered to it, forming thereby a column, in height about three times the diameter of the drop. The water forming the column appears to be taken entirely from the hollow; the ridge at first formed by the percussion meanwhile continuing its movement outwards from the centre of the percussion area. This column appears when it is reflected to about two-thirds of its full height, as shown in Fig. 222, below.

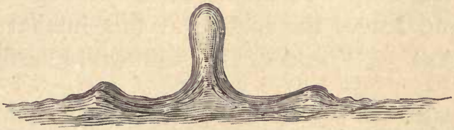

Fig. 222.—Ex.—Recoil of Drop after Contact upon Surface of Water. (Enlarged $1\frac{1}{2}$ diameters.)

h. As the drop rises the remaining third above the height shown enlarged above, the cohesion of the water can no longer maintain its continuous form; at this point, therefore, a sudden contraction

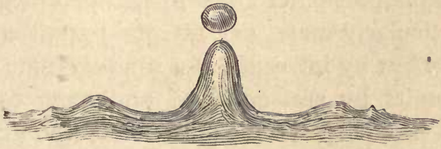

Fig. 223.—Ex.—Ejection of Drop upon Recoil from Surface of Water. (Enlarged $1\frac{1}{2}$ diameters.)

occurs at a distance of about one-third from the top of the column, and the head separates in the form of a separate drop, which is immediately ejected a short distance above the parted column. The lower part reforms beneath to the like figure the water had in rising. This is shown in the sketch, last page, Fig. 223.

i. During the whole of the short time occupied in the operation described, the hollow upon the surface of the water around the drop continues to enlarge in circumference, and the outer ridge or wave to become flattened out to conchoidal form; the first ridge moving at the same time constantly outwards from the centre.

j. The following instant the column sinks down, and immediately forms a ridge, as that of the first percussion of the drop, but this appears to be of greater volume, and takes at once a conchoidal form, containing the entire water of the column shown in the last engraving, Fig. 223. The appearance, as nearly as I could observe it in so short a period, by taking observation of fifty consecutive drops, was at this time nearly as in the sketch above, Fig. 224.

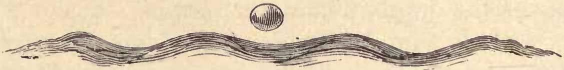

Fig. 224.—Ex.—Surface at the time of Fall of Reflex Drop. (Enlarged 1½ diameters.)

k. The ejected drop now finally falls upon the water with very slight recoil, and leaves the surface at this instant with three concentric waves A A′, B B′, and C′ C″, as shown in section in the figure (Fig. 225), of which the vertical scale is purposely exaggerated to make the diagram appear more clear.

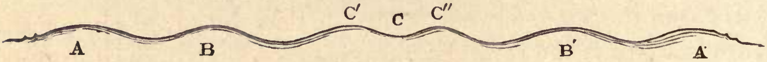

Fig. 225.—Ex.—Section of Wavy System produced by the Fall of a Drop on Water.

l. In these simple phenomena of propagation of waves, the motive causes that particularly merit our attention are, that we have here a series of *projections* by intrusion of a small volume of water which separates into units, by surface recoil and repercussion, so that a wavy surface is produced—the conditions of the genesis of which, we are able to define the cause. The drop in the first instance has no considerable projectile force derived from the short distance of 5 inches that it falls, to the surface of the water, to enable it to enter the surface or sink into this nearly incompressible fluid, except for a small part, as I will hereafter show; its force is therefore principally developed, or continued outwardly by deflections, to a horizontal direction from the point of contact, so that we find by observation that each wave, *in this case*, is derived from a single effort of percussion, or compression upon the central liquid area.

m. We have therefore for the three principal protuberant waves shown in section in the preceding diagram, Fig. 225:—

A A′ a percussionary wave formed by the water ejected from the first percussion, at the outer edge of which there is found a series of ripplets formed by an incipient splash, which produces altogether an imperfect circumscribing wave undulatory towards its vertex, but ragged in front.

B B′, secondary reflected compressile wave, containing a hollow in front drawn up in the recoil from adhesion of the surrounding water, as shown Fig. 222, this wave from falling by natural gravitation into the surface beneath, produces a perfect wave of a low conchoidal type (187 prop.).

C′ C″, third final projectile wave, caused by the detached drop, illustrated in Fig. 223; this drop follows closely the descending nipple which forms the previous wave B B′. It is produced with much less force, and is followed by the same secondary phenomenon as the first percussion from the drop falling 5 inches, § *c*; the wave formed is therefore imperfect, being followed by a series of very small undulations, which in miniature repeat the entire phenomenon.

n. From the above we may conclude that the entire phenomenon here discussed is the result of impression of force derived from percussion and surface reflection of two material bodies, the large mass of water and the drop. In this case the wavy surface is not a matter of chance, as it were, or of oscillation, but each wave is an individual protuberance derived from a single motive cause; and we may observe also that it has a definite individual action, capable of continuity, but incapable of engendering other forms of surface-motion, or of reproduction of its own form.

o. I may here mention that there may be a cause of failure in repeating this simple experiment. Thus if the drop falling the given distance of 5 inches be too large, it will sometimes eject two or three drops from the top, that will confuse the whole phenomenon. I have found another method of dropping very satisfactory, which is to point a piece of glass and dip it in water for a given distance, sufficient water will adhere for a perfect drop to be formed. This after a few trials will exactly answer in its motive effects the description here given.

p. After the series of waves have been formed by our drop, as shown Fig. 225, each by its distinct generic cause, these waves ride on smoothly outward from the centre, whence the motive forces are derived. And here it is most important to note the perfect smoothness of the water left after the waves have passed, that is clearly observable when the final wave about C has passed only 3 inches from the centre; which shows the perfect solidity of the water and freedom from any oscillatory motion from reaction. For it must be remembered that it is not the water which is moving outwards with the wave, but the impressed force acting upon the surface-water. It can be clearly observed that the water remains relatively stationary to the wave progression, and only moves upwards once and returns to its *original position.* Of the pendular motion of a balanced oscillation there appears to be no trace in this simple phenomenon.

q. By the general appearances we may imagine that the impulse bends up, as it were, the water into a superficial loop by its compression, and that the water falls to its surface by gravitation, the lift exhausting the original force, less the potential energy in the fall of the wave which presses it forward, as before demonstrated for compressile motive waves, 171 prop., the total loss thereby being principally caused by the necessary frictional mode of motion of a system under restraint of lower static water.

196. Separation of near circular waves by infractions in an open system.

a. The characters of the small waves formed by a drop may be defined by observation of their distribution; in which, from these waves being under restraint of lower inactive water, the same principle of infraction by overflowing of the elevated tops occurs as in the larger superficial oceanic waves already discussed, 187 prop. This is clearly demonstrated by the continued increase of distance apart of the waves as they leave the centre. Thus the three principal wave tops A B and C′ or A′ B′ and C″, Fig. 225, at the subsidence of the last reflected drop will be included in the space of about 1½ inches from the centre, as ascertained by the average of 20 measurements. At about one foot distance from the centre the same three waves will occupy about 4 inches; at two feet distance about 7 inches, and at three feet about 11 inches. The overflow from the top of each wave causes the retardation of the next following, and the acceleration of the wave in front. I may note that upon the principle of the pendular theory, waves should remain at equal distances apart according to periods of oscillation. They do not do so in the case of these small waves, and my experience tells me that waves do not in any case that I have been able to carefully observe, except they be maintained by undercurrents, wind, or other force, and are restricted in area of propagation by resistance; but that in all cases, where free from restraint, they spread out and diffuse themselves over the water surface, increasing in amplitude ? as these minute waves do, the water running down by natural gravitation to fill the hollows, on principles of infraction already discussed.

b. The velocity of the wave has nothing to do with the distance of hollows and tops of these small waves, and there is no oscillatory period. The wave-force appears to proceed with a velocity in constant equation with the particular constitution and elasticity of the fluid affected. In my experiments with a falling drop upon a surface of water 6 inches deep, the central wave B, Fig. 225, travelled with a velocity somewhat under 9 inches per second, that is, from 8 to 9 inches, the first wave being quicker, the second slower. The method of observation followed was to have a marked rod supported lengthwise at a short distance above the water surface; the surface of the under side of the rod was white and the figures black, so that they could be seen reflected on the water surface; if the sunshine fell upon the rod a shadow was produced in the water, and up the edge of the shadow the ripple or wave could be clearly traced as it passed along the deflected reflection.

c. By careful observations on a still day, taking the mean of 20 observations of the wave B, Fig. 225, its velocity of propagation was 8·4 inches per second for a distance of 5 feet from the centre; at this distance it could still be distinctly seen. This wave B was the best defined of the series, A and C were nevertheless quite * distinct at this distance. The mean velocity of A was about 9·1 inches per second; C about 7·9 inches. The extreme ripples in front of A about 9·3; the last ripple following C about 7·8 inches per second. The above dimensions are approximate; the vessel

* These waves seem to have been not strictly waves of gravity but capill

was not perfectly shaded, and any slight wind or draught made a small difference in the velocity of propagation.

197. Analysis of the projectile action of a drop falling upon a surface by whirls engendered within the lower water.

a. The effects of projection of drops *within* water have been discussed in 66 prop. By the conditions just proposed the larger element of the entire projectile force in the falling drop is active to develope surface waves. The evidence of percussion and recoil and repercussion, by which the separate waves are produced in the small experiments just discussed, should, however, be made evident by the separate projections of whirl-rings from each impulse. This collateral evidence of the separate geneses of waves from a falling drop I thought necessary to investigate, as the means were at hand without new apparatus. I therefore changed the water in the dropping-tube before described, and filled it with water coloured with a solution of carmine. In this experiment the first drop engendered a whirl-ring, but the reflex action that might be observed so readily in surface motions, was not of sufficient energy for the reflected nipple and final drop when falling upon the surface to engender separate distinct rings, without confusion from nearness. I therefore raised the dropping-tube and the supplying vessel 30 inches above the surface of water in the trough, instead of 5 inches as before, to increase the force of the fall. It was not desirable to increase the size of the drop, as its globular cohesive form would suffer thereby. The drop falling from greater elevation produced more splash and consequently more compression of the surface, and changed the exact conditions of the wave effects previously considered; but as the forms of the waves were not the subject now to be observed, this was of no consequence.

b. Having the above arrangement complete, with the dropping-tube raised 30 inches, immediately after the drop of coloured water had fallen on the surface of the water in the vessel, there appeared below the place a clear whirl-ring, at its first observable position, of about a quarter of an inch diameter. This moved slowly towards the bottom of the tank, expanding in its course as in the previous experiments given, 65 prop. *a.* When the ring had entered the water about half an inch, a second ring was formed by the descent of the reflected nipple shown, Fig. 222, which formed the wave B of experiments, Fig. 225. From the projection being greater in this case than in the previous experiment, the nipple rises in a columnar form to about one and a half inches as shown, Fig. 226; the second ring being formed by the descent of this nipple. From the time of continuous projection of this long descending nipple it does not produce within the water a perfect ring, but a whirl-ring united to the following less percussionary matter, so that it appears as a very fine cone of colour terminating at the apex as a fine thread; the whole cone being distorted by motions already engendered in the water upon which it is projected. After the second ring a third follows from the projected reflected drop which produces the wave C′ C″, Fig. 225, of previous experiments. In this case the drop falling on a disturbed surface is much more distorted, and sometimes so confused as not to be distinguishable as a whirl-ring. But in all cases there is a third detached system of colouring matter projected, so that upon the whole, this experiment gives evidence of the three projectile impressions producing the waves previously discussed, which is my present purpose to assure. The appearance within the water is represented in the diagram below, Fig. 227.

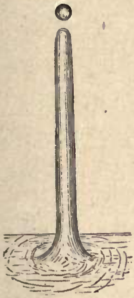

Fig. 226.—Ex.—Nipple produced by Drop Falling 30 inches.

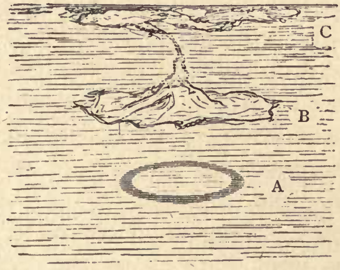

Fig. 227.—Ex.—Effects of Percussion and Repercussion in producing Whirl-rings.

A, Effect of first percussion, producing a whirl-ring.

B, Effect of descent of reflected column, after first percussion of the drop, producing an imperfect ring united by a cone.

C, Incipient ring from final drop, generally of irregular and broken form.

198. Production of larger waves by central forces.

a. The above is the limit of my experiments of central forces, except some desultory experiments which were followed with insufficient care to obtain reliable results. For larger waves we have some experiments and principles offered by M. Flaugergues, which I think should be discussed here to fill up the defect; for, although his experiments are somewhat rough, his arguments, founded upon the appearances produced, agree very well in some particulars with inferences that might be drawn from experiments that I have followed for the propagation of direct wave motions. The principles that M. Flaugergues adduces from these experiments are, I think, important for consideration of the subject, I therefore offer them somewhat in detail in place of experiments I intended to follow, if convenient opportunity had permitted.

b. M. Flaugergues commences his operations upon a smooth surface in a tank 12 feet square and 3 feet deep filled with clear water and placed in the shade protected from all winds. He then endeavours to follow the motion of each separate wave engendered by a single cause following no preconceived idea of an oscillatory or any other form of motion. In his observations he follows the motion of a single wave, or small group of waves, which he produces by striking the surface of the water with a cylinder of copper fixed upon the end of a stick. The copper cylinder is $\frac{3}{4}$ inch in diameter, and with the blow he causes it to sink about a full $\frac{1}{4}$ inch in the water. He notes the motion of the first single wave as it swims across the surface of the water, and is able to detect no oscillation of the surface, as this wave appears to be a simple protuberance passing over it. His opinion of the cause of this is as follows, for which I translate his words.[1]

c. "We may conclude from these experiments that a wave is not the effect of a movement in the particles of water by which these particles go up and down alternately in following a serpentine line, and spreading thus from the spot upon the surface where the shock was made. But that it is an intumescence which the shock produces around the place of the depression by which it is caused, and which intumescence propagates itself afterwards circularly in receding from the centre of pressure, by means of the portion of water

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[1] *Journal des Sçavans*, 1789, p. 682.

that is raised above the level of the stagnant water. And as a part of this water flows from every other part of the surface into the hollow formed at the place of the shock, this hollow becomes more than filled, until the water rises in such a manner as to produce around it an intumescence or new wave which is propagated circularly as the first. This effect being repeated in this manner several times, the surface of the water becomes divided into a great number of concentric circles raised and lowered in succession; which has given the idea of an oscillatory movement such as has been held heretofore."

d. M. Flaugergues here identifies the individuality of the wave, and it appears somewhat curious that having done so he should not have refined his mode of production. The stick which strikes the water is withdrawn and the hollow left, the whole motions from this rough experiment being accounted for in a concrete sentence. M. Flaugergues further developes his theory, but goes little further in the direction of experimental demonstration. His final conclusion, derived from arguments and mathematical demonstrations, is that the velocity of the wave is in all cases the same, whether large or small, and is derived from the molecular mobility of the water through which it passes, and not from any principle of oscillatory motion, nor in oscillatory periods. He throws stones, large and small, into the still clear water of a river, and finds no difference in the velocity of the wave produced, although there is great difference in its dimensions, and concludes by his experiments, that the single wave of any altitude travels outwards at the velocity of 30 French feet in 21 seconds. In going roughly over M. Flaugergues' experiment upon a pond of about 50 feet diameter, of depth varying from 5 to 2 feet, I have found the velocity of the wave approximate to M. Flaugergues' measurement, and what is important, of nearly equal velocity for all sizes of stone that I dropped to generate the wave, a little greater velocity only being observed from larger masses after deducting the mass radius.

e. Mr. J. Scott Russell, to whose work I have often referred, has given 8½ inches per second as the velocity of small surface waves that he terms capillary waves. M. Flaugergues gives 18 inches nearly, but the circumstances of production are very different. In all cases I have found that the separate waves of a group have a varying velocity by their constant separation (187 prop.), therefore no fixed velocity can be found, although the mobility of a liquid surface will cause the velocity to be approximately equal for very small waves.

f. There appears to me in the above experiments of M. Flaugergues a strong analogy to the principles shown in my drop experiments given in this chapter, in the reflex action observable in surface motions. But in the striking a surface with a heavy body, or in the projection of stones, we have clearly complicated phenomena derived from the *splash* which would be difficult to follow, to obtain exact results.

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